PHYSNET

MISN-0-382

WAVE NATURE OF PARTICLES

Quantum Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

WAVE NATURE OF PARTICLES
by
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Title: **Wave Nature of Particles**

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Input Skills:

1. Vocabulary: complex function (MISN-0-59), particle, momentum, energy (MISN-0-15).
2. Describe the diffraction pattern produced by collimated light incident on a single slit (MISN-0-235).
3. Describe the particle-like properties of electromagnetic radiation (MISN-0-381).

Output Skills (Knowledge):

K1. Vocabulary: wave function, probability density, normalized wave function, principle of superposition.

K2. Given the momentum and mass of a particle, write equations for its wavelength and frequency.

K3. Explain how Davisson and Germer found evidence for wave-like properties of electrons.

K4. Give a qualitative summary of the results of the particle "Young's double slit experiment," emphasizing why the results would not be expected classically.

K5. Show, for a non-relativistic particle, that the group velocity of a free particle wave is the same as the classical particle velocity.

K6. Using the principle of superposition, write down an expression for the observed particle density in the double slit experiment.

Output Skills (Problem Solving):

S1. Normalize and interpret simple wave functions.

External Resources (Required):

1. E. E. Anderson, *Modern Physics and Quantum Mechanics*, W. B. Saunders (1971).

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1. Introduction

In MISN-0-381 we studied the particle-like properties of a known wave phenomenon: electromagnetic radiation. We now take up the even more surprising wave-like properties of particles.

The idea that particles might have wave-like properties was suggested by de Broglie in 1925 in an attempt to explain the values of the Bohr orbits. He proposed, in analogy with the photon case, that a particle of momentum p and total energy E be represented by a wave of wavelength $\lambda = h/p$ and frequency $\nu = E/h$.

De Broglie's ideas were confirmed in a classic series of experiments by Davisson and Germer in which they studied the scattering of low energy electrons from crystals. They observed interference patterns which proved beyond doubt that the electrons were acting like x-rays in x-ray diffraction, i. e. exhibiting wave-like behavior.

In analogy with electromagnetic waves, we represent particle waves or "matter waves" by a function $\Psi(x,t)$, called the "wave function." In the electromagnetic case, the "wave function" was the electric field vector and its square gave the light intensity at each point in space and time. The function Ψ unlike the electric field, is not directly observable and is, in general, complex. The absolute square of $\Psi(x,t)$, $|\Psi(x,t)|^2$, is interpreted as a probability density, i. e. $|\Psi(x,t)|^2 dx$ is the probability of finding the particle between x and $x + dx$ at time t.

$\Psi(x,t)$ satisfies the principle of superposition in the same wave as other wave phenomena; and this property gives rise to fascinating matter-wave interference effects.

2. Procedures (Keyed to Output Skills)

1. Read section 4.4, omitting the remarks-on relativistic particles. Relativistic quantum mechanics is a very complicated subject replete with differences from the non-relativistic case. Here we are only concerned with *non-relativistic* quantum mechanics. Memorize equations (4.14)

and the definitions below them (in particular, $\hbar \equiv h/2\pi$ turns out to be more useful than h. Planck should have divided his constant by 2π!). What is the velocity of an electron whose de Broglie wavelength is 3000 Å?

2. The group velocity[1] is given by $d\omega/dk$. Recall from optics that this velocity represents the rate at which energy is transmitted. Using $E = p^2/2m$ and equations 4.14 to express ω in terms of k, show that

$$\frac{d\omega}{dk} = \frac{p}{m} = v$$

3. Read section 4.5. Review optical interference phenomena (from old notes or books) to be certain that you understand why the intesity at a given angle in Bragg reflection should depend critically on the wavelength of the incident radiation. You need not study any of the mathematics here but make sure you see why the Davisson - Germer experiments were incontrovertible evidence of wave behavior.

4. Read the first paragraph of section 4.6 and study figures 4.9 and 4.10. Look back at the optical Young's double slit pattern and compare.

5. Read the remainder of section 4.6. Note the dx in the expression $|\Psi(x,t)|^2 dx$ for the probability of finding the electron between x and $x + dx$ at time t.

   a. The wave function for a particle with no internal degrees of freedom (e. g. no spin) is the function of the coordinates about the state of that *particle*. *Encyclopedia of Physics*, Ed. R. G. Lerner and G. L. Trigg, Addison - Wesley, Reading (1981), p. 813.

   b. The probability density is the amplitude squared of the wave- function unless the latter is not normalized (see equation 4.18). Although we shall be chiefly concerned with normalized wave functions, in some cases wave functions cannot be normalized. (See 5c).

   c. Wave functions are sometimes written as $Af(x,t)$ where A is called the normalization constant.

[1] Here *group velocity* means the velocity of the physical center or average position of a wave packet, not the velocities of the individual single-wavelength waves that make up the packet.

(1) Show that $|A|^2 = 1/\int_{-\infty}^{\infty}|f(x,t)|^2 dx$ for a normalized wave function. What freedom does this condition still leave in the choice of A? Be able to integrate $\sin^2$ and $\cos^2$ without use of tables.

(2) Consider the wave function $\Psi(x,t) = Ae^{i(kx-\omega t)}$ where k and ω are constants. (For a free particle of mass m and energy E, what are the values of k and ω?) Can A be chosen so that Ψ is normalized? The answer to this question shows that this wave function is an idealization and does not represent a real particle (which certainly has unit probability to be found *somewhere* in space). For a proper treatment, one needs a *wave packet* to make Ψ vanish outside a finite region. Such a Ψ can always be normalized.

d. Study the middle paragraph of page 129. Make sure you can reproduce the three lines dealing with $\rho(x,t)$ and explain them. The "Principle of Superposition" is defined by this statement: "If Ψ_1 and Ψ_2 are the wave functions for two possible states of the system, then $N(\alpha\Psi_1 + \beta\Psi_2)$ is also a possible state of the system, where α and β are arbitrary constants and N is a number which guarantees that the wave fuction is normalized." *Encyclopedia of Physics*, Ed. R. G. Lerner and G. L. Trigg, Addison - Wesley Publ. Co. , Reading (1981), p. 813.

6. Do problems 4.8, 4.9 and 4.10. What information would you need to get a numerical answer for problem 4.8b?

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