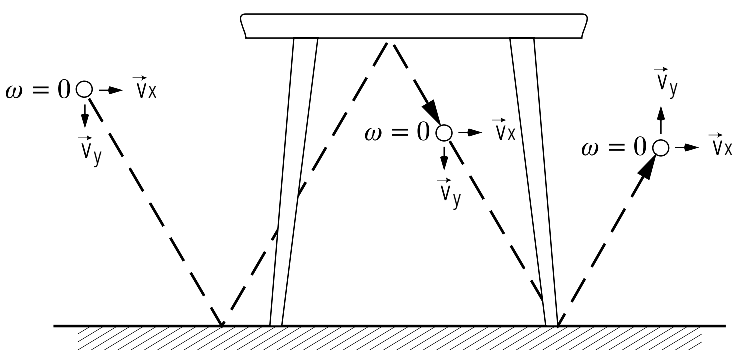
MISN-0-53

ORDINARY AND SUPER-BALL BOUNCES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ORDINARY AND SUPER-BALL BOUNCES

by

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Title: **Ordinary and Super-Ball Bounces**

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Input Skills:

1. Given impulsive forces on the surface of an object, write the equations for the changes in its center of mass momentum and angular momentum (MISN-0-36).

Output Skills (Knowledge):

K1. Derive, in complete detail, the equations relating the linear and angular velocities of a Superball before and after a collision with a flat surface.

K2. Using the equations from Skill K1, give an example that demonstrates a difference in what can happen when a Superball and a smooth ball are bounced.

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New authors, reviewers and field testers are welcome.

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1. Overview

Why does a Super-Ball[1] bounce so differently from an ordinary ball? How can one predict its trajectory, given an initial position, velocity, and spin? Here we help you answer those questions in a quite simple and satisfying manner, albeit assuming that the ordinary ball's bounce is frictionless while the Super-Ball's has lots of friction but no heat loss due to skidding.[2] In each case, then, mechanical energy is conserved. We use this and impulse relations to determine what a bounce does to a ball's linear and angular velocity components. Then we apply those velocity changes again and again, bounce after bounce, to plot trajectories.

2. Velocity Relations for a Bounce

2a. Nomenclature, Impulse Equations. A ball before and after a bounce can be described in terms of its linear velocity v and angular velocity about its center of mass. Here we will orient an x-y plane to coincide with the plane defined by the bounce so that $v_z = 0$ (see Fig. 1). We will also assume that any spin is about the z-axis. Under these circumstances the useful impulse equations, taken with respect to the ball's center of mass, are:[3]

$$M\Delta v_x = \int_{\Delta t} f_x\,dt = -\int |f_x|\,dt, \tag{1}$$

$$M\Delta v_y = \int_{\Delta t} f_y\,dt = -\int |f_y|\,dt, \tag{2}$$

$$I_z\Delta\omega = \int_{\Delta t} \tau_z\,dt = r\int |f_x|\,dt\,, \qquad \text{(upward bounce)} \tag{3}$$

[1]Registered trademark of the Wham-O Corporation, San Gabriel, CA.

[2]Possible reasons for this property of the Super-Ball are given by Garwin in "Super-Ball Bounces," R. L. Garwin, *American Journal of Physics* **37**, 88 (1969). Garwin calls this "ultraelasticity" and discusses many interesting trajectories. See also *Classical Mechanics, A Modern Perspective*, V. Barger and M. Olsson, McGraw-Hill Book Co., NY (1953).

[3]For background, see "Momentum: Conservation and Transfer" (MISN-0-15)-and "Rotational Motion of a Rigid Body" (MISN-0-36).

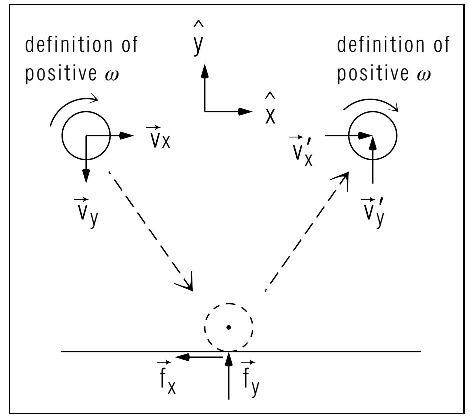


Figure 1. Symbols used to describe a bounce. Note: $\vec{v}_x \equiv v_x\hat{x}$, etc.

where M is the mass of the ball, r is its radius, I_z is its moment of inertia about the z-axis through its center, τ_z is the torque exerted on it about that axis, and it is the time interval it spends in contact with the floor. Note that a gravity impulse has not been included: its net effect is zero. For a perfectly smooth (frictionless) ball, f_x is zero. For the Super-Ball, with a high coefficient of surface friction, f_x is not zero.

2b. Conservation of Energy. We assume that neither the ordinary ball nor the Super-Ball loses mechanical energy during a collision. The collision forces are thus conservative and we can write:

$$\frac{1}{2}M(v_y{}')^2 = \frac{1}{2}Mv_y^2\,, \tag{4}$$

$$\frac{1}{2}I_z(\omega')^2 + \frac{1}{2}M(v_x{}')^2 = \frac{1}{2}I_z\omega^2 + \frac{1}{2}Mv_x^2\,. \tag{5}$$

These equations can be obtained, if desired, by integrating Eqs. (1) - (3), first noting that the net gravitational impulse is zero and that the frictional impulse can be eliminated by combining Eqs. (1) and (3). Strict conservation of mechanical energy is only an approximation to what happens in the real world, but it is sufficiently accurate for our purposes.[4]

2c. The Equations to Be Solved. In order to solve Eq. (5) we must find a second relationship between the two unknowns, $v_x{}'$ and ω'. A convenient relationship is obtained by eliminating the integral between Eqs. (1) and (3). Doing this, and substituting the value of I_z for a homogeneous

[4]See Garwin's article, cited earlier.

ball,[5] $I_z = (2/5)Mr^2$, we get the equations to be solved for an upward bounce:

$$v_x'^2 = v_y^2\,, \tag{6}$$

$$v_x'^2 - v_x^2 = (2/5)(r^2)(\omega^2 - \omega'^2)\,, \tag{7}$$

$$v_x' - v_x = (2/5)(r)(\omega - \omega')\,. \tag{8}$$

2d. Solutions to the Vertical Equation. Equation (6) has the immediate solutions $v_y' = \pm v_y$. Since the ball cannot continue through the surface, as it would have to for $v_y' = v_y$, v is required to change sign:

$$v_y' = -v_y. \tag{9}$$

2e. Solutions to the Horizontal Equations. The horizontal equation (7) is quadratic, so there will be two solutions. One simultaneous solution to (7) and (8) can be obtained by inspection:

$$v_x' = v_x\,, \tag{10}$$

$$\omega' = \omega\,. \tag{11}$$

An easy way to find the other solution is to factor each side of (7) into products of sums and differences, then to use (8) to simplify it. The resulting linear equation can then be easily solved simultaneously with the linear equation (8). The result is (for an upward bounce):

$$v_x' = (3/7)v_x + (4/7)r\omega\,, \tag{12}$$

$$\omega' = (-3/7)\omega + (10/7)v_x/r. \tag{13}$$

The horizontal angular velocity changes, (10) and (11) or (12) and (13), along with the vertical velocity change, (9), give us all possible upward-bounce trajectories consistent with conservation of energy and the impulse equations.

3. Trajectories, Successive Bounces

3a. The Ordinary Ball. The ordinary ball is assumed to have a frictionless surface, so its upward-bounce equations are (9), (10) and (11), not (9), (12) and (13). For the downward bounce under the table, you can rework Eqs. (3) and (8), and find a change of sign in each. We find it helpful to make a downward-bounce sketch similar to Fig. 1. Equations (10) and (11) are found to be unchanged. Application of the appropriate equations to successive bounces results in the trajectory shown in Fig. 2.

[5] See "Calculation of Moments of Inertia" (MISN-0-35).

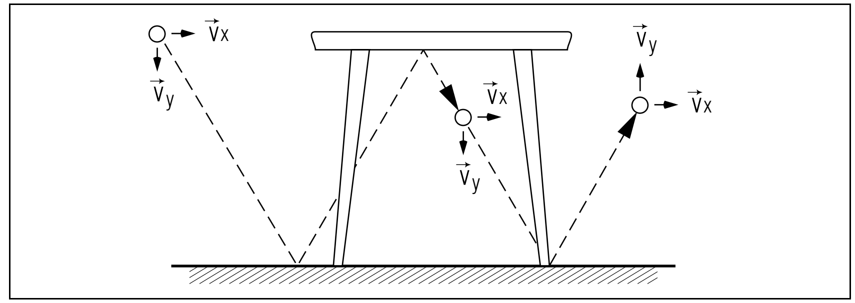


Figure 2. Successive bounces of an ordinary ball, assumed smooth. The missing second-leg situation can be filled in by use of Eqs. (9), (10) and (11).

3b. The Super-Ball Challenge. Suppose a Super-Ball is given the same first leg as in the trajectory shown in Fig. 2. What will its next three legs look like? Before turning the page and finding out, why not see if you can determine the trajectory yourself and sketch it properly. Watch out, though. Eqs. (3), (12) and (13) are only valid for an upward bounce. The second bounce is downward, so you must rework the equations for that case. This alters the right side of equation (3) and the second terms on the right side of equations (12) and (13). In calculating the v_x's and ω's after each successive bounce, we suggest that you put them in terms of the symbol v_x, which represents the x-component of velocity on the very first leg.

▷ Try it before turning the page!

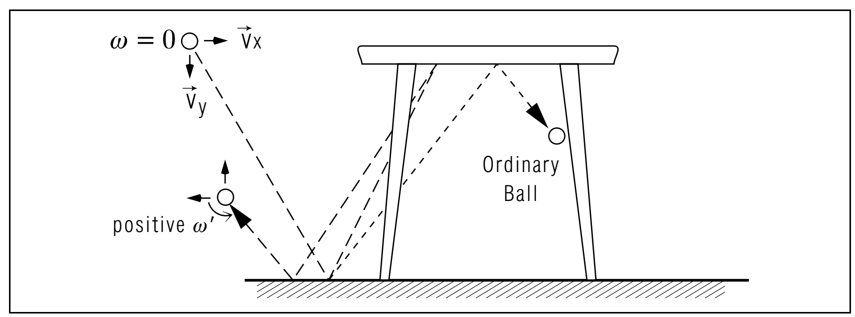


Figure 3. Successive bounces of a Super-Ball, assuming no sliding during a bounce. The rotation is as in Fig. 2.

3c. The Superball. Surprise! A Super-Ball, thrown to the floor in front of a table, comes back to you and picks up spin! The basic downward-bounce equation is just:

$$I_z \Delta\omega = \int_{\Delta t} \tau_z \, dt = -r \int_{\Delta t} |f_x| \, dt \, . \tag{14}$$

and this leads to these characteristics after the third bounce:

$$v_x''' = (-333/343) v_x; \qquad \omega''' = (-130/343)\,(v_x/r)\, .$$

Acknowledgments

I would like to thank Gwynne Stoddard and James Griffin for their very helpful reviews of this module's first and second versions. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

PROBLEM SUPPLEMENT

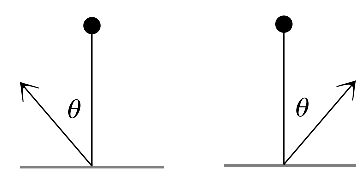
1. A Super-Ball with positive spin ω is dropped from a height h onto the floor. Determine its angle of rebound and sketch the situation. Check the angle as $\omega \to 0$ and ∞; as $h \to 0$ and ∞. What happens if the ball is again dropped, this time with opposite spin?

2. Find the relationship between ω and v_x such that a Super-Ball has a repeating single-leg trajectory between bounces. If the peak of the arcing trajectory is a height h above the floor, what is the ball's angle of rebound at each of the two bounce points? Sketch the situation. Check ω and v_x after the first and second bounce.

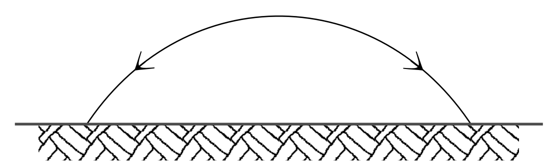
3. Other examples can be found in Garwin's article, cited in the *text*.

Brief Answers:

1. $\theta = \tan^{-1}\left[(4/7) r\omega/\sqrt{2gh}\,\right]$.



2. $\omega = -(5/2)(v_x/r)$; $\theta = \tan^{-1}\left[-(10/7) v_x/\sqrt{2gh}\,\right]$ to vertical ;
$\omega'' = -\omega' = \omega$; $v_x'' = -v_x' = v_x$.



MODEL EXAM

$$v_x{}' = \frac{3}{7}v_x + \frac{4}{7}r\omega; \qquad \omega' = -\frac{3}{7}\omega + \frac{10}{7}(v_x/r)$$

1. See Output Skill K1 in this module's *ID Sheet*.
2. See Output Skill K2 in this module's *ID Sheet*.

Brief Answers:

1. See this module's *text*.
2. See this module's *text*.