PHYSNET

MISN-0-496

DRIVEN OSCILLATIONS

Classical Mechanics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

DRIVEN OSCILLATIONS

by

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Title: **Driven Oscillations**

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Version: 2/1/2000 Evaluation: Stage B0

Length: 2 hr; 8 pages

Input Skills:

1. Solve problems involving linear oscillators with damping (MISN-0-495).

Output Skills (Knowledge):

K1. State Fourier's theorem.

K2. Define and explain the significance of the Q-value of an oscillator.

K3. Sketch the resonance curves for amplitude and phase for arbitrary Q-values of an oscillator.

Output Skills (Rule Application):

R1. Obtain the Fourier series expansion of a given periodic function.

Output Skills (Problem Solving):

S1. Set up and solve the differential equation of a mechanical oscillator subjected to a sinusoidal driving force. Identify the transient and steady state contributions to the solution and give the phase relationship between the driver and the oscillator. Determine the condition for the three types of resonance: amplitude, kinetic energy, power.

External Resources (Required):

1. J. Marion, *Classical Dynamics*, Academic Press (1988).

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1. Introduction

The previous unit reviewed the essential features of undriven oscillators. Sustained oscillations in a physical system, however, never actually occur in the absence of a driving mechanism because of ever present damping forces. Such damping forces always bring the oscillations to a halt after a finite interval of time. In order to continue the oscillations beyond the time allowed by the damping mechanism, it is necessary to replenish the oscillator's energy by a driving mechanism. The efficiency with which the driving mechanism supplies energy to the oscillator depends on the frequency of the driver as compared to the natural frequency of the oscillator. For certain driving frequencies, the efficiency can be quite high leading to a state of affairs known as a resonance. Such properties of driven oscillators will be covered in this unit.

2. Procedures

1. Read section 3.6 in Marion filling in details where necessary.

   Optional: Read section 4.3 in Wylie.

   ▷ Work these problems:

   a. Find, using the principle of superposition, the motion of an underdamped oscillator ($\beta = 1/3\omega_0$) initially at rest and subject, after $t = 0$, to a force

   $$F_d = A \sin \omega_0 t + B \sin 3\omega_0 t$$

   where ω_0 is the natural frequency of the oscillator.

   b. What ratio of B to A is required in order for the forced oscillation at frequency $3\omega_0$ to have the same amplitude as that at frequency ω_0?

   c. A weight of 64 lb hangs from a spring of modulus 36 lb/in. During the free motion of the system it is observed that the maximum displacement of the weight decreases to one-tenth of its value in

5 cycles of the motion. Find the amplitude of the steady-state motion produced by a force equal to $(6\,\text{lb}) \sin(15\,\text{sec}^{-1})t$. By what time interval does this steady-state motion lag the driving force in this case?

▷ Learn these definitions:

amplitude resonance: that frequency (ω_R) at which the amplitude is a maximum.

kinetic energy resonance: that frequency (ω_E) at which the average kinetic energy is a maximum.

power resonance: that frequency (ω_P) at which the average power supplied by the driver is a maximum.

▷ Work this problem:

Show that $\omega_R = \sqrt{\omega_0^2 - 2\beta^2}$ and $\omega_E = \omega_P = \omega_0$. Hint: Recall that the instantaneous power is the product of force and velocity.

▷ Answer this question:

How is the average power dissipated in the damper related to the average power supplied by the driver in steady state?

▷ Work these problems:

a. Show that equations 3.59 and 3.61 of Marion can be expressed in the form:

$$D = \frac{A\sqrt{1 + 2Q^2}/\omega_0^2}{\sqrt{1 + (\omega/\omega_0)^4 + 2Q^2[1 - (\omega/\omega_0)^2]^2}}$$

$$\tan\delta = \frac{2\omega/\omega_0}{[1 - (\omega/\omega_0)^2]\sqrt{2(1 + 2Q^2)}}$$

b. Study figure 3-14 of Marion in conjunction with these expressions.

c. Determine the vertical scale of the top graph in Figure 3-15 of Marion. In particular, determine the D-axis intercept, the values at $\omega = \omega_0$ and the hash-mark spacing.

Note: Kinetic energy and power resonances occur at ω_0 while amplitude resonance occurs very near ω_0 except for small Q.

Work problems 3-18 and 3-19 of Marion. The expression for Q in problem 3-18 should be:

$$Q \approx 2\pi \left[\frac{(\text{total energy})_{\text{avg}}}{(\text{energy lost per period})_{\text{avg}}} \right]_{\omega=\omega_0} .$$

2. Read section 3.9 in Marion - the Fourier Theorem is stated just before eq. (3.100) and includes Eqs. (3.100-3.102a).

   Optional - Read section 5.1 of Wylie.

   ▷ Work problems 3-32 and 3-33 in Marion.

3. Discuss the steady-state motion of the system in Fig. 1 in the absence of gravity.

Acknowledgments

The author would like to thank Illinois State University for support in the construction of this lesson. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

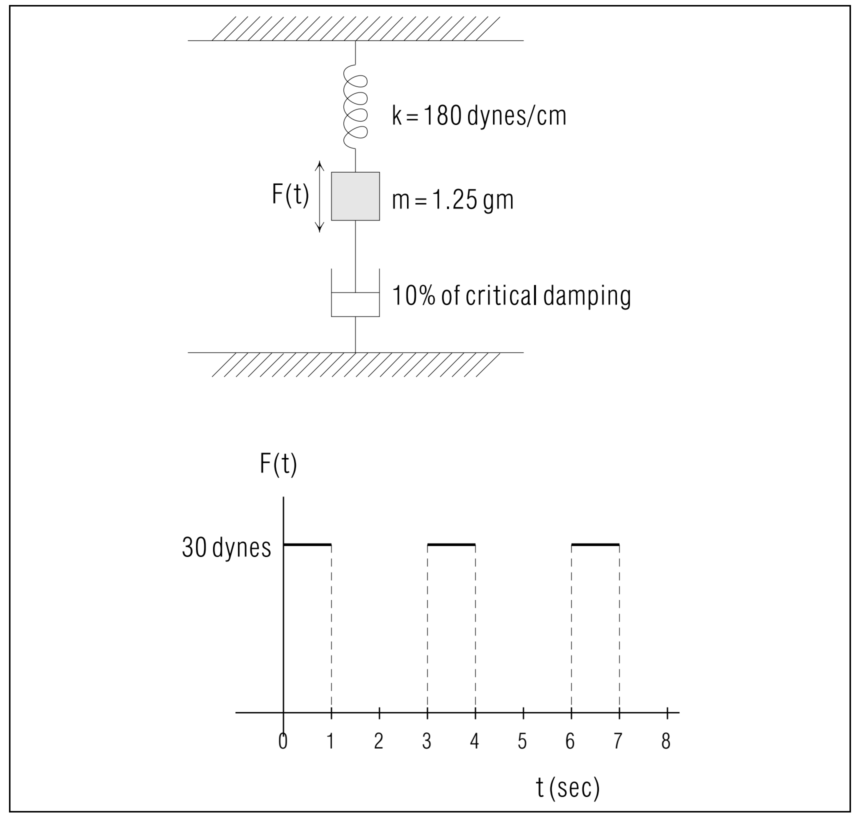

Figure 1.