



CAUCHY RESIDUE THEOREM AND DEFINITE INTEGRALS

Math Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CAUCHY RESIDUE THEOREM AND DEFINITE INTEGRALS

by

R. D. Young, Dept. of Physics, Illinois State Univ.

1. Introduction	1
2. Procedures	1
3. Supplementary Note	3
Acknowledgments	3

Title: **Cauchy Residue Theorem and Definite Integrals**

Author: R. D. Young, Dept. of Physics, Illinois State Univ.

Version: 2/1/2000

Evaluation: Stage B0

Length: 2 hr; 9 pages

Input Skills:

1. Vocabulary: conformal mapping, complex variables, singularities, residues, mapping, transformations: translation, rotation, stretching, inversion, linear.
2. Unknown: assume (MISN-0-489).

Output Skills (Knowledge):

- K1. Write the definition or explain the meaning of the Cauchy principle value of an integral.
- K2. Write down the Cauchy residue theorem when asked, and include the proper conditions on the functions involved.

Output Skills (Rule Application):

- R1. Evaluate various definite integrals using the Cauchy residue theorem.

External Resources (Required):

1. G. Arfken, *Mathematical Methods for Physicist*, Academic Press (1995).
2. Schaum's Outline: Murray Spiegel, *Theory and Problems of Advanced Mathematics for Scientists and Engineers*, McGraw-Hill Book Co. (1971).

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

PROJECT STAFF

Andrew Schnepf	Webmaster
Eugene Kales	Graphics
Peter Signell	Project Director

ADVISORY COMMITTEE

D. Alan Bromley	Yale University
E. Leonard Jossem	The Ohio State University
A. A. Strassenburg	S. U. N. Y., Stony Brook

Views expressed in a module are those of the module author(s) and are not necessarily those of other project participants.

© 2001, Peter Signell for Project PHYSNET, Physics-Astronomy Bldg., Mich. State Univ., E. Lansing, MI 48824; (517) 355-3784. For our liberal use policies see:

<http://www.physnet.org/home/modules/license.html>.

CAUCHY RESIDUE THEOREM AND DEFINITE INTEGRALS

by

R. D. Young, Dept. of Physics, Illinois State Univ.

1. Introduction

This is the last unit in your introduction to the theory of complex variables and its applications to physics and engineering. The Cauchy Residue Theorem is of powerful use in evaluating many of the integrals which regularly occur in physics and engineering.

2. Procedures

1. Review Procedure - Read Sec. 7.1, Singularities.
2. Read Sec. 7.2, Calculus of Residues in Arfken.
3. Read these sections in Spiegel:
 - Residue Theorem, pages 289 to 290
 - Evaluation of Definite Integrals, page 290
4. Write down the definition of Cauchy Principle Value as given in eq. (7.12) of Arfken.
5. Write down the Cauchy Integral Theorem 13.1 on page 290 of Spiegel. Note all conditions of $f(z)$.
6. Write down the formulas for evaluating these definite integrals:

a.

$$\int_0^{2\pi} f(\sin \theta, \cos \theta) d\theta$$

as in eqns (7.25) and (7.28) of Arfken.

b.

$$\int_{-\infty}^{\infty} f(x) dx$$

as in eq. (7.31) of Arfken. The conditions on $f(x)$ are listed immediately after eq. (7.29) of Arfken.

c.

$$\int_{-\infty}^{\infty} f(x)e^{i\alpha x} dx$$

as in eq. (7.44) of Arfken. The conditions on $f(x)$ are given in eq. (7.38) of Arfken.

- d. Integrals with a singularity on the contour of integration as in Example (7.2.3) of Arfken.

Note: These same integrals are discussed on page 290 of Spiegel as well as in the Solved Problems in Spiegel. The next procedure will refer you to these Solved Problems.

7. Read through the following Solved Problems of Spiegel:

13.23 (Proof of Cauchy Residue Theorem)

13.25 (Proof of Cauchy Residue Theorem)

13.29 (Evaluation of Definite Integrals)

13.30 (Evaluation of Definite Integrals)

13.31 (Evaluation of Definite Integrals)

13.32 (Evaluation of Definite Integrals)

13.34 (Evaluation of Definite Integrals)

13.35 (Evaluation of Definite Integrals)

13.37 (Evaluation of Definite Integrals)

8. Solve the following problems in Arfken:

7.2.8 (Evaluation of Definite Integrals)

7.2.9 (Evaluation of Definite Integrals)

7.2.12 (Evaluation of Definite Integrals)

7.2.14 (Evaluation of Definite Integrals)

9. Solve the following problems in the Supplementary Problem section of Spiegel:

13.88 (Evaluation of Definite Integrals)

13.90 (Evaluation of Definite Integrals)

13.91 (Evaluation of Definite Integrals)

13.100 (Evaluation of Definite Integrals)

13.103 (Evaluation of Definite Integrals)

3. Supplementary Note

At various times, the residues of the function $f(z) = 1/z^n + b$ at each of the poles are needed.

It is easy to calculate the residue using L'Hospital's rule. Thus, if z_0 is a pole of $f(z)$, then,

$$a_{-1} = \lim_{z \rightarrow z_0} \frac{(z - z_0)}{z^n + b} = \lim_{z \rightarrow z_0} \frac{d/dz(z - z_0)}{d/dz(z^n + b)}$$

$$a_{-1} = \lim_{z \rightarrow z_0} \frac{1}{nz^{n-1}} = \frac{1}{nz_0^{n-1}} = -\frac{z_0}{nb}$$

Since $z_0^n = -b$ so that $1/z_0^{n-1} = -z_0/b$.

Acknowledgments

The author would like to thank Illinois State University for support in the construction of this lesson. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

