PHYSNET

MISN-0-476

EINSTEIN'S FIELD EQUATIONS

Relativity

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

EINSTEIN'S FIELD EQUATIONS

by

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Title: **Einstein's Field Equations**

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Length: 2 hr; 8 pages

Input Skills:

1. Unknown: assume (MISN-0-475).

Output Skills (Knowledge):

K1. Justify the form of the matter tensor $M_{\mu\nu}$ for dust and verify its properties:
$M_{\mu\nu} = M_{\nu\mu}$;
$\partial_\nu M^{\mu\nu} = 0$.

K2. Describe briefly the general manner whereby the laws of physics (other than those dealing with gravitation) are usually generalized from special relativity to general relativity.

K3. Give a plausibility argument for Einstein's field equations.

K4. Summarize the fundamental ideas of general relativity in terms of the field equations, the geodesic equations and the equivalence principle.

External Resources (Required):

1. W. Rindler, *Essential Relativity*, van Nostrand (1977).

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1. Introduction

In the previous unit the field equations for vacuum were discussed and a solution of those equations was found for the special case of spherically symmetric static mass distribution. This unit is concerned with the form of the field equations even in the presence of matter. Once the form of these equations has been determined the formal structure of general relativity is complete.

2. Procedure

1. Read Rindler, section 5.13. The objective is to justify eq. 5.49 of Rindler and then prove eqs. 5.50 and 5.51.

   Comment - Many authors use the symbol $T^{\mu\nu}$ instead of $M^{\mu\nu}$ and refer to $T^{\mu\nu}$ as the energy-momentum tensor or the stress-energy tensor.

   Comment - Equation 5.49 is valid only for the very special case of dust (a perfect fluid). However, eq. 5.50 and 5.51 are assumed to be valid for an arbitrary medium.

2. Read Rindler, section 8.9. Of particular interest are equations 8.120, 8.121, 8.122 and 8.129.

   ▷ Exercise - Propose a generalization consistent with the scheme proposed by Rindler,for extending Maxwell's equations and the Lorentz force law to general relativity. Hint: Consult pages 6 and 8 of MISN-0-470. Also recall the last exercise on p. 3 of unit MISN-0-473.

3. Read Rindler unit 8.10 to the bottom of p. 180.

   ▷ Exercise - It can be shown (but you need not do it) that the curvature tensor satisfies Bianchi's identities.

   $$R^{\lambda}_{\mu\nu\rho,\sigma} + R^{\lambda}_{\mu\sigma\nu,\rho} + R^{\lambda}_{\mu\rho\sigma,\nu} = 0$$

   Use these identities to verify equation 8.134 in Rindler.

   Note - The coefficient k in Einstein's field equations (Rindler's eq. 8.136) is at this point completely undetermined. In the next unit it will be shown to be related to Newton's gravitational constant G.

   (Optional) - Read Rindler, bottom of p. 180 to end of chapter 8.

4. This objective is essentially a review which you are to carry out on your own.

Acknowledgments

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