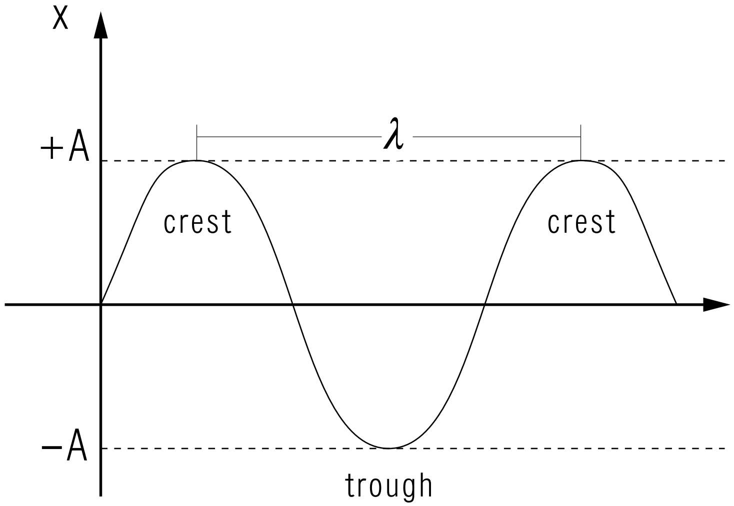
PHYSNET

MISN-0-430

BASIC PROPERTIES OF WAVES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

BASIC PROPERTIES OF WAVES

by
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CONTENTS

Title: **Basic Properties of Waves**

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Input Skills:

1. State the principle of conservation of energy in terms of macroscopic and internal energy (MISN-0-416).

Output Skills (Knowledge):

K1. Vocabulary: wave, amplitude, sinusoidal wave, phase, period, frequency, hertz, wavelength, wave front, intensity, interference.

K2. Describe at least three types of waves.

K3. State the frequency ranges of audible sound and visible light.

K4. Describe the wave fronts for spherical and plane waves.

K5. Relate intensity to amplitude.

K6. State the superposition principle for waves.

Output Skills (Problem Solving):

S1. Given its wave speed, use information about a wave at a given position or time to determine the wave at some other position or other time.

S2. Compare the intensities and amplitudes of two sinusoidal waves.

S3. Use the superposition principle to relate the values of two or more individual waves to the value of the resultant wave.

MISN-0-430

BASIC PROPERTIES OF WAVES

Abstract:
A wave is a disturbance that remains unchanged while it moves with some constant velocity. There are water waves, waves on strings, sound waves, earthquake waves, radio waves, light waves, radar waves, X-rays, etc.

In the present unit we shall discuss the most basic properties common to all kinds of waves. We shall then use the next several units to explore a wide variety of implications and practical applications.

SECT.
A WAVES ON A STRING

To illustrate some of the general properties of waves, let us begin by considering the particularly simple example of a wave traveling along a string.

► *Disturbance of a string*
Figure A-1a shows an undisturbed string stretched between two points so as to lie along a straight line parallel to the $\hat{x}$ direction. Figure A-1b shows a disturbance on this string. As indicated, this disturbance consists of a displacement of a small region of the string along a direction $\hat{y}$ perpendicular to the string. This disturbance may be described by the component, along the $\hat{y}$ direction, of the displacement of the string at any point.

► *Wave Motion*
The disturbance of Fig. A-1b can move with unchanged shape along the string with some constant velocity $\vec{V}$ (as shown in Fig. A-1b, Fig. A-1c, and Fig. A-1d, where this wave travels along the $\hat{x}$ direction). Such a moving disturbance is then called a "wave" moving (or "propagating") along the string.

Why does the disturbance move along the string? The basic reason is that a displacement of any point of the string along the $\hat{y}$ direction produces at a neighboring point of the string a force in this direction, hence an acceleration in this direction, and thus (slightly later) a correspond-

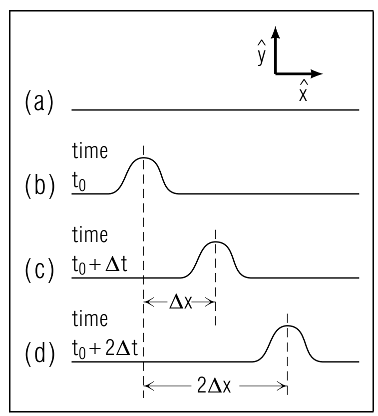


Fig. A-1: Disturbance moving along a stretched string: (a) Undisturbed string; (b), (c), (d) a disturbance shown at three successive times. This disturbance moves with a constant velocity $\vec{V} = (\Delta x)/(\Delta t)$ along the $\hat{x}$ direction.

ing displacement in this direction. Thus, a disturbance at any point of the string produces slightly later a similar disturbance at a neighboring point of the string; in turn, this disturbance produces then slightly later a similar disturbance at the next neighboring point; and so forth. The net result is that the disturbance moves along the string by a succession of locally generated effects (somewhat reminiscent of the successive collapse of a pile of dominoes).

► *Velocity of wave*

The velocity of the disturbance moving along the string depends on the properties of the string. Indeed, as shown in Problem H-1, this velocity depends on the magnitude of the tension force in the string and on the mass per unit length of the string. (To find this velocity, one need only focus attention on any small part of the string. Then one can use the equation of motion to analyze how the acceleration of this part of the string is related to the net force resulting from the displacements of the string.)

► *Wave vs. particle motion*

Note that the motion of the disturbance involves the successive motions of *different* particles in the string and is very different from the motion of any of these individual particles. Thus the disturbance moves along the string with some constant velocity $\vec{V}$ along the string. But this velocity is very different from the velocity $\vec{v}$ of any individual particle in the string. Indeed, any such particle moves along a direction *perpendicular* to the string and never moves very far from its original position (while the disturbance itself moves along the entire string).

► *Energy transport*

Energy is associated with a moving wave. Indeed, since a disturbance produces at a neighboring point of the string a force and causes this point to move, the disturbance can do work.

► *Transmission of waves*

A device used to produce a disturbance is called a "source." (For example, such a source might be simply a finger plucking the string to produce a displacement at a particular point.) A device used to detect the arrival of the disturbance at some point is called a "detector." (For example, such a detector might be simply a small piece of paper attached at a particular point of the string so that the displacement of this point can be easily observed.) A wave produced by the source and later arriving at the detector can be used to transmit signals (and thus information) from the source to the detector. Furthermore, work is done by the source to

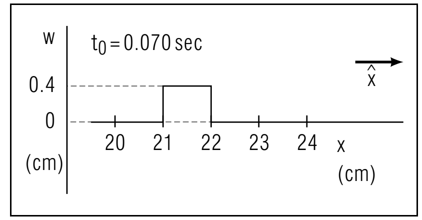


Fig. A-2.

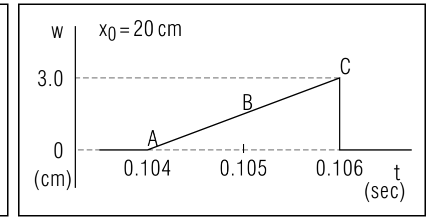


Fig. A-3.

produce the wave. The wave arriving at the detector can then later do work on the detector. Thus the wave can also be used to transmit energy from the source to the detector.

Motion of a Wave (Cap. 1)

A-1 A displacement wave moves along a long stretched string with a velocity of 6.0×10^3 cm/s along the $\hat{x}$ direction. The graph in Fig. A-2 illustrates, at the particular time $t_0 = 0.070$ s, the wave (described by the component w of the displacement of the string along the $\hat{y}$ direction perpendicular to $\hat{x}$) at various positions (described by the position coordinate x along the $\hat{x}$ direction). Draw the graph showing this wave as a function of position along the string at the later time $t = 0.072$ s. *(Answer: 4) (Suggestion: [s-3])*

A-2 As a different wave travels in the direction along the string described for the wave in the preceding problem, the displacement component w of the string, observed at the *fixed* position $x_0 = 20$ cm, varies with the *time* t in the manner indicated in the graph of Fig. A-3. We are interested in finding the position of this wave at the particular time $t_1 = 0.108$ s. (a) Consider the part C of the wave (where $w = 3.0$ cm at the time $t = 0.106$ s). Through what distance does this part of the wave travel until the time $t_1 = 0.108$ s? What then is the position of this part of the wave at the time t_1? (b) Similarly, consider the part B of the wave (where $w = 1.5$ cm at the time $t = 0.105$ s). Through what distance does this part of the wave travel until the time $t_1 = 0.108$ s? What then is the position of this part of the wave at the time t_1? (c) Answer the same questions for part A of the wave (where $w = 0$ at the time $t = 0.104$ s). (d) To show the positions of the various parts of the wave at the time t_1, draw a graph of w versus x at the time t_1. *(Answer: 7) (Suggestion: [s-5])*

A-3 *Particle vs. wave velocity :* (a) Use the information provided in Fig. A-3 to find the velocity of a particle in the string (at the position $x_0 = 20\,\text{cm}$) at the time $t = 0.105\,\text{s}$. (b) Is the magnitude of the velocity larger than, equal to, or smaller than the speed of the wave along the string? *(Answer: 2) (Suggestion: [s-2])*

More practice for this Capability: [p-1], [p-2]

SECT.

B TYPES OF WAVES

GENERAL PROPERTIES

There are many different kinds of waves, but all of them share the following general properties illustrated in the previous example of a wave on a string:

► *Disturbance in a medium*

(1) There is some kind of a medium (either a material medium or a vacuum) which is originally undisturbed. A disturbance can then be produced near some point in this medium. This disturbance can be described either by a number, by a vector (such as a displacement), or even by several vectors. If we specify any such vector by its numerical components, any disturbance can then be described by one or more numbers, each of which we shall simply denote by the letter w.

► *Wave motion*

(2) A disturbance near a particular point can produce slightly later a similar disturbance at a neighboring point, which in turn can produce slightly later a similar disturbance at a next neighboring point, and so forth. Under these conditions the disturbance can move as a "wave."

Def. **Wave**: A disturbance which remains unchanged (or nearly unchanged) while it moves through a medium with some constant velocity. (B-1)

► *Wave velocity*

(3) The velocity $\vec{V}$ with which a wave moves through a medium depends on the properties of the medium and can be found from a detailed analysis of the mechanism whereby a local disturbance produces a disturbance at neighboring points. If the medium is "isotropic" (i.e., if its properties are the same along every direction), the magnitude V of the wave velocity is the same along any direction. [The velocity $\vec{V}$ of a wave through a medium is quite different from the velocity $\vec{v}$ of any individual particle in the medium.]

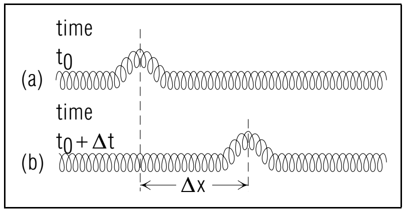


Fig. B-1: Transverse displacement wave in a spring.

► *Energy transport*

(4) A wave possesses energy since work must be done to produce a wave and the wave can then, in turn, do work on some other system. A moving wave can thus transport energy from one point to another.

► *Wave transmission*

(5) The preceding comments imply that a wave can be used to transmit signals and energy from some source to some detector. Since the wave moves with a finite velocity, there is a time delay between the instant the wave is emitted by the source and the instant it arrives at the detector.

► *Differences between waves*

(6) Although all waves share the preceding general characteristics, they can differ quite widely because they may involve very different disturbances moving through very different media. Let us then mention some of the various kinds of waves encountered most commonly.

ELASTIC WAVES

Consider a medium consisting of any material. The undisturbed state of such a medium is one where the atoms in the material are at their normal positions. A disturbance in the material can then be produced by displacing the atoms near some point from their normal positions. Since these atoms interact with neighboring atoms, these neighboring atoms then also begin to move. Thus a disturbance, consisting of a displacement of the atoms from their normal positions, moves through the material with a velocity depending on the properties of the material. Such a disturbance, called an "elastic" wave, consists thus of a displacement of the atoms from their normal positions in a material (despite the interatomic forces tending to restore these atoms to their normal positions).

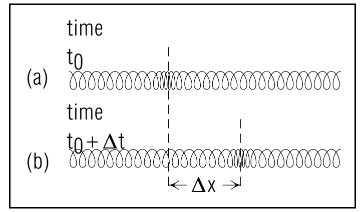


Fig. B-2: Longitudinal displacement wave in a spring.

► *Transverse waves*

As a specially simple example of an elastic wave, consider a stretched spring or string (like that discussed in the preceding section). One way to create a disturbance in such a spring is to displace the atoms near some point along a direction *perpendicular* to the spring, as shown in Fig. B-1a.

This disturbance then travels along the spring away from its original position (i.e., away from its source) in both directions. For example, Fig. B-1 illustrates the resulting displacement traveling to the right. The resulting wave is called a "transverse" elastic wave since the disturbance consists of a displacement in a direction perpendicular to (i.e., transverse to) the velocity of the wave along the spring.

► *Longitudinal waves*

Another way of creating a disturbance in the preceding spring is to displace the atoms near some point in the spring along a direction *parallel* to the spring, as indicated in Fig. B-2a, so as to create near this point either a compression or decompression of the spring (i.e., so as to either decrease or increase the average separation of the atoms near this point, compared to their normal separation). Such a disturbance can again travel along the spring in both directions. The resulting wave is called a "longitudinal" elastic wave since the disturbance now consists of a displacement in a direction *parallel* to (or along) the velocity of the wave along the spring.

► *Waves in solids*

The displacement associated with an elastic wave in a solid (e.g., the kind of wave produced in the earth by an earthquake) can have *any* direction relative to the velocity of the wave.

Since this displacement can always be regarded as consisting of component displacements parallel and perpendicular to the velocity, such a wave can be viewed as consisting of a mixture of longitudinal and transverse waves (which travel ordinarily with velocities of different magni-

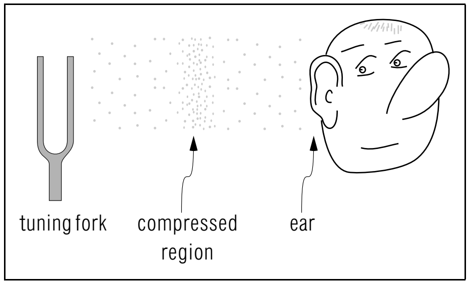


Fig. B-3: Sound wave traveling in air from a source (a tuning fork) to a detector (an ear.)

tudes).

► *Waves in fluids*

The displacement associated with an elastic wave in a fluid is longitudinal, i.e., parallel to the velocity of the wave.*

> * The reason is that the pressure forces responsible for the interaction between neighboring layers of atoms in a fluid are perpendicular to the surface between these layers (see text section A of Unit 417). Hence a displacement *parallel* to this surface produces no forces generating a neighboring disturbance and thus does not result in a traveling disturbance. But a displacement *perpendicular* to this surface generates a neighboring disturbance and thus results in a disturbance traveling perpendicular to the surface, i.e., parallel to the displacement.

Such a longitudinal wave is then associated with a corresponding local compression or decompression traveling through the fluid (just as the previously discussed longitudinal wave in a spring).

► *Sound waves*

A sound wave in air is a familiar example of such a longitudinal wave in a gas. As illustrated in Fig. B-3, the source of such a sound wave is some moving object (such as a vocal cord, a tuning fork, or the diaphragm of a loudspeaker) which produces a displacement in the air immediately adjacent to the moving object. After the wave has traveled some distance, it may then produce a corresponding displacement of some other object (such as an eardrum or a diaphragm in a microphone) which acts as a detector registering the arrival of the wave.

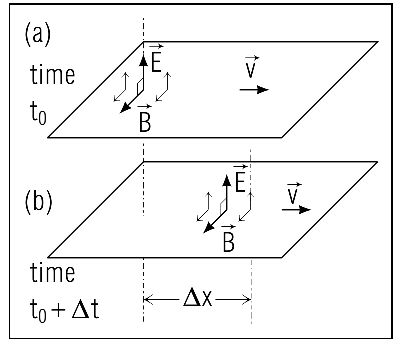


Fig. B-4: Electromagnetic wave consisting of an electric field $\vec{E}$ and a magnetic field $\vec{B}$ traveling through space with a velocity $\vec{V}$.

WAVES ON THE SURFACE OF A LIQUID

The undisturbed surface of a liquid near the surface of the earth is horizontal. The surface of such a liquid can be disturbed by displacing the atoms near some point of the liquid surface away from their normal positions. Such a disturbance can then travel along the surface and gives rise to a wave, such as the familiar waves observed on the surface of a lake. (Such a wave is *not* an elastic wave since the forces tending to restore the atoms to their normal positions along the horizontal surface are the gravitational forces due to the earth, and *not* interatomic forces.)

ELECTROMAGNETIC WAVES

A vacuum, or a region filled with some material, can be considered as an undisturbed medium (from an electromagnetic point of view) if there are no electric or magnetic fields in it. An "electromagnetic wave" consists then of electric and magnetic fields existing in some region and moving through the medium with some constant velocity (see Fig. B-4.)

► *Properties of e.m. waves*

The main properties of electromagnetic waves have already been discussed in text section C of Unit 429. Thus,the reason that an electromagnetic disturbance can move is that a time-varying electric field at a given point produces at a neighboring point a magnetic field, and that a time-varying magnetic field at the given point produces at the neighboring point an electric field. As a result, an electromagnetic disturbance at a given point can produce a similar disturbance at a neighboring point, and can thus move through the medium. Furthermore, it is shown in text section C of Unit 429 that the electric field $\vec{E}$ and magnetic field $\vec{B}$ associated with an electromagnetic wave are perpendicular to each other and perpendicular to the velocity $\vec{V}$ of the wave (see Fig. B-4.) The speed V of an electromagnetic wave moving through a *vacuum* is a fundamental constant (commonly denoted by the letter c) having the approximate value $c = 3 \times 10^8$ m/s. The speed of electromagnetic waves in other media is smaller than c.

As we shall discuss more fully in the next section, radio waves, radar waves, light, and x-rays are all merely different kinds of electromagnetic waves.

► *Sources and detectors*

The source of an electromagnetic wave is an accelerated charged particle which produces in its vicinity changing electric and magnetic fields. (For example, a radio-transmitting antenna consists basically of a wire in which electrons are made to move back and forth so that they have a large acceleration.) Similarly, the arrival of an electromagnetic wave can be detected by the fact the fields of such a wave accelerate charged particles by producing forces on them. (For example, the receiving antenna of a radio is merely a wire in which the electrons are accelerated by the arrival of a radio wave.)

Knowing About Types of Waves

B-1 *Longitudinal wave:* Fig. B-5 shows a row of undisturbed atoms in a solid, the separation between adjacent atoms being L. Suppose now that, as a result of a *longitudinal* elastic wave traveling along $\hat{x}$, the atom 3 is displaced by an amount $w = 0.001\,L$ along the $\hat{x}$ direction (as indicated in Fig. B-5b) while the other atoms remain in their normal positions. (a) What then is the separation L' between atoms 3 and 4? Express your answer in terms of L. (b) Is the separation L' larger than, smaller than, or equal to the normal atomic separation L? (c) As a result of this wave, is the density of the solid in the region between x_3 and x_5 then larger than, smaller than, or equal to the normal density of the solid? *(Answer: 5) (Suggestion: [s-1])*

B-2 *Transverse wave:* Consider again Fig. B-5a showing a row of undisturbed atoms in a solid. Suppose that, as a result of a *transverse* elastic wave traveling along $\hat{x}$, the atom 3 is displaced by an amount $w = 0.001L$ along the $\hat{y}$ direction perpendicular to $\hat{x}$ (as indicated in Fig. B-5c) while the other atoms remain in their normal positions. Answer the same questions a, b, and c of the preceding problem for the case of this transverse wave. *(Answer: 1) (Suggestion: [s-4])*

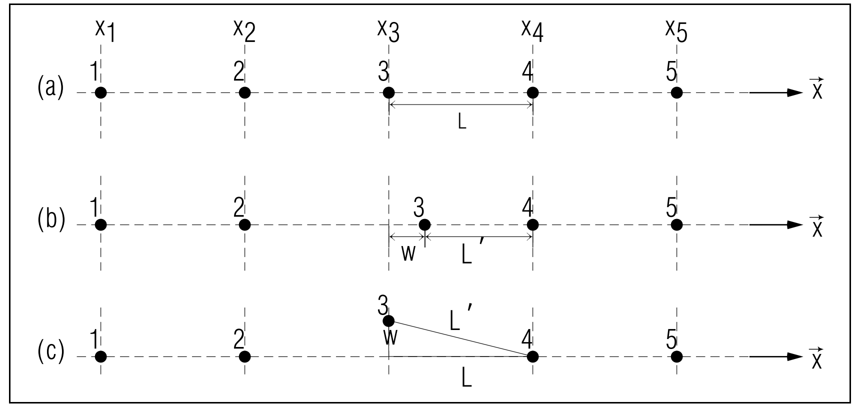


Fig. B-5.

B-3 *Electromagnetic waves:* (a) Can light waves travel through a vacuum (such as interplanetary space)? (b) Can radio waves travel through a vacuum? (c) If the answers to the preceding questions are affirmative, what is the speed of light waves in a vacuum and the speed of radio waves in a vacuum? (d) Can light waves and radio waves also travel through a solid or liquid, such as glass or alcohol? *(Answer: 6) (Suggestion: [s-7])*

B-4 *Sound waves:* (a) Can a sound wave travel through a vacuum? (b) Can a sound wave travel through a gas, such as air? (c) Can it travel through a liquid, such as water? (d) Can it travel through a solid such as steel? *(Answer: 9) (Suggestion: [s-9])*

B-5 *Bell in a jar:* An electric bell is suspended by two fine metal wires inside an air-filled jar made of transparent glass so that the bell can be seen from outside the jar. An electric current is now passed through the bell. (a) Can the ringing of the bell then be heard from outside the jar? Why? (b) Suppose that the air inside the jar is now pumped out. Can the bell still be seen from outside the jar? Can the ringing of the bell still be heard from outside the jar? Why? *(Answer: 3)*

B-6 *Lightning and thunder:* A lightning bolt produces a sound wave (thunder) as a result of the expansion of the air heated in the electric discharge constituting the lightning bolt. The speed of light in air is nearly the same as that in vacuum, i.e., 3×10^8 m/s. The speed of sound of air is approximately 340 m/s. (a) How long a time t_s does it take the sound wave produced by the lightning bolt to reach a point P 2.0 km away? (b) How long a time t_L does it take the light produced by the lightning bolt to reach this same point? Compare this time with the time t_s by computing the ratio t_L/t_s. (c) At the point P, what is the time elapsed between the instant the lightning is seen and the instant thunder is first heard? *(Answer: 10)*

SECT.

C SINUSOIDAL WAVES

Waves which are repetitive, so that the same sequence of events is merely repeated indefinitely, are simplest to study. The reason is that such a wave has no beginning or end, so that one need not worry about how such a wave starts or stops. Throughout the following units we shall, in particular, deal almost entirely with waves which change repetitively in a "sinusoidal" manner (i.e., in a manner similar to how a sine or cosine changes with angle). An understanding of such sinusoidal waves is sufficient to understand all other kinds of waves. Indeed, as discussed in Unit 433, any wave, no matter how complex, can always be expressed as a sum of simple sinusoidal waves.

► *Sinusoidal change*

To specify more precisely what is meant by a sinusoidal change, we recall the general definition of the cosine of an angle: $\cos\phi$ is the numerical component, along a direction $\hat{x}$, of a *unit* vector $\hat{u}$ making an angle ϕ relative to $\hat{x}$. Values of $\cos\phi$ for successively larger values of ϕ can then be simply read off a diagram, such as that in Fig. C-1a, showing the unit vector $\hat{u}$ rotating around so that its angle ϕ increases.*

> * Similarly, $\sin\phi$ is defined as the numerical component of $\hat{u}$ along the $\hat{y}$ direction in Fig. C-1a.

Note that the set of values assumed by $\cos\phi$ repeats itself whenever $\hat{u}$ has rotated by one complete revolution, i.e., whenever ϕ changes by 360° or 1 "cycle." The graph in Fig. C-1b shows explicitly how $\cos\phi$ varies with ϕ.

Let us now examine in greater detail the properties of a sinusoidal wave and introduce some important definitions used throughout the following units.

WAVE AT VARIOUS TIMES AT A FIXED POINT

Consider some fixed point P_0. A sinusoidal wave at this point can then be described by a disturbance w which varies with the time t in a manner described by

$$w = A\cos\phi \qquad \text{(C-1)}$$

where A is some positive constant and ϕ depends on the time t. The

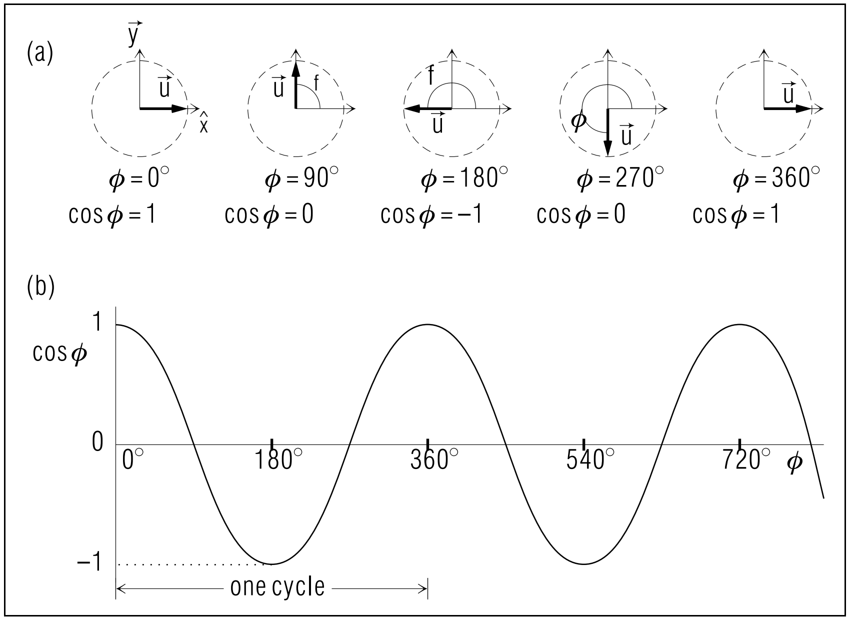


Fig. C-1: Cosine of an angle ϕ: (a) Definition of $\cos\phi$ as the component of a unit vector $\hat{u}$ along some direction $\hat{x}$; (b) Graph of $\cos\phi$ versus ϕ.

constant A is called the "amplitude" of the wave and ϕ is called the "phase angle" (or simply the "phase") of the wave. A graph showing how w varies with the time t is shown in Fig. C-2.

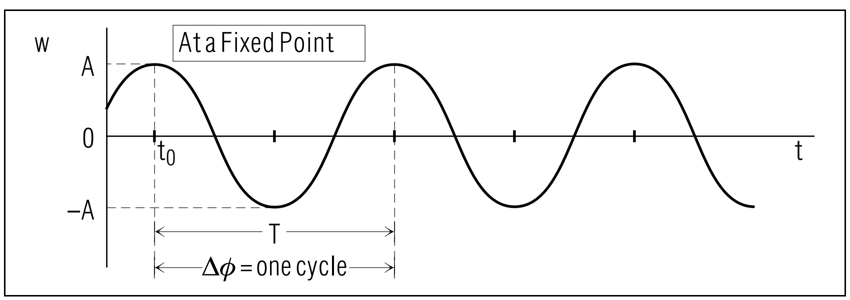


Fig. C-2: Sinusoidal wave varying with time at a fixed point.

► *Amplitude*

The wave w is seen to assume all values between a maximum value $+A$ and a minimum value $-A$. Thus the meaning of the amplitude A is specified by this definition:

Def. | **Amplitude**: The maximum magnitude of a sinusoidal wave. | (C-2)

► *Phase*

The meaning of the "phase" of a wave is simply this:

Def. | **Phase**: The quantity ϕ specifying the value of a sinusoidal wave varying like $\cos\phi$. | (C-3)

In particular, such a wave repeats itself whenever its phase ϕ changes by 1 cycle (or 360°).

► *Period*

The set of values assumed by the wave w is seen to repeat itself whenever the time changes by some amount T (corresponding to the phase of the wave changing by 1 cycle). The time T is called the "period" of the wave in accordance with this definition:

Def. | **Period**: The period T of a sinusoidal wave is the time elapsed between successive repetitions of the same set of values of the wave at a *fixed position*. | (C-4)

For example, the period T is the time between two successive maxima or two successive minima of the wave (as indicated in Fig. C-2).

► *Time variation*

The period T is a very useful quantity for describing how the wave at a fixed point repeats itself in the course of time. Thus, as illustrated in Fig. C-2, if the time changes by T (so that the phase of the wave changed by 1 cycle), the wave assumes the same value as before. But if the time changes by $(1/2)\,T$ (so the phase of the wave changes by (1/2) cycle), the wave assumes a value of the same magnitude but opposite sign.

For example, consider some time t_0 when the wave w assumes its maximum value $w = A$ (as indicated in Fig. C-2). At a time $(1/4)\,T$ after t_0, the value of the wave is then $w = 0$. At a time $(1/2)\,T$ after t_0, the value of the wave is then $w = -A$. At a time $(3/4)\,T$ after t_0, the value of the wave is then $w = 0$. And a time T after t_0, the value of the wave is again equal to its original maximum value $w = A$.

► *Frequency*

Instead of using the period T to describe how a sinusoidal wave repeats itself in the course of time, one can equally well use the "frequency" ν, defined this way:

Def. **Frequency**: The frequency of a sinusoidal wave is the number of repetitions *per unit time* of the same set of values of the wave. (C-5)

The frequency ν is simply related to the period T. Indeed, the number of repetitions occurring during some long time t is t/T (i.e., it is simply the time t divided by the time T required for a single repetition). To find the number of repetitions *per unit time* (i.e., divided by the corresponding time), one need then only divide (t/T) by t, thus obtaining the result $1/T$. Hence the frequency ν is simply related to the period so that

$$\nu = \frac{1}{T} \tag{C-6}$$

► *Unit of frequency*

The relation (C-6) implies that the SI unit of frequency is

$$\text{unit of } \nu = \frac{1}{\text{second}} = \text{hertz} \tag{C-7}$$

where the unit "hertz" (abbreviated as "Hz") is simply a convenient abbreviation for (second)$^{-1}$. [This unit is named in honor of Heinrich R. Hertz (1857-1894), the German physicist who first demonstrated experimentally the existence of radio waves.] For example, if the period of a wave is 0.1 second, Eq. (C-6) implies that the frequency of this wave is 10 hertz (corresponding to 10 repetitions per second).

WAVE AT VARIOUS POINTS AT A FIXED TIME

► *Sinusoidal spatial variation*

Consider a wave which travels with a constant speed V along some direction and let x be the component of the position vector of any point along this direction. If the wave at some fixed point varies sinusoidally with *time* and then travels from this point with the speed 6V, the resulting wave produced at any fixed time must then also vary sinusoidally with *position*. To show why this is so, we illustrate in Fig. C-3 how the wave varies with position at various times differing by $T/4$ (where T is

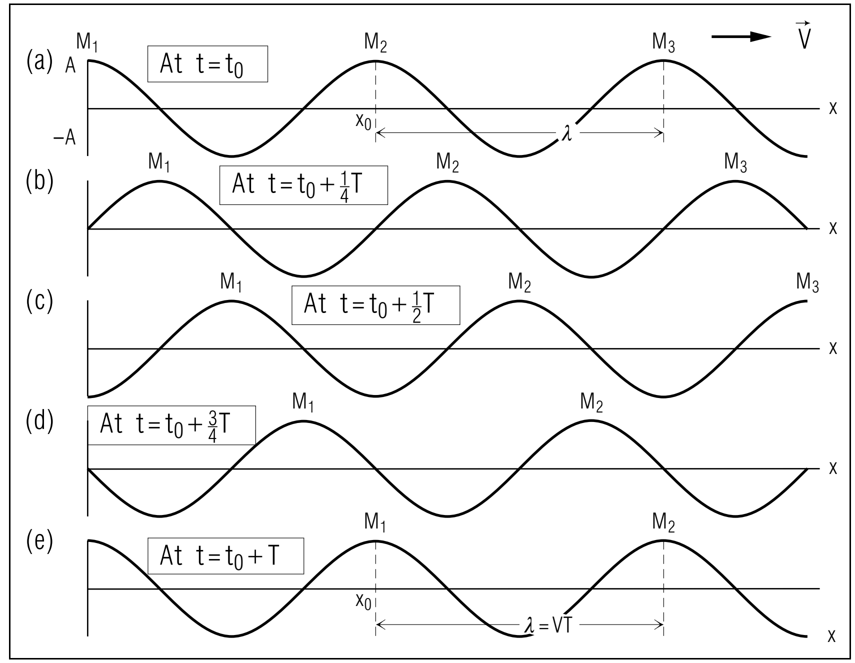


Fig. C-3: Sinusoidal wave varying with position, specified by x, at successive fixed time.

the period of the wave). At the point specified by x_0, the wave varies sinusoidally with the time t as indicated in Fig. C-2 and correspondingly indicated by the wave at x_0 in the successive graphs of Fig. C-3. Because the wave in Fig. C-3 moves to the right with a speed V, any value of the wave appearing x_0 (or any other point) in one graph appear in the next graph, at a time later by $T/4$, displaced to the right by the same amount $V(T/4)$. This fact has been used to construct the successive graphs in Fig. C-3. In particular, we see that, at any fixed time, the wave varies sinusoidally with the position specified by x.

► *Wavelength*

At any fixed time, the wave repeats itself (i.e., its phase changes by one cycle) when the distance along the wave changes by some amount λ, called the "wavelength."

Def.	**Wavelength**: The wavelength λ of a sinusoidal wave is the distance, along the velocity of the wave, between successive repetitions of the same set of values of the wave *at a fixed time*.	(C-8)

For example, the wavelength is the distance between two successive maxima or two successive minima of the wave (as indicated in Fig. C-3).

RELATION BETWEEN WAVELENGTH, FREQUENCY, AND SPEED

As is seen from Fig. C-3 during a time equal to a period T, any disturbance (such as the maximum M_2 of the wave) moves by a distance equal to a wavelength λ. Thus the speed V of the wave is simply equal to the distance λ divided by the time T. Hence:

$$V = \frac{\lambda}{T} \qquad \text{(C-9)}$$

or

$$\boxed{V = \lambda\nu} \qquad \text{(C-10)}$$

where the last relation is obtained by using Eq. (C-6) to express $(1/T)$ in terms of the frequency ν. According to Eq. (C-10), the product of the wavelength λ multiplied by the frequency ν is thus always equal to the speed V of the wave.

APPLICATIONS

The observable effects of a sinusoidal wave depend crucially on its frequency or wavelength. We mention several examples.

► *Detectors*

A detector (such as an ear or an eye) is located at some fixed point of space and thus responds to how rapidly a wave changes at this point, i.e., it responds to the *frequency* of the wave. For example, the ear (and associated nervous system) perceives sound waves of different frequencies as having different "pitch." Similarly, the eye (and associated nervous system) responds to electromagnetic waves of different frequencies as having different "color."

► *Sound waves*

The human ear can only detect sound waves in the approximate frequency range between 20 Hz and 20,000 Hz. In this range, sound waves of higher frequency are perceived as having higher pitch. Sound waves with frequencies smaller than about 20 Hz are called "infrasonic" waves, while those with frequencies larger than about 20,000 Hz are called "ultrasonic" waves. Such waves are not audible to the human ear, but can be detected by various non-physiological devices. In particular, ultrasonic waves have many practical applications in medicine and in other fields.

► *Light waves*

The human eye can only detect electromagnetic waves in the approximate frequency range between 4×10^{14} Hz and 7×10^{14} Hz. Waves of increasingly larger frequency in this range are perceived by the eye as having different colors ranging from red, to yellow, to green, to blue, and to violet (i.e., ranging through colors of the rainbow). Electromagnetic waves of frequencies somewhat below those detectable by the eye are called "infrared" waves, while electromagnetic waves of frequencies somewhat above those detectable by the eye are called "ultraviolet" waves. Such waves, as well as all other electromagnetic waves not detectable by the eye, can be detected by various non-physiological devices.

► *Electromagnetic waves*

The many different kinds of electromagnetic waves differ only in frequency (or thus also in wavelength), but have correspondingly widely different observable properties. Figure C-4 illustrates the various names give to electromagnetic waves in different frequency ranges. For example, AM radio waves have frequencies near 10^6 Hz, FM radio waves have frequencies near 10^8 Hz, radar waves have frequencies near 10^{10} Hz, light waves have frequencies near 5×10^{14} Hz, and x-rays have frequencies near 10^{17} Hz or above.

Understanding ($\nu = 1/T$) and ($V = \lambda\nu$) (Cap. 2)

C-1 *Properties of sinusoidal waves:* At a particular point, the component E_y of the electric field (along the $\hat{y}$ direction) of a sinusoidal AM radio wave (traveling along the $\hat{x}$ direction perpendicular to $\hat{y}$) varies with the time in the manner indicated in Fig. C-5. At the time t_0, the field E_y assumes its minimum value of -10^{-4} volt/meter. The time elapsed between t_0 and the time when E_y assumes its next maximum value is 0.50 μs (where 1 μs = 1 microsecond = 10^{-6} second). (a) What is the period of

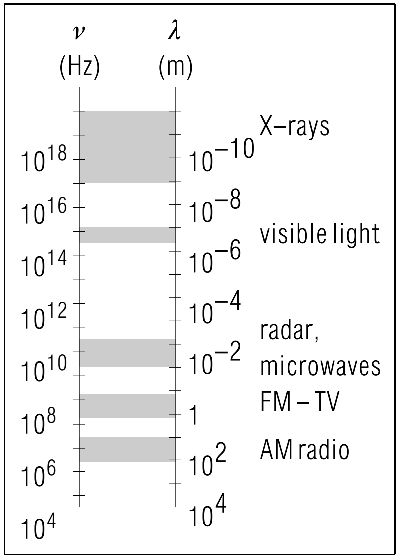


Fig. C-4: Frequencies and corresponding wavelengths (in vacuum) of various kinds of electromagnetic waves.

this radio wave? (b) What is the frequency of this wave? (c) What is the amplitude of this wave? (d) What is the value of E_y 0.75 μs after the time t_0 and 1.50 μs after the time t_0? (e) What is the wavelength of this radio wave? (f) At a fixed instant of time, what is the distance along the $\hat{x}$ direction between a point where E_y is maximum and the next point where E_y is minimum? *(Answer: 12) (Suggestion: [s-6])*

C-2 *Sound waves in air:* The sound wave producing the tone having the pitch of a violin "A-string" has a frequency of 440 Hz. (a) What is the period of this sound wave? (b) What is the wavelength of this sound wave in air and in water? (The speed of a sound wave is 340 m/s in air and 1500 m/s in water.) (c) Is the perceived pitch of this sound wave in water higher, lower, or the same as in air? (d) The lowest and highest frequencies of audible sound are 20 Hz and 20×10^3 Hz. What

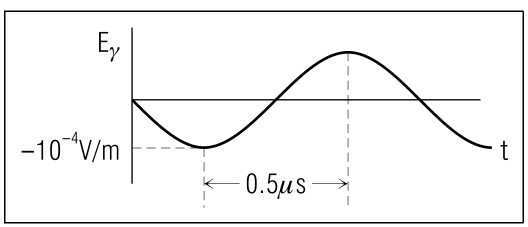


Fig. C-5.

are the corresponding smallest and largest wavelengths of audible sound waves in air? (e) Is the smallest wavelength of audible sound larger than, about equal to, or smaller than the size of an atom (10^{-10} m)? the size of a bacterium (about 10^{-6} m)? the size of the human eardrum? *(Answer: 8)*

C-3 *Light waves:* Yellow light (of the kind emitted by a sodium lamp) has a wavelength of approximately 5.9×10^{-7} m in vacuum. (a) What is the frequency of such a light wave? (b) The frequency of a wave (i.e., the number of repetitions per second) is the same at any point, irrespective of the medium in which this point may be located. Thus the frequency of the yellow light passing through glass is the same as in vacuum. If the speed of light in a glass is 2.0×10^8 m/s, what is the wavelength of this light in the glass? (c) Is the wavelength of visible light (such as yellow light just discussed) larger than, about equal to, or smaller than the size of an atom? the size of a bacterium? the size of the pupil of the human eye? *(Answer: 13)*

C-4 *Observations with sound waves:* As we shall discuss later, waves can only be used to make detailed observations of an object if the wavelength is smaller than the object. Thus visible light, having a wavelength in vacuum of about 5×10^{-7} m, can be used to make observations of bacteria (which are about 10^{-6} m in size). (a) Suppose that one wants to use *sound* waves, having in water the same wavelength of 5×10^{-7} m, to make similar observations of bacteria. What would have to be the frequency of these sound waves if the speed of sound in water is 1500 m/s? (b) Is the frequency of these waves larger than, about equal to, or smaller than the frequency of audible sound waves? *(Answer: 11)*

More practice for this Capability: [p-3], [p-4], [p-5]

SECT.
D WAVEFRONTS

Elastic waves along a stretched string or spring travel simply along a line. But most waves (such as sound waves or electromagnetic waves) are more complex because they travel through three-dimensional space. Hence we should like to introduce a simple way to describe and visualize such waves.

► *Def. of wavefront*

To achieve our aim, we can focus our attention on any "wavefront" of a sinusoidal wave:

Def. **Wavefront**: A wavefront is a set of adjacent points along which a wave at a specified time, has everywhere the same value. (D-1)

Most conveniently, we may consider a set of adjacent points where the wave has its *maximum* value. (Such a wavefront can be indicated on a diagram by a *solid* line.) Similarly, we may consider a set of adjacent points where the wave has its *minimum* value. (Such a wavefront can be indicated on a diagram by a *dashed* line.)

► *Waves on a Surface*

As a simplest example of wavefronts, consider the familiar case of waves on the horizontal surface of water. Suppose that a sinusoidal wave is produced by a source (such as a finger moving up and down through the surface at some point). Then sinusoidal waves move outward from this source, traveling along the surface of the water with the same constant speed V in all directions. At any time t_0, the wavefronts are then concentric circles on the surface of the water, as illustrated in Fig. D-1a. Here the solid lines indicate the "crests" of the waves (i.e., wavefronts along which the displacement of the water above the undisturbed surface is maximum.) Similarly, the dashed lines indicate the "troughs" of the wave (i.e., wavefronts along which the displacement of the water above the undisturbed surface is minimum, so as to be of maximum magnitude but *negative).*

The distance between neighboring wavefronts corresponding to maxima of the wave (or between neighboring wavefronts corresponding to minima of the wave) is just equal to the wavelength λ of the wave.

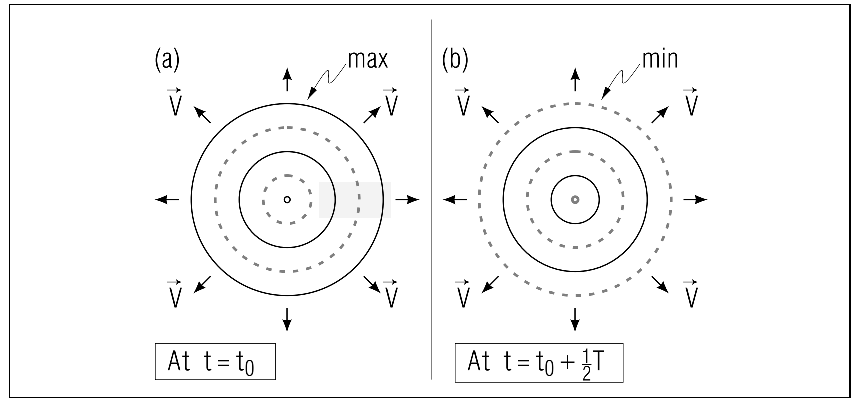


Fig. D-1: Wavefronts of a wave emanating from a small source. The velocity $\vec{V}$ of the waves is everywhere outward, perpendicular to the wavefronts.

► *Motion of wavefronts*

After a time $T/2$, where T is the period, each of the wavefronts in Fig. D-1a has moved outward by a distance $\lambda/2$. The results are the wavefronts indicated in Fig. D-1b. This figure shows properly that, after the time $T/2$, the wave at any point reverses its sign. Thus, at every point where the wave is maximum in Fig. D-1a, the wave is minimum in Fig. D-1b, and vice versa.

WAVES IN SPACE

► *Motion of wavefronts*

Consider a sinusoidal wave which travels in space away from some small source (e.g., a sound wave traveling in air away from a tuning fork). If the speed of the wave is the same in all directions, the wavefronts at any time t_0 are then simply concentric *spherical* surfaces, as indicated in Fig. D-1 (if this figure is interpreted as a two-dimensional representation of a three-dimensional situation).*

* For example, in the case of a sound wave, the wavefronts indicated by solid lines may represent spherical surfaces where the compression in the air is maximum, while the wavefronts indicated by dashed lines may represent spherical surfaces where the compression is minimum.

Such a wave, whose wavefronts are spherical surfaces, is called a "spherical

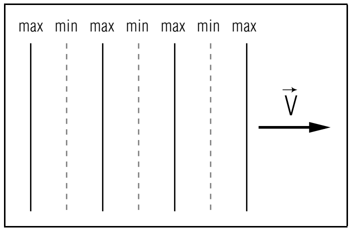


Fig. D-2: Wavefronts of a plane wave. (Each line represents a plane perpendicular to the paper.)

wave."

In the course of time, the wavefronts in Fig. D-1a move outward. For example, after a time $T/2$, the wavefronts have moved outward a distance $\lambda/2$, as shown in Fig. D-1b.

► *Plane wave*

A very simple kind of a wave in space is a "plane wave," i.e., a wave whose wavefronts are planes, as shown in Fig. D-2. (For example, a plane sound wave might be generated in the air adjacent to a large flat plate vibrating back and forth.) Note that, in a limited region of space (such as the gray region indicated in Fig. D-1a) a spherical wave approximates a plane wave since a spherical wavefront is nearly flat in a region of sufficiently small lateral extent.

Knowing About Wave Fronts

D-1 A sound wave emanates from a tuning fork and travels in air where the speed of sound is 340 m/s. Figure D-3 indicates the spherical wavefronts of this wave at a particular time t_0.

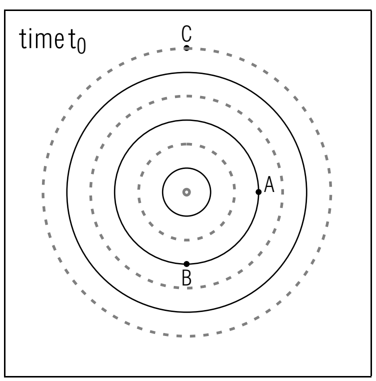


Fig. D-3.

Here the solid lines indicate locations where the excess density of the air (i.e., the actual density minus the normal density) is maximum, while the dashed lines indicate the locations where the excess density is minimum. The distance between a solid line and an adjacent dashed line is 0.34 m. (a) What is the wavelength of this wave? (b) What is the frequency of this wave? (c) What is the period of this wave? (d) At the time t_0, the excess density at the point A is ρ_0. What then is the excess density at the point B in Fig. D-3? (e) At the time t_0, is the excess density at the point C maximum or minimum? (f) At a time 1.0×10^{-3} second after the time t_0, what is the excess density at the point B? Is the excess density at the point C then maximum or minimum? (g) At a time 1.5×10^{-3} second after the time t_0, what is the excess density at the point B? What is the excess density at the point C? *(Answer: 15)*

SECT.

E INTENSITY

The work done by a source in producing a wave is converted into energy transported by the wave moving away from the source. The wave arriving at some detector can then give up its energy to do work on the detector.

► *Def. of intensity*

To describe the energy transported by a sinusoidal wave moving through space, consider at any point P a small surface, of area $\mathcal{A}$, perpendicular to the velocity of the wave at this point. Then a certain amount of energy *per unit time* (or power $\mathcal{P}$) is transported through this area as a result of the wave moving through this area. This power $\mathcal{P}$ fluctuates in time, since the wave varies sinusoidally with time, but has some non-zero average value $\bar{\mathcal{P}}$ if the amplitude of the wave is non-zero.*

> * The average value of a sinusoidal wave is zero since the wave varies in time so as to be as often positive as negative. Correspondingly, the average values of both the sinusoidal force, and of the sinusoidal displacement produced by the wave, are also zero. But, whenever the force changes its direction, the displacement also changes its direction. Hence the *work* done by the wave has always the *same* sign. Therefore the average value of the work, and thus also that of the power, is *not* zero.

The ratio $\bar{\mathcal{P}}/\mathcal{A}$ is independent of the area $\mathcal{A}$ of the surface (if this surface is small enough). This ratio is called the "intensity" I of the wave.

Def. **Intensity**: $I = \dfrac{\bar{\mathcal{P}}}{\mathcal{A}}$ (E-1)

In other words, the intensity I of a wave at a point P is the average power, per unit area, passing through a small surface located at P and perpendicular to the velocity of the wave.

► *Examples*

If light arriving at an eye has larger intensity, it is perceived as having greater "brightness." Similarly, if sound arriving at an ear has larger intensity, it is perceived as having greater "loudness." Sound with

an intensity of 10^{-12} watt/meter^2 is barely audible to the human ear, while sound with an intensity larger than 1 watt/meter^2 is perceived as painfully loud.

► *Measurement of I*

To measure the intensity I at some point P, one can place at this point a detector, of small area $\mathcal{A}$, which absorbs all of the energy of the wave incident on the surface of the detector (so that none of the wave is reflected back from the detector, nor transmitted into the region behind the detector). The average power of the wave arriving at the detector is then just equal to the average work per unit time done by the wave on the particles in the detector.

► *Intensity and amplitude*

How does the intensity of a sinusoidal wave depend on its amplitude? Suppose that we are interested in the intensity I of the wave at some point P where the sinusoidal wave has an amplitude A. This intensity I is then proportional to the average work per unit time done on the particles in a detector placed at P. Suppose now that the amplitude A of the wave were 3 times as large. At any instant, the magnitude of the force produced by the wave on a particle in the detector would then also be 3 times as large. Similarly, the magnitude of the displacement produced by the wave on this particle during some short time would also be 3 times as large. During any short time the work done (obtained by multiplying the magnitude of the force by the component of the displacement along the force) would then be $3 \times 3 = 9$ times as large. Thus we see that the intensity I is proportional to the *square* of the amplitude, i.e., we can write

$$\boxed{I = \gamma A^2} \qquad \text{(E-2)}$$

where γ is a constant which does not depend on A (although it may depend on the properties of the medium and on the frequency of the wave).*

> * If a wave is described by several numbers w (e.g., by the several components of a vector), its intensity is given by a sum of terms like Eq. (E-2), each involving the square of a different amplitude. We shall ignore such complications throughout the following unit.

INTENSITY AND CONSERVATION OF ENERGY

► *Energy dissipation*

As a wave travels through a material medium, some of the energy associated with the wave may gradually be "dissipated," i.e., it may gradually be transformed into increased random internal energy of the medium (so that the temperature of the medium increases). In many cases, such energy dissipation is negligibly small. In particular, when electromagnetic waves travel through a *vacuum*, the energy associated with the wave cannot be given to any particles since none are present. Hence an electromagnetic wave traveling in a vacuum retains its energy, without any dissipation.

► *Spherical wave*

Consider a small source emitting a sinusoidal spherical wave which travels outward in all directions without any dissipation of energy (see Fig. D-1.) By conservation of energy, the average power $\mathcal{P}_s$ emitted by the source must then be equal to the average power passing outward through any spherical surface centered at the source. This average power, passing uniformly through all parts of such a surface of radius R (or corresponding area $4\pi R^2$) then produces at any point of this surface an intensity I which is, by Def. (E-1), equal to

$$\boxed{I = \frac{\bar{\mathcal{P}}_s}{4\pi R^2}} \qquad \text{(E-3)}$$

Thus the intensity I is smaller at larger distances from the source (since the same average power in the wave is then distributed over a larger area). In particular, I decreases so that it is inversely proportional to the square of the distance. For example, at a distance 3 times as far from the source, the intensity of the wave is only $1/3^2 = 1/9$ as large.*

* Since $I = \gamma A^2$ by Eq. (E-2), the amplitude A of the wave must then decrease proportionately to $\sqrt{I}$, i.e., proportionately to $1/R$.

If some of the energy of the wave is also dissipated to the medium as the wave travels through it, the intensity of the wave decreases with increasing distance even more rapidly than indicated by Eq. (E-3).

► *Plane wave*

Consider the special case of a *plane* wave traveling through a medium with negligible dissipation of energy. (See Fig. D-2.) By conservation of energy, the average power passing through any such plane must then be the same. But since every such plane has the *same* area, the intensity at every such plane must also be the same. Hence the intensity (and correspondingly also the amplitude) of a plane wave remains constant as the wave travels through the medium.

Of course, if there is some dissipation of energy, the intensity (and correspondingly also the amplitude) of the plane wave gradually decreases as the wave travels through the medium.

E-1 *Intensity and energy from the sun:* The intensity of electromagnetic radiation (averaged over day and night) reaching the earth from the sun is 1.34×10^3 watt/meter^2. (a) What then is the average power delivered by the sun to New York City which has an area of 945 km^2? (b) What then is the average solar energy delivered to New York City during one year? (1 year = 3.15×10^7 second) (c) How much larger is this energy than the energy of about 2.5×10^{18} joule consumed by the population of New York City during one year? *(Answer: 17)*

Understanding ($I = \bar{\mathcal{P}}_s/4\pi R^2$)

E-2 *Power emitted by the sun:* The intensity of electromagnetic waves arriving at the earth from the sun is 1.34×10^3 km^2. The average distance of the earth from the sun is 1.49×10^8 km. On the basis of this information, what is the average power of electromagnetic waves emitted by the sun? *(Answer: 14)*

E-3 *Power from a spacecraft:* The Viking spacecraft, sent in 1975 to explore the possibility of life on Mars, sent back information to earth by radio waves emitted by a 16 watt radio transmitter. If these waves are sent out uniformly in all directions and if the distance between the earth and Mars at the time of the exploration is 3×10^8 km, what is the average power of the radio waves arriving at the large receiving antenna in Goldstone, California? (This antenna has the shape of a circular disk with a diameter of 64 m or corresponding area of 3.2×10^3 m^2)? (b) is the power arriving at this antenna large enough to be easily detected? (The Goldstone antenna can detect powers as small as 2×10^{-21} watt.) *(Answer: 19) (Suggestion: [s-10])*

E-4 *Variation of intensity with distance:* Sound emanates uniformly in all directions from a large bell. At a distance of 3 m from the bell, the intensity of the sound is 1 km^2 and is thus painfully loud. How far would one have to be from this bell so that the intensity is 10^{-4} km^2? (Neglect dissipation of energy in the air.) *(Answer: 16) (Suggestion: [s-13])*

More practice for this Capability: [p-6]

Comparing Intensities and Amplitudes

E-5 *Transmitted power and electric field:* The current in the transmitting antenna of a radio station is increased so that the electric field of the radio wave emitted by the antenna is everywhere 3 times as large as before. (a) How much larger than before is the intensity of the radio wave at any point? (b) How much larger than before is the total power emitted by the radio station? *(Answer: 23)*

E-6 *Excess densities of sound waves:* The intensity of a painfully loud sound is 1 km^2, while the intensity of a barely audible sound is 10^{-12} km^2. By what factor is the amplitude of the painfully loud sound larger than the amplitude for the barely audible sound? *(Answer: 21)*

SECT. F SUPERPOSITION

Suppose that a wave emitted by one source produces at some point a disturbance w_1, and that a wave emitted by another source produces at this point a disturbance w_2. What then is the resulting disturbance w produced at this point by both waves present simultaneously?

In the case of an electromagnetic wave, the disturbance traveling as a wave is just an electric or magnetic field. By the superposition principle for fields, the resultant field produced by both waves is then simply the sum of the individual fields produced by these waves. In the case of elastic waves, or waves on the surface of a liquid, the disturbance traveling as a wave is a displacement. The resultant displacement produced by both waves is then again simply the sum of the individual displacements produced by these waves (as long as these displacements are not too large). Similar statements hold then for any numerical component of these disturbances. Thus we conclude that:

$$\boxed{w = w_1 + w_2\,.} \qquad \text{(F-1)}$$

This conclusion can be stated in words:

> Superposition principle for waves: At any point, the wave resulting from several waves presented simultaneously is equal to the sum of the individual waves. (F-2)

IMPLICATIONS FOR INTENSITY

Up to now we have focused attention on the moving disturbances, i.e., on the waves themselves. Let us now consider the energies associated with these disturbances, i.e., the *intensities* of the waves. Is it true that the intensity I resulting from the simultaneous presence of two waves is equal to the sums of the intensities I_1 and I_2 of the individual waves?

► *Resultant intensities*

The answer to this question is definitely *no*. As a particular example, consider at some point P two sinusoidal waves w_1 and w_2 of the same frequency, each wave having the same amplitude A_1 and thus correspondingly the same intensity $I_1 = \gamma A_1^2$. Suppose that both of these waves have the same phase (so that the maxima of both waves occur at

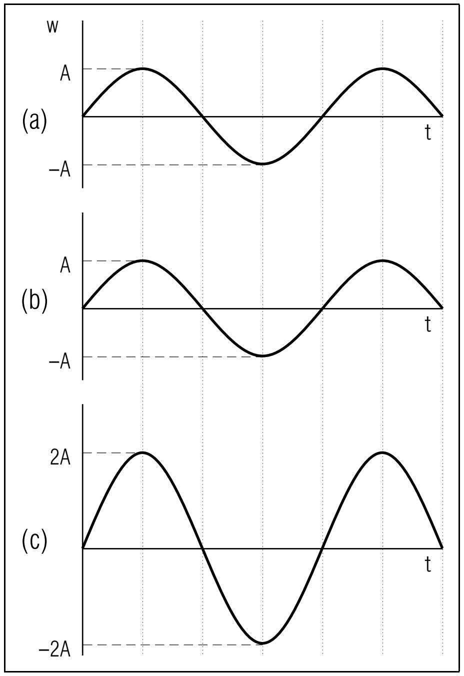


Fig. F-1: (a), (b) Two waves with the same amplitude and the same phase. (c) Resultant wave equal to the sum of these waves.

the same time, as indicated in Fig. F-1.) At any instant, the waves have then the same value $w_2 = w_1$. By the superposition principle Eq. (F-1), the value of the resultant wave is then at any instant $w = 2w_1$, so that the amplitude of the resulting wave is correspondingly $A = 2A_1$ (see Fig. F-1c.) Hence the intensity I of the resultant wave is:

$$I = \gamma A^2 = \gamma(2A_1)^2 = 4\gamma A_1^2 = 4I_1 \qquad \text{(F-3)}$$

which is twice as large as the sum $2I_1$ of the intensities I_1 of each of the two individual waves.

As another even more striking example, consider the same two waves as before, but suppose that there is a phase difference of one-half cycle between the waves (so that the maximum of one wave occurs at the same time as the minimum of the other wave, as indicated in Fig. F-2.) At any instant, the waves have then opposite values so that $w_2 = -w_1$. By the superposition principle (see Fig. F-1), the resultant wave has then at any instant a value equal to $w = 0$. In other words, the two waves cancel each other completely so as to give rise to a resultant wave with amplitude $A = 0$ and corresponding intensity $I = 0$. (See Fig. F-2c.) This

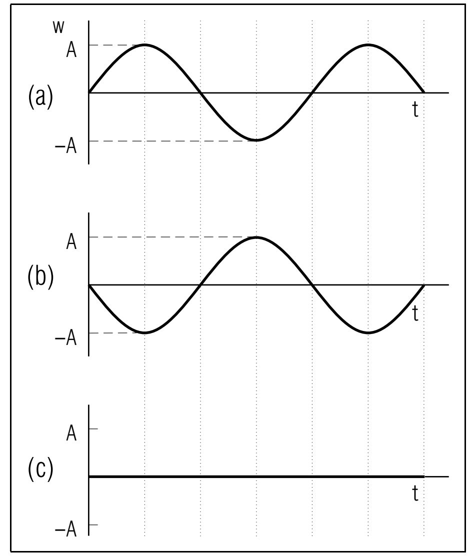


Fig. F-2: (a), (b) Two waves with the same amplitude and with phases differing by one-half cycle. (c) Resultant wave equal to the sum of these waves.

zero intensity of the resultant wave is obviously not only smaller than the sum 2 I_1 of the intensities of the individual waves, but smaller than the intensity I_1 of either one of these individual waves!

► *Interference*

Thus the superposition principle Eq. (F-1) or Rule (F-2) for waves does *not* imply a corresponding superposition principle for the intensities of these waves. Indeed, the intensity I of the resultant wave is ordinarily different from the sum of the intensities of the individual waves, i.e.,

$$I \neq I_1 + I_2 \qquad \text{(F-4)}$$

Thus one can ordinarily expect to observe "interference" between two waves, in this sense:

Def. **Interference**: A situation where the resultant intensity of several waves present simultaneously is different from the sum of the intensities of the individual waves. (F-5)

In particular, the interference is said to be "constructive" if the resultant intensity I is *larger* than the sum of the individual intensities (as in the case of Fig. F-1). Conversely, the interference is said to be "destructive" if the resultant intensity I is *smaller* than the sum of the individual intensities (as in the case of Fig. F-2).

The superposition principle for waves, and the resulting possibility of interference between waves, is the most striking property of all waves. This property leads to remarkable conclusions of the greatest importance. Indeed, we shall spend the next several units discussing the phenomenon of interference and some of its many practical implications.

Superposition Principle for Waves

F-1 Two sinusoidal waves have the same frequency, but the amplitude A_1 of the first wave is 3 times as large as the amplitude of the second wave, so that $A_1 = 3A_2$. To compare the intensities of these waves, express the intensity I_1 of the first wave in terms of the intensity I_2 of the second wave. *(Answer: 24) (Suggestion: [s-15])*

F-2 *Waves in phase:* Suppose that the two waves described in the preceding problem are present simultaneously at the same point and have the same phase. (The relationship between the phases of the waves is then the same as that illustrated in Fig. F-1.) (a) Express the amplitude A of the resultant wave in terms of the amplitude A_2 of the second wave. (b) Express the intensity I of the resultant wave in terms of the intensity I_2 of the second wave. (c) Is the intensity I of the resultant wave equal to, larger than, or smaller than the sum $(I_1 + I_2)$ if the intensities of the two individual waves? To make the comparison quantitative, find the ratio $I/(I_1 + I_2)$. *(Answer: 20) (Suggestion: [s-12])*

F-3 *Waves 1/2 cycle out of phase:* Suppose now that the two waves described in Problem F-1 are present simultaneously at the same point, but differ in phase by 1/2 cycle. (The relationship between the phases of the waves is then the same as that illustrated in Fig. F-2.) For this case, answer the same questions *a*, *b*, and *c* of the preceding Problem F-2. *(Answer: 18)*

SECT.
G SUMMARY

DEFINITIONS

wave; Def. (B-1)

amplitude; Def. (C-2)

phase; Def. (C-3)

period; Def. (C-4)

frequency; Def. (C-5)

hertz; Eq. (C-7)

wavelength; Def. (C-8)

wavefront; Def. (D-1)

intensity; Def. (E-1)

interference; Def. (F-5)

IMPORTANT RESULTS

Relation between frequency and period: Eq. (C-6)

$\nu = 1/T$.

Relation between speed, wavelength and frequency: Eq. (C-9), Eq. (C-10)

$V = \lambda/T = \lambda\nu$

Relation between intensity and amplitude: Eq. (E-2)

$I = \gamma A^2$

(where γ is some constant).

Intensity of spherical wave at distance R from a source: Eq. (E-3)

$I = \bar{\mathcal{P}}_s/4\pi R^2$

(where $\bar{\mathcal{P}}_s$ is the average power emitted by a source).

Superposition principle for waves: Eq. (F-1), Rule (F-2)

$w = w_1 + w_2$

USEFUL KNOWLEDGE

Types of waves (elastic, liquid surface, electromagnetic.) (Sec. B)

Properties of sinusoidal waves. (Sec. C)

Frequencies of audible sound and visible light. (Sec. C)

Wave fronts for spherical and plane waves. (Sec. D)

NEW CAPABILITIES

(1) Knowing the velocity of a wave, use information about the wave at a given position or a given time to find the wave at some other position or other time. (Sects. A, D; [p-1], [p-2])

(2) Understand the relations $\nu = 1/T$ and $V = \lambda/T = \lambda\nu$ relating the wavelength λ, frequency ν (or period T), and speed V of a sinusoidal wave. (Sec. C; [p-3], [p-4], [p-5])

(3) Understand the relation $I = \bar{\mathcal{P}}_s/4\pi R^2$ relating the intensity I of a spherical wave at a distance R from a source emitting an average power $\bar{\mathcal{P}}_s$. (Sec. E; [p-6])

(4) Compare the intensities and amplitudes of two sinusoidal waves. (Sec. E)

(5) Use the superposition principle to relate the values of two or more individual waves to the value of the resultant wave. (Sec. F)

G-1 *Pitch and wavelength:* The pitch of a first tone is perceived to be an "octave" higher than the pitch of a second tone if the frequency of the sound wave producing the first tone is twice as large as the frequency of the sound wave producing the second tone. (a) What then is the relationship between the wavelength λ_1 of the first sound wave compared to the wavelength λ_2 of the second sound wave? (b) Does the answer to the preceding question depend on whether the sound waves are traveling in air, water, or any other medium? *(Answer: 26)*

G-2 *Wavelengths of radio waves:* The radio waves sent out by AM stations have frequencies near 1 MHz, while the radio waves sent out by FM stations have frequencies near 100 MHz. (1 MHz = 1 megahertz = 10^6 Hz.) (a) What then are the typical wavelengths of AM and FM radio waves? (b) Are these wavelengths larger than, smaller than, or about equal to the size of a transistor radio? *(Answer: 22)*

G-3 *Properties of sinusoidal waves:* Two waves of the same frequency and amplitude travel in the same direction. How do these waves differ if the difference between the phases of these waves is (a) (1/2) cycle? (b) 1 cycle? (c) (3/2) cycle? *(Answer: 28)*

SECT.

H PROBLEMS

H-1 *Speed of waves along a string:* The speed V of waves traveling along a stretched string depends on the magnitude F_t of the tension force in the string and on the mass per unit length m' of the string. (a) What are the SI units of the quantities V, F_t, and m'? (b) How must V be related to the other two quantities so that the units in this relation are consistent? (this relation is unambiguously determined except for some constant C without any units.) (c) Suppose that two strings have the *same* mass per unit length, but that the magnitude of the tension force in the first string is twice as large as that in the second string. What then is the ratio V_1/V_2 of the speed V_1 of the wave along the first string compared to the speed V_2 of the wave along the second string? (d) Suppose that two strings are subjected to the same tension force, but that the mass per unit length of the first string is twice as large as that of the second string. What then is the ratio V_1/V_2? *(Answer: 30)*

H-2 *Wave number:* Consider a wave traveling along a particular direction. The "wave number" b of this wave is then, at any particular time, the number of waves per unit length (e.g., the number of waves per meter) along the direction of travel of the wave. How is the wave number b related to the wavelength λ *(Answer: 27) (Suggestion: [s-14])*

H-3 *Simple way of measuring the speed of sound:* To measure the speed of sound, a person stands at a distance L in front of a large flat wall and claps his hands regularly at a rate of N claps per unit time such that each echo (produced by the sound reflected back from the wall) is heard by the person precisely halfway between every two successive claps. To show that this information is sufficient to deduce the speed V of sound, express V in terms of L and N. *(Answer: 32) (Suggestion: [s-17])*

H-4 *Decibel loudness scale:* One can compare the intensity I of any sound with some standard intensity I_0 which is conventionally chosen to be equal to $I_0 = 10^{-12}$ watt/m^2 (i.e., the intensity corresponding to barely audible sound). A convenient measure of loudness is then given by the logarithm (to the base 10) of the intensity ratio I/I_0. Thus the loudness β of a sound (measured in "decibels," abbreviated as "dB") is *defined* as $\beta = 10\,\log(I/I_0)$. [The decibel is named in honor of Alexander

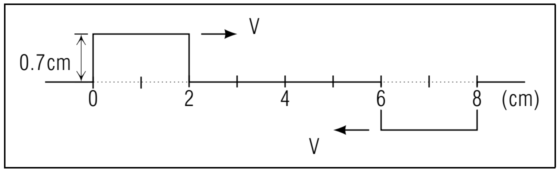


Fig. H-1.

G. Bell, the inventor of the telephone.] (a) What then is the intensity, expressed in dB, of a sound having the intensity I_0 of barely audible sound? (b) What is the intensity, expressed in dB, of a sound having an intensity 100 times larger than I_0? (c) What is the intensity, expressed in dB, of a sound with an intensity $I = 1\,\text{watt}/m^2$ at the threshold of pain? (d) The intensity of sound of busy street traffic is about 70 dB. What is the corresponding intensity expressed in watt$/m^2$? *(Answer: 29)*

H-5 *Waves moving in opposite directions:* Figure H-1 shows, at a particular time t_0, two rectangular displacement waves moving with speeds of 5.0 m/s in opposite directions along a stretched string. Draw sketches showing the resultant shape of the string 4, 5, 6, 7, 8, and 10 millisecond after the time t_0. *(Answer: 34)*

Note: Tutorial section H contains additional problems on the "Doppler effect."

TUTORIAL FOR H

Additional Problems

h-1 **DOPPLER SHIFT: MOVING SOURCE:* A wave emitted from a source at a time t_1 arrives at a detector at a time t'_1. A second wave emitted from the source at a time t_2 arrives at the detector at a time t'_2. When both the source and the detector are at rest relative to the medium in which the wave is traveling, the time $T' = t'_2 - t'_1$ between the *arrival* of the two waves at the detector is equal to the time $T = t_2 - t_1$ between the *emission* of the waves from the source. But if either the source or the detector is moving relative to the medium, T' is different from T. The existence of this difference is called the "Doppler effect."

For example, suppose that the source moves *toward* the detector with a constant speed v. Then the distance traveled by the first wave is larger than the distance traveled by the second wave by an amount equal to the distance traveled by the source toward the detector during the time $t_2 - t_1$ between the emission of the two waves. (a) Express the preceding relation between the distances as an equation in terms of v, the speed V of the wave in the medium, and the various times. (b) Solve this relation to express the time T' between the arrivals of the waves in terms of the time T between the emission of the waves and the speeds v and V. Is T' larger or smaller than T? (c) Suppose that t_1 and t_2 are the times corresponding to two successive maxima of a sinusoidal wave emitted by the source, and that t'_1 and t'_2 are the times corresponding to the arrivals of these maxima at the detector. Then part (b) gives the relation between the period T of the sinusoidal wave emitted by the source and the period t' of the wave arriving at the detector. What then is the relation between the frequency ν' of the wave registered at the detector and the frequency ν of the wave emitted by the source? Is ν' larger or smaller than ν? (d) What would be the answers to the preceding questions (a), (b), and (c) if the source moved away from the detector with a speed v? (e) If a train blowing its whistle approaches a stationary observer, is the pitch of the whistle heard by the observer higher or lower than the pitch heard if the train were stationary? What is the answer to this question if the train is moving away from the stationary observer? *(Answer: 59) (Suggestion: s-16)*

h-2 **DOPPLER SHIFT: MOVING DETECTOR:* Consider again the situation described in the preceding frame [h-1], but for the case where

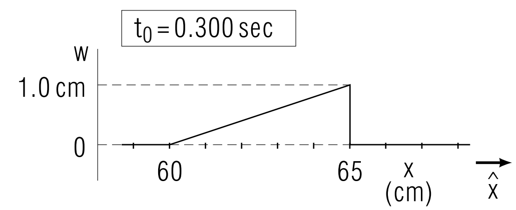
th source is stationary relative to the medium while the *detector* moves *toward* the source with a constant speed v. Then the distance traveled by the first wave is larger than the distance traveled by the second wave by an amount equal to the distance traveled by the detector during the time $t'_2 - t'_1$ between the arrival of the two waves at the detector. (a) Express this relation between the distances as an equation in terms of v, the speed V of the wave in the medium, and the various times. (b) What then is the answer to part (b) of frame [h-1]? (c) What then is the answer to part (c) of frame [h-1]? (d) What would be the answers to the preceding questions (a), (b), and (c) if the detector moved *away* from the source with a speed v? *(Answer: 61)*

h-3 **DOPPLER SHIFT: REFLECTION FROM A MOVING OBJECT:* An object moves toward a source of waves with a speed v relative to a medium in which the speed of the waves is V. A first wave, leaving the source at a time t_1, then arrives at the moving object at a time t'_1 when it is reflected and then travels back to the source so as to arrive there at a time t''_1. Similarly, a second wave, leaving the source at a time t_2, arrives at the moving object at a time t'_2 when it is reflected and then travels back to the source so as to arrive there at a time t''_2. (a) Write an equation expressing the fact that in traveling toward the moving object, the distance traveled by the first wave is larger than that traveled by the second wave by an amount equal to the distance traveled by the moving object toward the source during the time $T' = t'_2 - t'_1$. Express this relation in terms of v, V, and the elapsed times $T = t_2 - t_1$ and T'. (b) Write an equation expressing the fact that, in traveling back toward the source after reflection, the distance traveled by the first wave is also larger than that traveled by the second wave by an amount equal to the distance traveled by the moving object toward the source during the time $T' = t'_2 - t'_1$. Express this relation in terms of v, V, T' and the elapsed time $T'' = t''_2 - t''_1$ between the arrival of the waves back to the source. (c) Use the preceding relations to express T'' in terms of T and the speeds v and V. (d) In the case of sinusoidal waves, suppose that T and T'' are the periods (or times between two successive maxima) of the waves emitted by the source and arriving back at the source. What then is the relation between the frequency ν' of the waves arriving back at the source and the frequency ν of the waves originally emitted by the source? *(Answer: 63)*

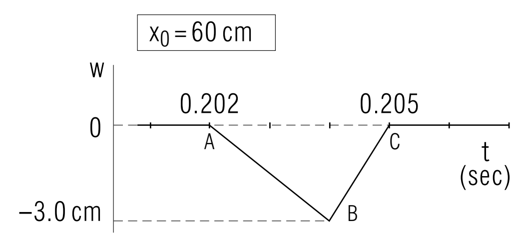
h-4 **DOPPLER DETECTION OF A FETAL HEARTBEAT:* Ultrasonic waves can be useful to observe the fetal heartbeat of an unborn child at an early stage of the gestation period, even when the fetus is no more than 3 months old. To show how this can be done, imagine that an ultrasonic wave, having a frequency of 5.00×10^6 Hz, is sent through the skin of the mother's abdomen and then reflected by the moving heart muscle of the fetus. Suppose that the heart wall moves toward the abdomen skin with a speed of 7 cm/s and that the speed of sound in the abdomen is 1500 m (i.e., the same as that in water). (a) In this case (using the results obtained in frame [h-3] is the frequency of the reflected ultrasonic waves larger or smaller than that of the original ultrasonic waves? (b) What is the change $(\nu'' - \nu)$ in the observed frequency ν'' of the reflected ultrasonic wave compared to the frequency $\nu = 5.00 \times 10^6$ Hz of the original ultrasonic wave sent into the abdomen? *(Answer: 60)*

PRACTICE PROBLEMS

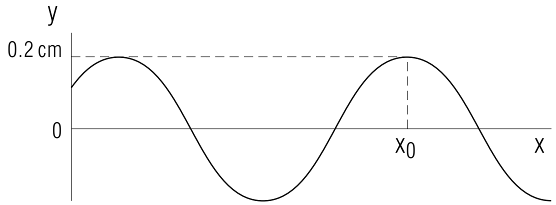
p-1 *MOTION OF A WAVE:* A displacement wave moves along a long stretched string with a velocity of 4.0×10^3 cm/s *opposite* to the $\hat{x}$ direction. The graph shows, at the particular time $t_0 = 0.300$ s, the wave (described by the component w of the displacement of the string along the $\hat{y}$ direction perpendicular to $\hat{x}$) at various positions (described by the position coordinate x along the $\hat{x}$ direction). Draw the graph showing the wave at various positions along the string at the later time $t = 0.305$ s. *(Answer: 55) (Suggestion: Review text problem A-1.)*



p-2 *MOTION OF A WAVE:* A wave travels with a velocity of 4.0×10^3 cm/s opposite to the $\hat{x}$ direction along the stretched string described in practice problem [p-1]. The displacement component w of the string, observed at the *fixed* position $x_0 = 60$ cm, then varies with the time t in the manner indicated in the graph. We are interested in finding the position of this wave at the particular time $t_1 = 0.210$ s. (a) Consider the part A of the string (where $w = 0$ at the time 0.202 s). Through what distance does this part of the wave travel until the time $t_1 = 0.210$ s? What then is the position of the part of the wave at the time t_1? (b) Answer the same questions for parts B and C of the string. (c) To show the positions of the various parts of the wave at the time t_1, draw a graph of w versus x at the time t_1. *(Answer: 52) (Suggestion: Review text problem A-2.)*



p-3 *RELATION BETWEEN V, λ, AND ν: Properties of sinusoidal waves*: The graph shows the component y of the transverse displacement of a wave along a string at various positions (described by the coordinate x along the string). This wave, which has a frequency of 25 Hz, travels to the right with a velocity of 1500 cm/s. At the particular time t_0 indicated in the graph, y has its maximum value of 0.2 cm at the point x_0. (a) What is the value of y at the point x_0, 0.03 second after the time t_0? (b) At the time t_0, what is the value of y at a point 150 cm to the right of the point x_0? *(Answer: 51) (Suggestion: See [s-11] and review text problem C-1.)*



p-4 *RELATION BETWEEN V, λ, AND ν: Useful electromagnetic waves* (a) Electromagnetic waves commonly used for radar have a wavelength (in vacuum) of 3.0 cm. What is the frequency of these waves? (b) "Diathermy" machines employed in medicine for heating tissues well below the skin use electromagnetic waves having a frequency of about 2.5×10^9 Hz. What is the wavelength of these waves in vacuum? *(Answer: 56)*

p-5 *RELATION BETWEEN V, λ, AND ν: Speed of water waves*: An electric tooth brush, held so as to touch the surface of water in a basin, vibrates up and down at a rate of 10 vibrations per second. Water waves are then observed to spread out in all directions from the vibrating toothbrush. The distance between the successive crests of these water waves is observed to be 3 cm. What then is the speed of the waves traveling on the water surface? *(Answer: 54)*

p-6 *INTENSITY AND EMITTED POWER:* As mentioned in test problem E-3, the Viking spacecraft had a radio transmitter emitting 16 watts of power. (a) Assuming that this power is emitted uniformly in all directions, how far from the earth could this spacecraft be and still be detected by the Goldstone radio antenna? This antenna has a diameter of 64 m (or area of $3.2 \times 10^3\,\text{m}^2$) and can detect a power as small as

2×10^{-21} watt. (b) How much larger is this distance than the distance of 1.5×10^8 km between the sun and the earth? *(Answer: 53) (Suggestion: See [s-8] and text problem E-3.)*

SUGGESTIONS

s-1 *(Text problem B-1):* Part c: As a result of the displacement of atom 3, there are more atoms than normal in the region between x_3 and x_5 (i.e., the solid in this region has been compressed). Hence the mass contained in the volume of this region has become larger so that the density (or mass per unit volume) in this region has become larger.

s-2 *(Text problem A-3):* Figure A-3 indicates that a particle (located at $x_0 = 20\,\text{cm}$) moves during the time from 0.104 s to 0.106 s at a uniform rate so that its component of displacement along the $\hat{y}$ direction (perpendicular to the string) is $w = 0$ at the earlier time and $w = 3.0\,\text{cm}$ at the later time. What then is the velocity of the particle at the time 0.105 s?

s-3 *(Text problem A-1):* What is the time elapsed between the initial time t_0 and the final time t? During this time, through what distance does any part of the wave in Fig. A-2 travel? At the time $t = 0.072\,\text{s}$, what then is the position of the part of the wave originally located at $x = 33\,\text{cm}$? Similarly, what then is the position of the wave originally located at $x = 34\,\text{cm}$? Use this information to sketch the graph showing the location of the entire wave at the time $t = 0.072\,\text{s}$.

s-4 *(Text problem B-2):* Part a: By the Pythagorean theorem, $L'^2 = L^2 + w^2$. But $w = 0.001\,L$. Hence $w^2 = 0.000001L^2$ is utterly negligible compared to L^2. Thus, to excellent approximation, $L'^2 = L^2$ or $L' = L$.

s-5 *(Text problem A-2):* Part a: The part C of the wave travels from the time $t = 0.106\,\text{s}$ to the time $t_1 = 0.108\,\text{s}$. During this time interval of 0.002 s, the wave (traveling with a velocity of $6.0 \times 10^3\,\text{cm/s}$ along $\hat{x}$) then travels a distance of $(6.0 \times 10^3\,\text{cm/s})(0.002\,\text{s}) = 12\,\text{cm}$ along the $\hat{x}$ direction. Since this part of the wave was initially located at $x_0 = 20\,\text{cm}$, what is the value of x specifying the position of this part of the wave at the later time t_1?

s-6 *(Text problem C-1):* Review the definitions of period, frequency, and wavelength. All questions can then be answered by using the relations $\nu = 1/T$ and $V = \lambda\nu$. Remember that the speed V of a radio wave in vacuum (and thus very nearly in air) is $3.00 \times 10^8\,\text{m/s}$.

s-7 *(Text problem B-3):* Electric and magnetic waves can exist in vacuum as well as in materials. Hence electromagnetic waves can travel in

all media (whether vacuum or material media). In a *vacuum*, the speed of *any* electromagnetic wave is always the same, equal to the fundamental constant $c = 3 \times 10^8\,\text{m/s}$. Light and radio waves are merely different kinds of electromagnetic waves.

s-8 *(Practice problem [p-6]:* What is the minimum *intensity* which the Goldstone antenna can detect? Use this information to find the farthest possible distance of the spacecraft.

s-9 *(Text problem B-4):* A sound wave is an elastic wave involving the displacement of atoms from their normal positions. Such a wave can then *not* exist if there are no atoms which can be displaced, i.e., it *cannot* exist in a vacuum. On the other hand, such a sound wave can exist in *any* material since any material contains atoms which can be displaced from their normal positions.

s-10 *(Text problem E-3):* What is the intensity of the radio waves arriving at the antenna? What then is the average power arriving at the antenna with the specified area?

s-11 *(Practice problem3:* Part a: How is the elapsed time related to the period of the wave?

Part b: How is the distance between the points related to the wavelength of the wave?

s-12 *(Text problem F-2):* Part a: Draw the two waves, in a manner similar to that in Fig. F-1, but remembering that in this case the amplitude A_1 of the first wave is 3 times as large as the amplitude A_2 of the second wave. Remembering that the resultant wave at any time is simply equal to the sum of the individual waves, how then is the amplitude of the resultant wave related to the amplitudes A_1 and A_2 of the individual waves?

Part c: Express the sum $I_1 + I_2$ in terms of I_2, using the answer to problem F-1 where you found how I_1 is related to I_2.

s-13 *(Text problem E-4):* The relation $I = \bar{\mathcal{P}}_s/4\pi R^2$ implies that the product IR^2 = constant for waves emitted by a given source. Thus the intensities I and I' at two different distances R and R' from the source are related so that $IR^2 = I'R'^2$.

s-14 *(Text problem H-2):* Review how the frequency (i.e., the number of waves per unit *time*) is related to the period. The wave number b is related to the wavelength λ in a similar manner.

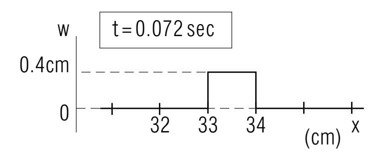
s-15 *(Text problem F-1):* Here $I_1 = \gamma A_1^2 = \gamma(3A_2)^2$ and $I_2 = \gamma A_2^2$. What then is the relation between I_1 and I_2?

s-16 *(Tutorial from H-1):* (a) Express the distance traveled by the first wave in terms of the speed V of the wave and the time $(t_1' - t_1)$ traveled by this wave. (b) Express the distance traveled by the second wave in terms of V and the time $(t_2' = t_2)$ traveled by this wave. (c) Express the distance traveled by the source, during the time $(t_2 - t_1)$ between the emission of the waves, in terms of this time and the speed V of the source. (d) Hence write the relation expressing the fact that the distance traveled by the first wave is larger than the distance traveled by the second wave by an amount equal to the distance traveled by the source between the emission of the waves. (e) Simplify this relation by expressing it in terms of the time $T = t_2 - t_1$ between the emission of the waves and the time $T' = t_2' - t_1'$ between the arrival of the waves at the detector. *(Answer: 57)*

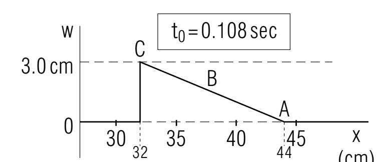
s-17 *(Text problem H-3):* What is the time required for the sound from a clap to travel from the person to the wall and then back again? (Express this result in terms of L and V.) What is the time between a clap and the return of the echo halfway between the next clap? (Express this result in terms of the clapping rate described by N.)

ANSWERS TO PROBLEMS

1. a. $L' \approx L$ (See [s-4])
   b. nearly equal
   c. nearly equal
2. a. 1.5×10^3 cm/s along $\hat{y}$
   b. smaller
3. Yes. Sound can travel through the air in the jar.
   b. Bell can be seen, but not heard. (Light can travel through vacuum, but sound cannot.)
4.



5. a. $L' = 0.999L$
   b. smaller
   c. larger
6. a. yes
   b. yes
   c. 3×10^8 m/s for both waves
   d. yes
7. a. 12 cm, $x = 32$ cm
   b. 18 cm, $x = 38$ cm
   c. 24 cm, $x = 44$ cm
   d.



(graph is correct!)

8. a. 2.27×10^{-3} second

   b. 0.77 m in air, 3.4 m in water

   c. same

   d. 0.017 m, 17 m

   e. much larger than atom or bacterium, about equal to eardrum

9. a. no

   b. yes

   c. yes

   d. yes

10. a. 5.9 second

    b. 6.7×10^{-6} second; $t_l/t_s = 1.1 \times 10^{-6}$

    c. 5.9 seconds

11. a. 3×10^9 Hz

    b. much larger

12. a. 1.00 μs= 1.00×10^{-6} s

    b. 1.00×10^6 Hz

    c. 10^{-4} volt/meter

    d. 0, $+10^{-4}$ volt/meter

    e. 300 meter

    f. 150 meter

13. a. 5.1×10^{14} Hz

    b. 3.9×10^{-7} m

    c. much larger than atom, about equal to bacterium, much smaller than pupil

14. a. 3.74×10^{26} watt

15. a. 0.68 m

    b. 500 Hz

    c. 2.0×10^{-3} s

    d. ρ_0

    e. minimum

    f. $-\rho_0$, maximum

    g. 0,0

16. 3×10^2 m

17. a. 1.27×10^{12} watt

    b. 3.99×10^{19} joule

    c. 16

18. a. $A = 2A_2$

    b. $I = 4I_2$

    c. smaller, 0.4

19. a. 4.5×10^{-20} watt

    b. yes

20. a. $A = 4A_2$

    b. $I = 16I_2$

    c. larger; 1.6

21. 10^6

22. a. 300 m, 3 m

    b. larger

23. a. 9

    b. 9

24. $I_1 = 9I_2$

25. BLK

26. a. $\lambda_1 = \lambda_2/2$

    b. no

27. $b = 1/\lambda$

28. a., c. waves are always opposite

b. waves are the same

29. a. 0

b. 20

c. 120

d. 10^{-5} watt/m^2

30. a. m/s, kg m/s^2, kg/m

b. $V = C\sqrt{F_t/m'}$

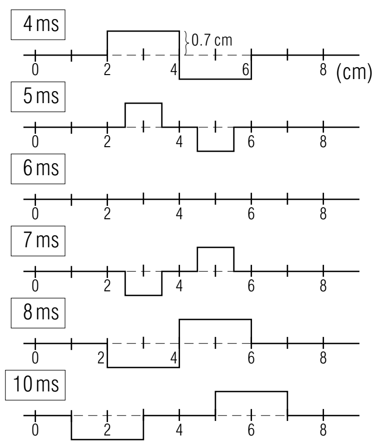
c. $\sqrt{2} = 1.41$

d. $1/\sqrt{2} = 0.707$

31. BLK

32. $V = 4LN$

34.

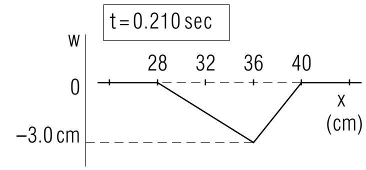


51. a. 0

b. -0.2 cm

52. a. 32 cm (opposite $\hat{x}$), $x = 28$ cm

b. B: 24 cm (opposite $\hat{x}$), $x = 36$ cm C: 20 cm (opposite $\hat{x}$), $x = 40$ cm

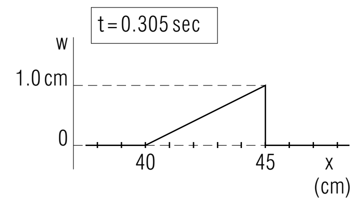


53. a. 1.4×10^{12} m

b. 9.5, but 9.3 is OK.

54. 30 cm/s

55.



56. a. 1.0×10^{10} Hz

b. 0.12 m

57. a. $V(t_1' - t_1)$

b. $V(t_2' - t_2)$

c. $V(t_2 - t_1)$

d. $V(t_1' - t_1) - V(t_2' - t_2) = v(t_2 - t_1)$

e. $V(T' - T) = -vT$

58. BLK

59. a. $V(t_1' - t_1) - V(t_2' - t_2) = v(t_2 - t_1)$

b. $T' = T(1 - v/V)$, smaller

c. $\nu = \nu(1 - v/V)$, larger

d. Preceding answers, with $-v$ replaced by $+v$. Thus $T' > T$ and $\nu' < \nu$.

e. higher, lower

60. a. larger

b. 467 Hz

61. a. $V(t_1' - t_1) - V(t_2' - t_2) = v(t_2' - t_1')$

b. $T' = T/(1 + v/V)$, smaller

c. $\nu' = \nu(1 - v/V)$, larger

d. Preceding answers with v replaced by $-v$ Thus $T' > T$ and $\nu' < \nu$.

62. BLK

63. a. $V(t_1' - t_1 - V(t_2' - t_2) = v(t_2' - t_1)$ or $V(-T' + T) = vT'$

b. $V(t_1'' - t_1') - V(t_2'' - t_2') = v(t_2' - t_1')$ or $V(-T'' + T') = vT'$

c. $T'' = T[(V - v)/(V + v)]$

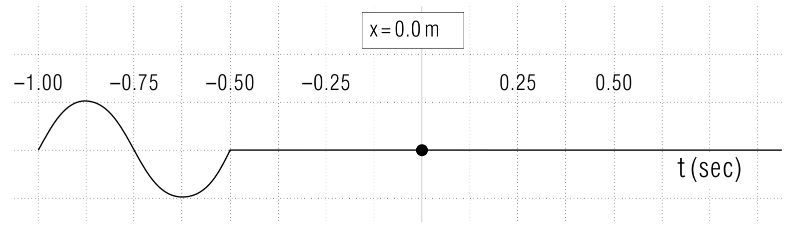
d. $\nu'' = \nu[(V + v)/(V - v)]$

64. BLK

MODEL EXAM

GIVEN INFORMATION: $c = 3.0 \times 10^8$ meter/second

1. **Motion of a wave.** A displacement wave moves along a string with a velocity of 48 meter/second along the $\hat{x}$ direction. The following graph illustrates the displacement component w of the wave, as a function of time, at the fixed position $x = 0.0$ meter.



Draw a graph showing how the displacement component varies with time at the fixed position $x = -12$ meter.

2. **Frequencies and wavelengths of visible light.**

a. What are the approximate frequencies between which electromagnetic radiation is perceived as visible light?

b. What are the wavelengths corresponding to these frequency limits?

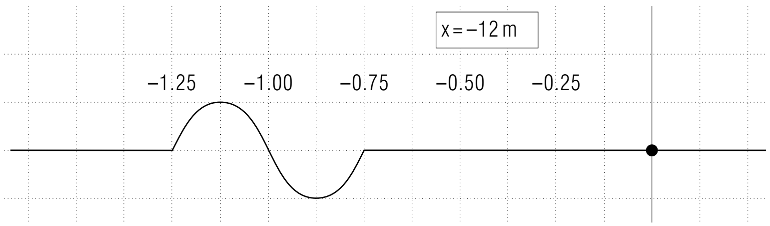
3. **A method of estimating the intensity of solar radiation.** A student points her head toward the sun, with her eyes closed, and notes the approximate brightness of the sun through her *closed* eyelids. Then she goes inside, and finds that she must hold a 200 watt clear lightbulb 0.02 meter from her eye to yield the same approximate brightness.

If only 2.5 percent of the energy consumed by the lightbulb is converted into visible radiation and on the assumption that the sun and the lightbulb are legitimately comparable what is the intensity of solar radiation implied by these observations?

4. **Amplitudes and intensities of sound waves.** A sound wave of displacement amplitude 2.0×10^7 meter has an intensity of 3.0×10^3 watt/meter^2. If the amplitude of this wave is increased by 10 percent, what is the corresponding increase (in percent) in the intensity of the wave?

Brief Answers:

1.



2. a. 4×10^{14} hertz, 7×10^{14} hertz
   b. 7.5×10^{-7} m, 4.3×10^{-7} m

3. 1.0×10^{3} watt/meter^2

4. 20%