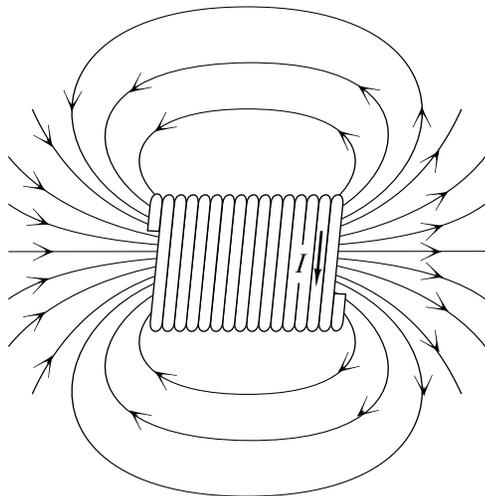


## PRODUCTION OF MAGNETIC FIELDS



## PRODUCTION OF MAGNETIC FIELDS

by  
F. Reif, G. Brackett and J. Larkin

### CONTENTS

- A. Magnetic Field Produced by Charged Particles
- B. Magnetic Field Lines
- C. Magnetic Interaction and Force Constant
- D. Interaction between Current-Carrying Coils
- E. Summary
- F. Problems

Title: **Production of Magnetic Fields**

Author: F. Reif, Dept. of Physics, Univ. of Calif., Berkeley.

Version: 4/30/2002

Evaluation: Stage 0

Length: 1 hr; 40 pages

**Input Skills:**

1. Vocabulary: magnetic field, magnetic moment (MISN-0-426).
2. State the magnitude and direction of the magnetic force on a moving charged particle (MISN-0-426).

**Output Skills (Knowledge):**

- K1. Vocabulary: magnetic field lines.
- K2. State how the coulomb is defined in terms of a magnetic force.
- K3. State the relative magnitude of electric and magnetic forces between two moving charged particles.

**Output Skills (Problem Solving):**

- S1. Given the velocities and charges of any number of particles, use the superposition principle to determine the net magnetic field due to the particles.
- S2. Given the velocities and charges of two particles, determine the magnetic force on one particle due to the other.
- S3. Given the currents in two conductors (long straight wires, loops, or coils), determine the direction of the magnetic force on one conductor due to the other.

*Post-Option:*

1. "Magnets" (MISN-0-366).

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION  
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

PROJECT STAFF

Andrew Schnepf	Webmaster
Eugene Kales	Graphics
Peter Signell	Project Director

ADVISORY COMMITTEE

D. Alan Bromley	Yale University
E. Leonard Jossem	The Ohio State University
A. A. Strassenburg	S. U. N. Y., Stony Brook

Views expressed in a module are those of the module author(s) and are not necessarily those of other project participants.

© 2002, Peter Signell for Project PHYSNET, Physics-Astronomy Bldg., Mich. State Univ., E. Lansing, MI 48824; (517) 355-3784. For our liberal use policies see:

<http://www.physnet.org/home/modules/license.html>.

## MISN-0-427

### PRODUCTION OF MAGNETIC FIELDS

- A. Magnetic Field Produced by Charged Particles
- B. Magnetic Field Lines
- C. Magnetic Interaction and Force Constant
- D. Interaction between Current-Carrying Coils
- E. Summary
- F. Problems

#### Abstract:

At the beginning of the preceding unit, we showed that the magnetic interaction between moving charged particles can conveniently be discussed by examining successively these two questions: (1) How does a moving charged particle produce a magnetic field? (2) How does such a magnetic field produce a magnetic force on another charged particle? In the last unit we discussed the second of these questions. We now turn our attention to the first question concerned with the production of magnetic fields. By answering this question, we shall complete our understanding of the interaction between moving charged particles. Hence we shall also be able to discuss various practical applications involving the magnetic interaction between current-carrying wires or between permanent magnets.

SECT.

### **A** MAGNETIC FIELD PRODUCED BY CHARGED PARTICLES

Consider a particle  $X_1$  with charge  $q_1$ , moving with a velocity  $\vec{v}_1$  as indicated in Fig. A-1a. What is the magnetic field produced by this particle at a point  $P$  whose position relative to the particle is specified by the position vector  $\vec{R}$ ?

#### DIRECTION OF $\vec{B}$

As already mentioned in text section A of Unit 426, the direction of  $\vec{B}$  depends on the velocity  $\vec{v}_1$  of the particle and on the position vector  $\vec{R}$  of the point  $P$  relative to this particle. Furthermore, the direction of  $\vec{B}$  is reversed if either the sign of the charge  $q_1$  or the direction of the velocity  $\vec{v}_1$  is reversed.

In text section B of Unit 426 we found that the magnetic force on any particle  $X$  located at the point  $P$  is perpendicular to both the velocity  $\vec{v}$  of this particle and to the magnetic field  $\vec{B}$  at  $P$ . To be consistent with this relation and with the observed properties of the magnetic force, the magnetic field  $\vec{B}$  produced at  $P$  by the particle  $X_1$  in Fig. A-1a must similarly be perpendicular to both the velocity  $\vec{v}_1$  of  $X_1$  and to the position vector  $\vec{R}$  of  $P$  relative to  $X_1$ . To be precise,  $\vec{B}$  must have a direction specified by this right-hand rule, as illustrated in Fig. A-1b:

*Right-hand rule for  $\vec{B}$ :* If the velocity  $\vec{v}_1$  of *positively* charged particle points along the thumb of the right hand and the fingers of this hand point along the vector  $\vec{R}$  from the particle to the point  $P$ , then the magnetic field  $\vec{B}$  at  $P$  points out of the palm of this hand. (A-1)

If the particle is *negatively* charged, the direction of  $\vec{B}$  is *opposite*. \*

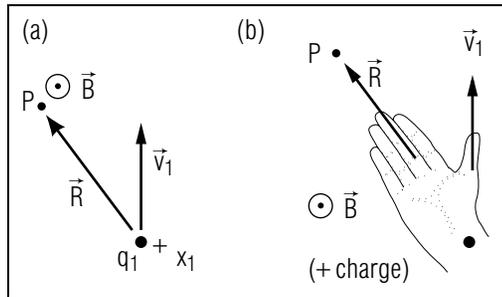


Fig. A-1: (a) Point  $P$  having a position vector  $\vec{R}$  relative to a charged particle moving with velocity  $\vec{v}_1$ . (b) Right-hand rule for direction of magnetic field  $\vec{B}$ .

\* This right-hand rule for the magnetic field is different from our previous right-hand rule (part “b” of Figure (B-4) of Unit 426) for the magnetic force. However, in both rules the thumb of the right hand points along the velocity of the relevant particle (a *different* particle in both cases) and in both rules the direction of the vector to be found (either  $\vec{B}$  or  $\vec{F}$ ) is out of the palm of the hand.

In the special case where the velocity  $\vec{v}_1$  of the particle is parallel to  $\vec{R}$  (so that the particle moves directly toward or away from the point  $P$ ), the magnetic field  $\vec{B}$  is zero. \*

\* The reason is that  $\vec{B}$  must then be perpendicular to every one of the many possible planes which contain both  $\vec{v}_1$  and  $\vec{R}$ .

## MAGNITUDE OF $\vec{B}$

Because a particle moving with a velocity  $\vec{v}_1$  parallel to  $\vec{R}$  produces no magnetic field, only the component vector  $\vec{v}_{1\perp}$  of the particle’s velocity *perpendicular* to  $\vec{R}$  (i.e., perpendicular to the line from the particle to the point  $P$ ) contributes to the magnetic field. The Relation (A-1) of Unit 426 for the magnitude of the magnetic force then suggests (in agreement with observations) that the magnetic field  $\vec{B}$  at the point  $P$  has a magnitude given by

$$|B| = k_m \left| \frac{q_1 v_{1\perp}}{R^2} \right| \quad (\text{A-2})$$

where  $R$  is the distance from the particle to the point  $P$ , and where  $k_m$  is a constant of nature, the “magnetic force constant.” (As we shall discuss

in Sec. C, the value of this constant is:  $k_m = 10^{-7} \text{ N s}^2/\text{C}^2$ .)

## SUPERPOSITION PRINCIPLE

By the superposition principle, the magnetic field due to any number of charged particles is simply the vector sum of the magnetic fields produced by the individual charged particles separately.

### ► $\vec{B}$ due to current

For example, the magnetic field  $\vec{B}$  produced at a point  $P$  by a short piece of wire, through which there flows a current  $I_1$ , is just the vector sum of the magnetic fields produced by all the moving charged particles contributing to this current. Thus the direction of  $\vec{B}$  is that obtained by the right-hand rule of Fig. A-1b if the current is imagined to be due to positively charged particles moving with an average velocity along the wire in the sense of the current. (If the current is actually due to *negatively* charged particles moving in the *opposite* direction, the direction of the magnetic field is the same.)

### ► $|B|$ due to current

One can relate the average velocity of the moving particles in the wire to the current  $I$  in a manner similar to that used in text section D of Unit 426. Then Eq. (A-2) implies that the magnitude of the magnetic field  $\vec{B}$  produced by the piece of wire is

$$|B| = k_m \left| \frac{I_1 \ell_{1\perp}}{R^2} \right| \quad (\text{A-3})$$

where  $\ell_{1\perp}$  is the component, perpendicular to  $\vec{R}$ , of a vector directed along the piece of wire and having a length equal to that of this piece.

## Understanding Magnetic Field due to Particle (Cap. 1)

**A-1** *Example:* Figure A-2 shows at some instant a particle traveling with a velocity  $\vec{v}$  in the  $-\hat{x}$  direction. (a) If the charge  $q$  of the particle is positive, what is the direction of the magnetic field produced by the particle at the points  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$  at the same distance  $R$  from the particle? (b) What is the direction of  $\vec{B}$  at these points if the charge  $q$  is negative? (c) How would the magnitude of  $\vec{B}$  at these points change if the velocity of the particle were 5 times as large? (d) What would be the magnitude of  $\vec{B}$  at these points if the particle were at rest? (*Answer:* 4)

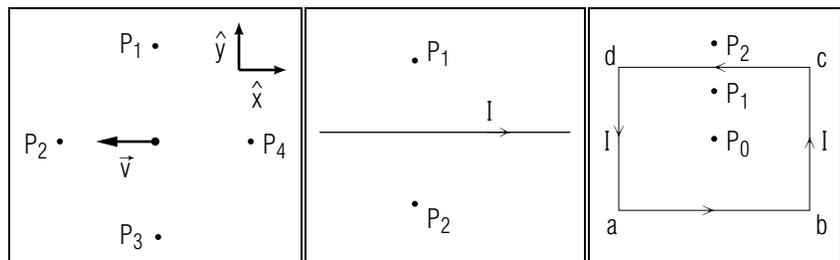


Fig. A-2.

Fig. A-3.

Fig. A-4.

**A-2** *Comparison with electric field:* What is the *electric* field  $\vec{E}$  produced by the particle at the four points in Fig. A-2? Answer the questions (a), (b), (c), (d) of Problem A-1 for this electric field. (*Answer: 1*) (*Practice: [p-1]*)

### Magnetic Field due to Many Particles (Cap. 2)

**A-3** *Long wire:* In Fig. A-3, a current  $I$  flows through a long wire in the indicated sense. (a) What is the direction of the magnetic field  $\vec{B}$  at the two points  $P_1$  and  $P_2$  equidistant from the wire? (b) Compare the magnitudes of the magnetic fields at these points. (*Answer: 6*) (*Suggestion: [s-1]*)

**A-4** *Loop:* Figure A-4 shows a rectangular loop of wire in which a current  $I$  flows in a counter-clockwise sense. (a) What is the direction of the magnetic field produced at the center  $P_0$  of the loop by the currents in each of the four sides of the loop? (b) What then is the direction of the total magnetic field produced by the loop at  $P_0$ ? (c) Is the direction of this field along or opposite to the direction of the magnetic moment of the loop [as in Definition (F-1) of Unit 426]? (d) What is the direction of the total magnetic field at the points  $P_1$  and  $P_2$  close to the side  $cd$  of the loop? (The field there is predominantly due to the current flowing in this side of the loop.) (*Answer: 3*) *More practice for this Capability: [p-2], [p-3]*

SECT.

## **B** MAGNETIC FIELD LINES

In text section C of Unit 419 we saw that the direction of the electric field at various points can be easily visualized by drawing electric field lines. In a similar way, the direction of the magnetic field at various points can be easily visualized by drawing “magnetic field lines” defined as follows:

Def.	<b>Magnetic field line:</b> A line every small segment of which is directed along the magnetic field at the position of this segment.	(B-1)
------	---	-------

Let us then examine the magnetic field, and the corresponding magnetic field lines, produced by current-carrying wires of various common shapes.

### ► *Straight wire*

What is the direction of the magnetic field  $\vec{B}$  produced at various points in the vicinity of a short piece of wire in which there flows a current  $I$  in the sense indicated in Fig. B-1a? The direction of this field is the same as that due to positively charged particles moving with a velocity  $\vec{v}$  along the wire in the sense of the current. To use the right-hand rule of Fig. A-1b, we thus place our right hand at the position of the wire, with the thumb pointing along the wire in the sense of the current and the fingers pointing toward the point where we want to find the magnetic field  $\vec{B}$ . The direction of  $\vec{B}$  is then out of the palm of the hand, i.e., in Fig. B-1a out of the paper at the point  $P_1$  and into the paper at the point  $P_2$ . If we look down end-on upon the wire, the situation looks as illustrated in Fig. B-1b. Here the magnetic field in a plane perpendicular to the wire is at every point directed along a counter-clockwise direction (because the palm of the right hand always points in this direction as the hand pivots around the wire with the thumb sticking out of the paper). Thus any magnetic field line, indicating the direction of the magnetic field at a set of adjacent points, is merely a circle centered around the wire, as indicated in Fig. B-1c. [Note that, if the thumb of the right hand points along the wire in the sense of the current, the direction of the magnetic field along a field line is everywhere along the direction indicated by the curled up fingers of the right hand (i.e., counterclockwise in Fig. B-1c).]

Since all points on each of the circular magnetic field lines are at the same distance from the piece of wire, the magnetic field has the same magnitude at all points of a field line. Of course, the magnetic field has a

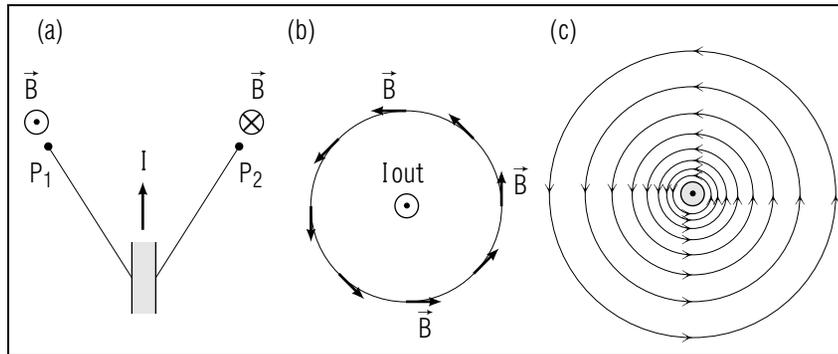


Fig.B-1: Magnetic field near a straight current-carrying wire: (a) side view of a short piece of the wire; (b) top view; (c) magnetic field lines.

smaller magnitude at points farther from the wire, i.e., along a field line of larger radius.

In the case of a *long* straight wire, the magnetic field due to *each* short piece of the wire has the directions indicated in Fig.B-1. Thus Fig.B-1c indicates also the magnetic field lines due to a long straight wire perpendicular to the plane of the paper.

► *Circular loop*

Figure B-2 shows the magnetic field lines near a circular loop of wire in which there flows a current  $I$ . The magnetic field  $\vec{B}$  at any point is the vector sum of the magnetic fields due to all the short pieces of the loop. At points very close to a piece of wire in the loop, the magnetic field is predominantly due to the current in this piece. Hence the magnetic field lines there are nearly circular, similar to those shown in Fig.B-1c for a single short piece of wire. At the center  $C$  of the loop, the current in each piece of the loop produces a magnetic field in the same direction (by the right-hand rule, to the right in Fig.B-2). Hence the total magnetic field at the center points also to the right [i.e., along the direction of the loop's magnetic moment, as defined in Definition (F-1) of Unit 426].

► *Coil*

Figure B-3 shows the magnetic field lines near a long current-carrying coil consisting of many turns of wire wound in a tight helix around a circular cross-section. (Such a coil is then nearly equivalent to many identical circular current loops placed coaxially next to each other.) The magnetic field at any point is again the vector sum of the magnetic

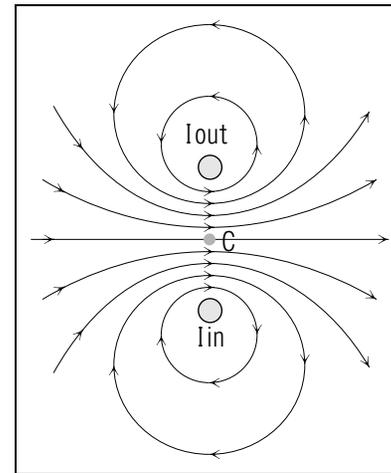


Fig.B-2: Magnetic field lines near a current-carrying circular loop (shown in cross-sectional view).

fields due to all short pieces of the wire of the coil. Thus the magnetic field is large inside the coil where the magnetic fields due to all these pieces have nearly the same direction (along the magnetic moment of the coil).

**Field Lines and Direction Of  $\vec{B}$  (Cap. 3)**

**B-1** Comparing  $\vec{B}$  and  $\vec{E}$  lines: (a) Does Fig. B-4a or Fig. B-4b show the magnetic field lines around a long current-carrying wire? Does the current in this wire flow out of, or into, the paper? (b) Which of these

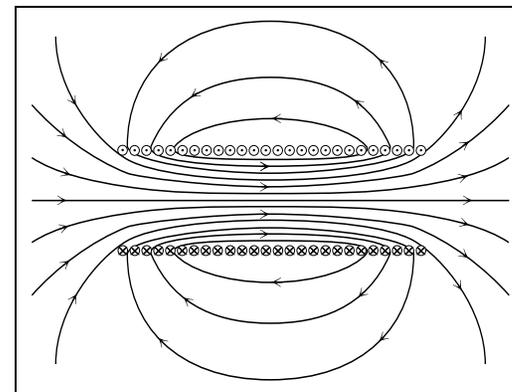


Fig.B-3: Magnetic field lines near a current-carrying circular coil (shown in cross-sectional view).

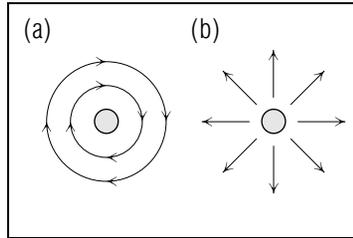


Fig. B-4.

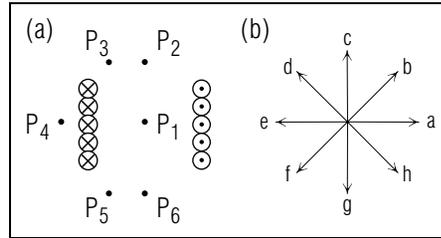


Fig. B-5.

figures shows the electric field lines due to a long wire with uniformly distributed charged particles? Is the net charge on this wire positive or negative? (c) Is the magnetic force on a moving charged particle parallel or perpendicular to the magnetic field line at the position of the particle? Is the electric force on this particle parallel or perpendicular to the electric field line at the position of the particle? (*Answer: 7*) (*Suggestion: [s-2]*)

**B-2** *Magnetic field due to coil:* Figure B-5 shows a cross-sectional view of a circular coil. The current flows into the paper in the wires on the left side and out of the paper in the wires on the right side. Sketch a few magnetic field lines by comparing this coil with that in Fig. B-3. What are the approximate directions of the magnetic field produced by this coil at the six points  $P_1, P_2, \dots, P_6$  indicated in the diagram? Specify these directions in terms of the possible directions labeled by  $a, b, \dots, h$  in Fig. B-5b. (*Answer: 2*) (*Suggestion: [s-3]*)

SECT.

## C

**MAGNETIC INTERACTION AND FORCE CONSTANT**

### MAGNETIC INTERACTION BETWEEN MOVING CHARGED PARTICLES

By using the results of Sec. A, we can find the magnetic field  $\vec{B}$  produced by moving charged particles at any point  $P$ . By text section B of Unit 426, we can then use this field  $\vec{B}$  to find the magnetic force on any particle located at  $P$ . Thus we can now discuss completely the magnetic interaction between charged particles.

As an especially simple example, consider two particles  $X$  and  $X_1$ , with *positive* charges  $q$  and  $q_1$ , which move side by side with velocities  $\vec{v}$  and  $\vec{v}_1$  in the *same* direction, as shown in Fig. C-1a. What then is the magnetic force  $\vec{F}_m$  exerted on the particle  $X$  by the particle  $X_1$ ?

► *Direction of force*

By the right-hand rule for the magnetic field (Fig. A-1b), the particle  $X_1$  produces at the position of particle  $X$  a magnetic field pointing into the paper. By the right-hand rule for the magnetic force (part “b” in Figure (B-4) of Unit 426), this field then produces on the particle  $X$  a magnetic force  $\vec{F}_m$  acting to the left. Thus this magnetic force is directed toward the other particle  $X_1$ , i.e., it is *attractive*. (If the sign of the charge, or the direction of the velocity of either particle were opposite, this magnetic force would be opposite; i.e., it would then be repulsive.)

► *Magnitude of force*

By Eq. (A-2), the magnitude  $B$  of the magnetic field produced by particle  $X_1$  at the position of particle  $X$  is  $B = k_m q_1 v_1 / R^2$  if the particles are separated by a distance  $R$ . \*

\* Since the velocity  $\vec{v}_1$  is perpendicular to the line joining the two particles, the magnitude of the component of  $\vec{v}_1$  perpendicular to this line is simply equal to the magnitude  $v_1$ .

By Relation (B-3) of Unit 426, the magnitude of the magnetic force  $\vec{F}_m$  produced by this field on the particle  $X$  is then

$$F_m = qvB = qv \left( k_m \frac{q_1 v_1}{R^2} \right) = k_m \frac{qq_1 v v_1}{R^2} \quad (\text{C-1})$$

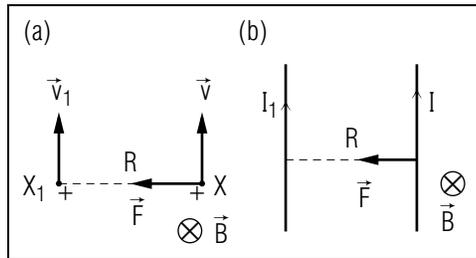


Fig. C-1: Magnetic interaction between moving charged particles. (a) Magnetic force exerted on one particle by another. (b) Magnetic force exerted on one current-carrying wire by another.

► *Interaction of currents*

As another example, consider two long straight parallel wires with currents  $I$  and  $I_1$  flowing in the same sense, as indicated in Fig. C-1b. Then the magnetic interaction between these wires is the same as that between positively charged particles moving along these wires in the senses of the currents. In particular, the direction of the magnetic force exerted on one wire by the other is similar to that discussed for the two particles in Fig. C-1a. Thus this magnetic force is attractive if the senses of both currents are the same, but is repulsive if these senses are opposite. \*

\* To find the magnitude of the magnetic force, one must, however, first calculate the magnetic field produced at the position of one wire by the *entire* other wire.

## MAGNETIC FORCE CONSTANT AND DEFINITION OF THE COULOMB

We know that the magnetic force constant  $k_m$  is some constant of nature, but have not yet specified its numerical value. This value can, of course, be found from any relation where all quantities, other than  $k_m$ , are known. For example, according to Eq. (C-1), the constant  $k_m$  should have the units of  $\text{Ns}^2/\text{C}^2$  if all other quantities in this relation are expressed in SI units. Furthermore, the value of  $k_m$  can be found from Eq. (C-1) by measuring the magnetic force exerted on one particle (with known charge and known velocity) by another particle (with known charge and known velocity) when the particles are separated by a known distance. \*

\* In practice, greater precision can be achieved by dealing with many charged particles, i.e., by measuring the magnetic force on one wire (with known current) by another wire (with known current).

► *Definition of coulomb*

Let us, however, remember that we have never yet specified the international convention used to define the amount of charge called the “coulomb.” The convention actually adopted *defines* the coulomb so that the magnetic force constant is *assigned* this simple value:

$$k_m = 10^{-7} \text{ N s}^2/\text{C}^2, \text{ exactly.} \quad (\text{C-2})$$

The coulomb is then defined to be that amount of charge which yields the correct value of the magnetic force [in relations such as Eq. (C-1) when  $k_m$  has the value specified by Eq. (C-2)].

► *Connection with  $k_e$*

Once the magnitude of the coulomb has been determined from measurements of magnetic forces, one can make other experimental measurements on particles with known charges, expressed in terms of the coulomb. For instance, by measuring the coulomb *electric* force exerted on one such particle by another, one can determine experimentally that the *electric* force constant has the approximate value  $k_e \approx 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$ .

## COMPARISON OF MAGNETIC AND ELECTRIC FORCES

The typical magnitude  $F_m$  of the *magnetic* force exerted in Fig. C-1a on one charged particle by the other can be found from Eq. (C-1). On the other hand, the magnitude  $F_e$  of the coulomb *electric* force exerted on one charged particle by the other is

$$F_e = k_e \frac{qq_1}{R^2} \quad (\text{C-3})$$

To compare the magnitudes of the electric and magnetic forces, let us find the ratio  $F_e/F_m$  by dividing Eq. (C-3) by Eq. (C-1). Thus

$$\frac{F_e}{F_m} = \frac{k_e}{k_m v v_1} \quad (\text{C-4})$$

where both the charges of the particles, and the distance  $R$  between them, have cancelled. But the ratio of the force constants in Eq. (C-4) is

$$\frac{k_e}{k_m} = \frac{9.0 \times 10^9 \text{ N m}^2/\text{C}^2}{10^{-7} \text{ N s}^2/\text{C}^2} = 9 \times 10^{16} \text{ m}^2/\text{s}^2 = c^2 \quad (\text{C-5})$$

where  $c = 3 \times 10^8$  m/s is a quantity having the units (m/s) of a speed. In fact, this quantity  $c$  has the value of the observed speed of light! [As we shall see later, this fact has profound significance since it indicates that light is an electromagnetic phenomenon.] Considering the simple case where both interacting particles move with the same speed  $v = v_1$ , the ratio of the forces in Eq. (C-4) is thus equal to

$$\frac{F_e}{F_m} = \frac{c^2}{v^2} = \left(\frac{c}{v}\right)^2 \quad (\text{C-6})$$

or  $F_m/F_e = (v/c)^2$ . Thus we see that, unless the interacting charged particles move with extremely large speeds close to the speed of light ( $3 \times 10^8$  m/s), the magnitude of the magnetic force is much smaller than that of the electric force.

► *Forces between wires*

When a current flows through a wire, the magnitude of the average velocity of the electrons responsible for this current is typically about  $10^{-3}$  m/s, *very* much smaller than the speed of light. Hence the magnetic force exerted on a moving electron in one wire by a moving electron in another wire is *extremely* small. Why then are magnetic forces between current-carrying wires so important in practical applications, such as electric motors? The reason is that each current consists of enormously many electrons (of the order of Avogadro's number, i.e.  $10^{23}$ ). Thus the magnetic force on *all* the electrons in one wire by all the electrons in the other wire can be quite large, despite the small magnitude of the magnetic force between individual electrons. On the other hand, the coulomb electric force exerted on one wire by the other is essentially zero since the total charge of each wire is always nearly zero (because each wire contains always nearly as many positively as negatively charged atomic particles).

### Magnetic Force between Particles Or Currents (Cap. 4)

**C-1** Two positively charged particles travel side-by-side with velocities along the same direction, as shown in Fig. C-2a. (a) What is the direction of the magnetic force  $\vec{F}_{12}$  exerted on particle 1 by particle 2? (b) What is the direction of the magnetic force  $\vec{F}_{21}$  exerted on particle 2 by particle 1? (c) Do these mutual forces have opposite directions, just as the non-magnetic forces discussed in Unit 408? (d) A beam of protons consists of many protons traveling side by side. Do the mutual magnetic forces between the protons in the beam tend to increase or decrease the diameter of the beam? (*Answer: 9*) (*Suggestion: [s-4]*)

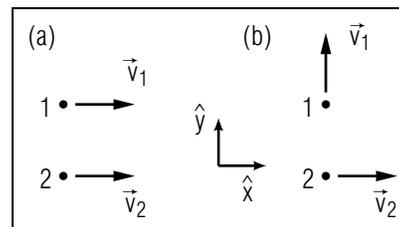


Fig. C-2.

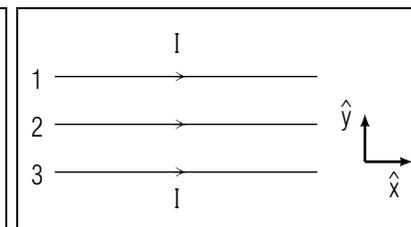


Fig. C-3.

**C-2** Figure C-2b shows the positively charged particles 1 and 2 of the preceding problem, but traveling with mutually perpendicular velocities. For this situation, answer again the questions (a), (b), and (c) of the preceding problem. (*Answer: 5*) (*Suggestion: [s-4]*)

**C-3** *Forces between wires:* Figure C-3 shows three long parallel wires 1, 2, and 3, where the middle wire 2 is equidistant from the other two wires. Currents of the same magnitude  $I$  flow in the same sense in all these wires. (a) What is the direction of the magnetic force on each of these wires due to its magnetic interaction with the other two wires? (b) Suppose that the sense of the current in wire 3 were opposite to that shown in Fig. C-3. What then would be the direction of the magnetic force on each wire due to its interaction with the other two wires? (*Answer: 12*) (*Suggestion: [s-5]*)

SECT.

## D INTERACTION BETWEEN CURRENT-CARRYING COILS

In the preceding section we discussed the magnetic interaction, between two straight current-carrying wires. Let us now examine another frequently occurring case, the magnetic interaction between two current-carrying coils.

### ► Coaxial coils

Consider two coils  $C$  and  $C_1$  aligned along the same axis, as shown in Fig. D-1. Suppose that currents  $I$  and  $I_1$  flow around these coils in the *same* sense, as indicated in the figure. (The magnetic moments  $\vec{M}$  and  $\vec{M}_1$  of the coils have then the *same* direction: see Unit 426.) Under these conditions, what is the direction of the magnetic force exerted on the coil  $C$  by the other coil  $C_1$ ?

### ► Force on coil

The question is readily answered by a simple qualitative argument. The current  $I_1$  flowing around the coil  $C_1$  produces in its vicinity a magnetic field indicated by the magnetic field lines of Fig. B-3. Thus this magnetic field has, at the position of the coil  $C$ , directions indicated in Fig. D-1 by the few field lines and by the directions of the vectors marked  $\vec{B}$ . By the right-hand rule for the magnetic force, the magnetic forces on the wires in the coil have then the directions indicated by the vectors marked  $\vec{F}$ . The component vectors of these forces along the axis of the coils are directed *toward* the coil  $C$ . Hence the total magnetic force on the coil  $C$  is also directed *toward* the coil  $C_1$ . In other words, the magnetic forces in Fig. D-1 are such that the coil  $C$  is *attracted* toward the coil  $C_1$ .

Suppose now that the sense of the current in one of the coils was reversed. (The magnetic moment of the coil would then correspondingly also be reversed.) Then the directions of all magnetic forces would also be reversed. Thus the coil  $C$  would then be repelled from coil  $C_1$ .

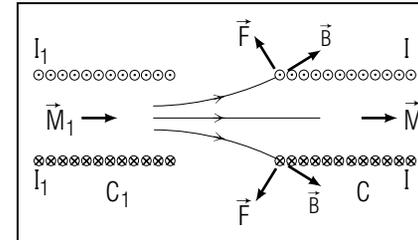


Fig. D-1: Magnetic force exerted on one current-carrying coil by another.

### ► Summary

The preceding comments can be summarized:

If two coils are aligned along the same axis, they attract each other if their magnetic moments have the same direction, and repel each other if their magnetic moments have opposite directions.

(D-1)

## Magnetic Force between Particles Or Currents (Cap. 4)

**D-1** *Force between coils:* Figure D-2 shows cross-sectional views of a large coil and of a small coil (or loop) located at four possible positions. The current in each of these coils flows in the same sense (out of the paper in the top wire and into the paper in the bottom wire) so that the magnetic moments of these coils are along the same direction. The magnetic field produced at the two sides of the small coil by the current in the large coil is indicated by the arrows labeled by  $\vec{B}$  (in accordance with the magnetic field lines illustrated in Fig. B-3). For each of the following four positions of the small coil, draw arrows indicating the magnetic forces produced on the two sides of the small coil. Then find the direction of the resultant total magnetic force  $\vec{F}_{\text{tot}}$  on this coil and state whether the small coil is repelled by, or attracted to, the large coil: (a) The small coil is at the center of the large coil where the magnetic field has the same direction and magnitude at both sides of the small coil. (b) The small coil is just outside the right end of the large coil. (The directions of the magnetic field at the sides of the small coil are then different). (c) The small coil is just outside the left end of the large coil. (d) The magnetic field is just outside the large coil near its center. (The magnitude of the magnetic field at the side closer to the large coil is then larger than at the farther side.) (*Answer: 8*) (*Suggestion: [s-6]*)

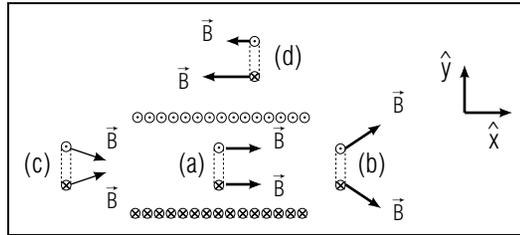


Fig. D-2.

**D-2** (a) How would the answers to the preceding problem be changed if the sense of the current in the large coil remained the same, but the current in the small coil were flowing in the opposite sense (so that the magnetic moments of the coils had opposite directions)? (b) In the preceding problem, what would happen to the magnitude of the total magnetic force on the small coil if the current in the large coil were 3 times larger? If the current in the small coil were 3 times larger? If the currents in both coils were 3 times larger? (*Answer: 10*)

SECT.

**E** SUMMARY**DEFINITIONS**

Magnetic field line; Def. (B-1)

**IMPORTANT RESULTS**

Magnetic field produced by a charged particle: Rule (A-1), Fig. A-1b, Eq. (C-2)

$$\vec{B} = k_m |q_1 v_{1\perp} / R^2|, \text{ direction of right-hand rule of Fig. A-1b } (v_1 \text{ along thumb, } \vec{R} \text{ along fingers, } \vec{B} \text{ out of palm for } + \text{ charge})$$

$$k_m = 10^{-7} \text{ N s}^2 / \text{C}^2$$

**USEFUL KNOWLEDGE**

Magnetic field lines due to currents in straight wires, loops, and coils (Sec. B)

Definition of the coulomb (Sec. C)

Relative magnitude of electric and magnetic forces (Sec. C)

**NEW CAPABILITIES**

- (1) Understand the relation between the magnetic field at a point and the charge, velocity, and position of a particle producing this field. (Sec. A, relations A-1 and A-2; [p-1])
- (2) Use the superposition principle to find the magnetic field due to any number of particles. (Sec. A; [p-2], [p-3])
- (3) Describe qualitatively the direction of the magnetic field  $\vec{B}$  near a current-carrying wire, loop, or coil (a) by using the right-hand rule to find the direction of  $\vec{B}$  at a few points, or (b) by using a diagram showing the magnetic field lines. (Sec. B, [p-4])
- (4) Find the direction of the magnetic force on one charged particle (or current-carrying wire or coil) due to another. (Sects. C and D)

**E-1** *Field due to a pair of coils (Caps. 1 and 2):* In the pair of identical coils illustrated in Fig. E-1, the current in the left coil flows into the paper in the top part of the left coil and out of the paper in the bottom part of this coil. (a) What is the direction of the magnetic field  $\vec{B}$  produced by the current in this left coil at the point  $P$  in the center between the two coils? (b) What must be the magnitude and sense of

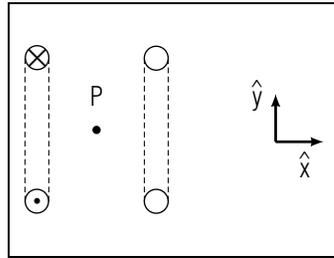


Fig. E-1.

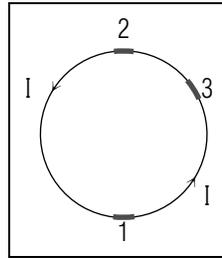


Fig. E-2.

the current flowing in the right coil so that the total magnetic field at the center point  $P$  is equal to  $2\vec{B}$ ? (*Answer: 16*) (*Suggestion: [s-13]*)

**E-2** *Current through a spring (Cap. 4):* A weight hangs at rest, supported from the lower end of a vertical spring. What happens when a current flows through the spring? (*Answer: 21*) (*Suggestion: [s-15]*)

**E-3** *Mutual forces in loop (Caps. 1, 2, 4):* A current  $I$  flows counter-clockwise around a circular loop (or coil) of wire, as illustrated in Fig. E-2. (a) What is the direction of the magnetic field produced at the position of any short wire piece 1 by the current flowing in the short wire piece 2 which is on the opposite side of the loop and parallel to the piece 1? (b) What is the direction of the magnetic field at the position of the wire piece 1 by the current flowing in any other wire piece, such as the piece 3? (c) What then is the direction of the total magnetic field produced at the position of the wire piece 1 by all the other parts of the wire loop? (d) What then is the direction of the magnetic force exerted on the wire piece 1 by all the other parts of the wire loop? (e) What would be the answer to the preceding question if the current were flowing around the loop in the opposite sense? (f) Does the magnetic force exerted on any part of the wire loop by all the other parts of the wire loop tend to contract or expand the loop? (g) If the current  $I$  through the loop is very large (e.g., if the loop is used to produce a very large magnetic field), the mutual forces exerted on any part of the loop by other parts of the loop can become so large as to destroy the loop. To prevent such destruction, should the loop be wound around a rigid core to prevent the loop from collapsing, or should the loop be surrounded by a rigid outside casing to prevent the loop from exploding? (*Answer: 19*)

SECT.

## F PROBLEMS

**F-1** *Field and power dissipation:* A current flowing in a coil is used to produce a large magnetic field. If the coil is water-cooled, the power dissipated in the coil as a result of its electric resistance can be doubled without danger of overheating the coil. By what factor can one thus increase the magnetic field produced by the coil? (Assume that the resistance of the coil remains unchanged.) (*Answer: 23*) (*Suggestion: [s-17]*)

**F-2** *Field and number of turns:* A circular coil, consisting of  $N$  turns of wire, is connected to an emf source of negligible resistance. A magnetic field is then produced by the coil as a result of the current flowing through the coil. How would the magnitude of this magnetic field be changed if the number of turns of wire in the coil were 3 times as large (the diameter and length of the coil remaining unchanged)? (*Answer: 25*) (*Suggestion: [s-14]*)

**F-3** *Field at center of loop:* A current  $I$  flows around a circular wire loop of radius  $a$ . Starting from the basic relation (A-3) for the magnetic field produced by a short piece of wire, find the magnitude  $B$  of the magnetic field produced at the center of the loop. (*Answer: 22*) (*Suggestion: [s-16]*)

**F-4** *Field on axis of loop:* A current flows around a circular loop of radius  $a$ . What is the magnitude of the magnetic field at any point  $P$  on the axis of the loop at a distance  $b$  from the plane of the loop? (*Answer: 27*) (*Suggestion: [s-21]*)

**F-5** *Measurement of the coulomb:* The magnetic field at a distance  $D$  from a long straight wire, carrying a current  $I$ , has a magnitude  $B = 2k_m I/D$ . In Fig. F-1 a current of the same magnitude  $I$  flows also through a rectangular coil, consisting of  $N$  turns of wire, which is suspended vertically from a balance. The lower side of this coil, of width  $w$ , is at a distance  $D$  from the wire, while its upper end is so far from the wire that the magnetic field there is negligibly small. (a) What then is the magnitude  $F$  of the total magnetic force on the coil? Express your answer in terms of  $k_m$ ,  $N$ ,  $I$ ,  $w$ , and  $D$ . (b) Express the current  $I$  in terms of the measured magnitude  $F$  of this magnetic force and the other quantities. (c) Suppose that  $N = 200$ ,  $L = 0.25$  m,  $D = 0.010$  m, and the measured value of  $F$  is 0.10 newton. Using the value of  $k_m = 10^{-7}$  N s<sup>2</sup>/C<sup>2</sup> adopted

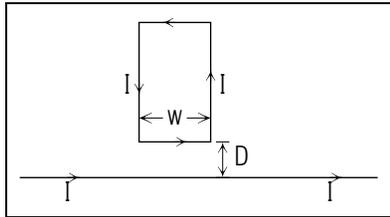


Fig. F-1.

by international convention, determine the numerical value of the current  $I$ . (d) What then is the amount of charge which flows through the long wire during a time of 30 second? Express your answer in terms of the unit "coulomb." (Answer: 24) (Suggestion: [s-20])

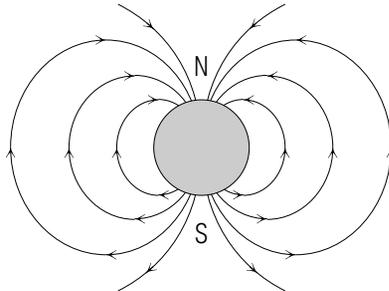
*Note: Tutorial section F contains additional problems.*

## TUTORIAL FOR F

### ADDITIONAL PROBLEMS

**f-1** *TOTAL MAGNETIC MOMENT:* In Fig. E-1 of the text, each atomic magnetic moment is represented as equivalent to an atomic current  $I$  flowing around a small loop of area  $A_0$ . By definition, the atomic magnetic moment is then equal to  $\vec{M}_0 = IA_0$  (directed out of the paper). (a) If the cross-sectional area of the rod is  $A$ , what is the number  $N$  of adjacent atomic current loops within this area of the rod? (b) The magnetic moment of the rod is, by definition,  $\vec{M} = IA$  (directed out of the paper) where  $I$  is the net current flowing around the periphery of the rod as a result of the atomic current loop. How then is the magnetic moment of the entire magnetized rod related to the number  $N$  of atoms and the magnetic moment  $\vec{M}_0$  of each such atom? (*Answer: 54*)

**f-2** *MAGNETIC MOMENT OF THE EARTH:* The diagram shows a sketch of the magnetic field lines in the vicinity of the earth, where N indicates the north pole and S the south pole of the earth. (a) At the surface of the earth, does the magnetic moment of a compass needle point toward the north pole or the south pole of the earth? (b) Does the magnetic moment of the earth itself point from its south pole toward its north pole, or from its north pole toward its south pole? (*Answer: 63*) (*Suggestion: s-23*)

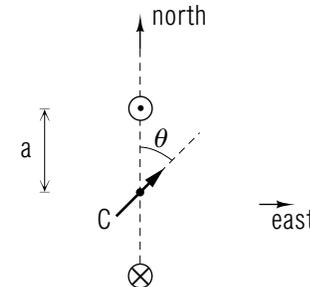


**f-3** *FIELD PRODUCED BY A POWER LINE:* The magnitude  $B$  of the magnetic field produced at a distance  $D$  from a long straight wire, carrying a current  $I$ , is equal to  $B = 2k_m I/D$ . (a) What then is the magnitude of the magnetic field produced at the surface of the earth by a current of 200 ampere flowing in a power line at a height of 6 meter above

the surface? Express your answer in terms of gauss. (b) Would this magnetic field be sufficiently large to affect the reading of a compass used by a person standing near this power line? (Assume that the magnitude of the earth's magnetic field near this power line is 0.2 gauss.) (*Answer: 65*)

**f-4** *MEASURING CURRENT BY USING THE EARTH'S FIELD:* The diagram shows a cross-sectional view of a circular coil, of radius  $a$ , consisting of  $N$  turns of wire through which there flows a current  $I$ . The magnitude of the magnetic field  $\vec{B}_c$  produced by this current at the center  $C$  of this coil is then equal to  $B_c = 2\pi N k_m I/a$  (as shown in text problem F-3). In the diagram, this coil is oriented so that its axis is perpendicular to the northern direction. The magnetic field  $\vec{B}_e$  produced at  $C$  by the earth has a northern direction. When the current  $I$  flows through the coil, a compass needle placed at  $C$  is found to rotate so that it comes to rest when its magnetic moment makes an angle  $\theta$  with the northern direction.

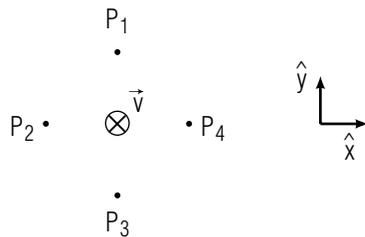
To show that this angle  $\theta$  can be used to measure an unknown current  $I$  flowing through the coil, express this current in terms of the known quantities  $N$ ,  $a$ ,  $B_e$ , and  $\theta$ . (*Answer: 62*) (*Suggestion: s-22*)



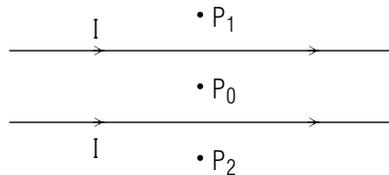
**f-5** *FORCE ON ELECTRON NEAR A WIRE:* The magnitude of the magnetic field at a distance  $D$  from the center of a long straight wire, carrying a current  $I$ , is  $B = 2k_m I/D$ . Suppose that  $I = 10$  ampere and that an electron at a distance of 2 cm from the center of the wire moves with a speed of  $5 \times 10^6$  m/s. (a) What is the force on the electron if it moves outward away from the wire? (b) What is the force on this electron if it moves parallel to the wire in a direction  $\hat{x}$  along the arrow specifying the sense of the current in the wire? (*Answer: 64*)

## PRACTICE PROBLEMS

**p-1** *MAGNETIC FIELD DUE TO A PARTICLE (CAP. 1)*: The diagram shows a negatively charged particle moving with a velocity  $\vec{v}$  into the paper. (a) What is the direction of the magnetic field  $\vec{B}$  at the points  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$  equidistant from the particle? (b) Is the magnitude of the magnetic field at  $P_4$  larger than, equal to, or smaller than that of the magnetic field at  $P_1$ ? (*Answer: 53*) (*Suggestion: Review text problem A-1.*)

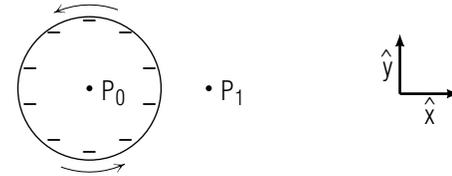


**p-2** *MAGNETIC FIELD DUE TO MANY PARTICLES (CAP. 2)*: The diagram shows two long parallel wires in each of which flows a current  $I$  to the right. (a) What is the direction of the resultant magnetic field at a point  $P_0$  which is equidistant from both wires? (b) What is the direction of the magnetic field at the points  $P_1$  and  $P_2$ ? (*Answer: 55*) (*Suggestion: Review text problems A-3 and A-4.*)

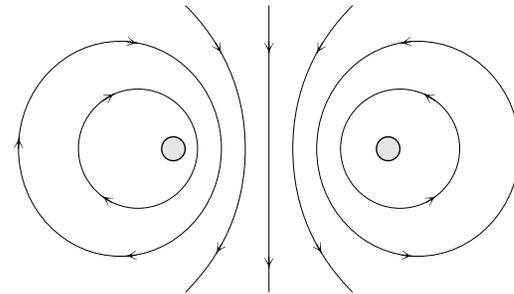


**p-3** *MAGNETIC FIELD DUE TO MANY PARTICLES (CAP. 2)*: *Rotating charged wheel*: Electrons are uniformly distributed at fixed positions along the rim of a wheel, illustrated in the diagram, which rotates in a counter-clockwise sense. (a) What is the resulting direction of the magnetic field at the center  $P_0$  of the wheel and at the point  $P_1$ ? (b) *Comparison*: What is the resulting direction of the *electric* field at

these two points? (*Answer: 52*) (*Suggestion: Review text problems A-3 and A-4.*)



**p-4** *FIELD LINES AND DIRECTION OF  $\vec{B}$  (CAP. 3)*: The diagram shows a few magnetic field lines sketched in the vicinity of a circular loop of wire (shown in cross-section). By comparing the direction of the magnetic field indicated by these field lines with the direction of the field expected from a current flowing in a particular sense around the loop, determine whether the current in the loop flows out of the paper on the left side and into the paper on the right side, or vice versa. (*Answer: 57*) (*Suggestion: Review text problems B-1 and B-2.*)



## SUGGESTIONS

**s-1** (*Text problem A-3*): At the point  $P_1$ , what is the direction of the magnetic field due to any small segment of the wire? What then is the direction of the total magnetic field equal to the vector sum of the magnetic fields due to all these segments?

**s-2** (*Text problem B-1*): If you have forgotten about electric field lines, you might want to review Unit 419.

**s-3** (*Text problem B-2*): The coil in Fig. B-5 is similar to that of Fig. B-3, but rotated by  $90^\circ$ . The field lines can thus be sketched by comparing them with those in Fig. B-3. (Alternatively, the directions of the magnetic field at the specified points may also be found approximately by determining the approximate magnetic field due to a wire segment on the left side and due to a wire segment on the right side of the coil, and then finding roughly the direction of the vector sum of these fields.)

**s-4** (*Text problems C-1 and C-2*): Part (a): To find the direction of the magnetic force on particle 1 due to particle 2, find first the direction of the magnetic field produced at the position of particle 1 by particle 2. Then find the magnetic force produced on particle 1 by this field (using the right-hand rule for the magnetic force).

Part (b): To find the direction of the magnetic force on particle 2 due to particle 1, find first the direction of the magnetic field produced at the position of the particle 2 by particle 1. Then find the magnetic force produced on particle 2 by this field.

**s-5** (*Text problem C-3*): The magnetic force on any one wire can be found from the magnetic field produced at this wire by the other two wires. By the superposition principle, this magnetic field is the vector sum of the magnetic fields produced by the other two wires separately. You know how to find the direction of each of these fields. Remember also that the magnetic field produced by a more distant wire is smaller than that produced by a closer wire.

**s-6** (*Text problem D-1*): In each case, use the sense of the current in each side of the small coil, and the direction of the magnetic field at that position, to draw an arrow indicating the magnetic force on this side (using the right-hand rule for the magnetic force). The total magnetic force on both sides of the coil is then the vector sum of these forces. (The total force on any other pair of opposite sides of the small coil has similarly the same direction. Thus the direction of the total magnetic force on the small coil is the same as the direction of the total magnetic force on *any* pair of opposite sides of this coil.)

NOTE: numbers s-7 through s-12 are not in use.

**s-13** (*Text problem E-1*): If the total magnetic field is  $2\vec{B}$ , the magnetic field due to the right coil must be the same as that due to the left coil. Furthermore point  $P$  is equidistant from both coils. What then must be the magnitude of the current in the right coil, compared to the magnitude of the current in the left coil? What must be the sense of the current in the right coil, compared to the sense of the current in the left coil?

**s-14** (*Text problem F-2*): Suppose that each turn of wire in the coil has a resistance  $R_0$ . What then is the resistance of the entire coil consisting of  $N$  turns? How is the current  $I$  through the coil related to the emf of the source and to the resistance of the coil? If  $N$  is doubled, what happens to the current  $I$ ?

If  $N$  is doubled, but the current  $I$  remains *unchanged*, what would happen to the magnetic field produced by the coil? Taking into account that  $I$  *does* change when  $N$  is doubled, what happens to the magnetic field?

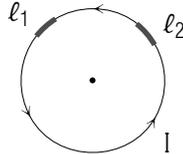
**s-15** (*Text problem E-2*): Does the current in adjacent turns of the spring flow in the same sense or the opposite sense? Is the magnetic force exerted on one such turn of the spring by an adjacent turn of the spring repulsive or attractive? As a result of this force, is the spring extended or compressed?

**s-16** (*Text problem F-3*): In the diagram, what is the direction and magnitude of the magnetic field  $\vec{B}_1$  produced at the center of the loop by the short piece of wire of length  $\ell_1$ ? (Note that the line from this piece to the center of the loop is simply perpendicular to the piece itself.) (See answer 59.)

Is the direction of the magnetic field produced at the center of the loop by any other piece of wire, of length  $\ell_2$  the same or different? What is

the magnitude of this field?

If the magnetic fields due to all the short pieces of the loop are added to find the total magnetic field at the center of the loop, what is the result? (If you need further help, go to [s-19].)



**s-17** (Text problem F-1): How is the power dissipated in the coil related to the resistance  $R$  of the coil and to the current  $I$  flowing through the coil? (Review text section C of Unit 425.) If this power is doubled, by what factor is the current  $I$  increased? Correspondingly, by what factor is the magnetic field produced by this current increased?

**s-18** (Suggestion [s-21]): Part c: The perpendicular components from wire pieces on opposite sides of the loop are opposite. Hence their sum is equal to zero. Thus the sum of all perpendicular components is zero, since contributions to this sum cancel each other in pairs.

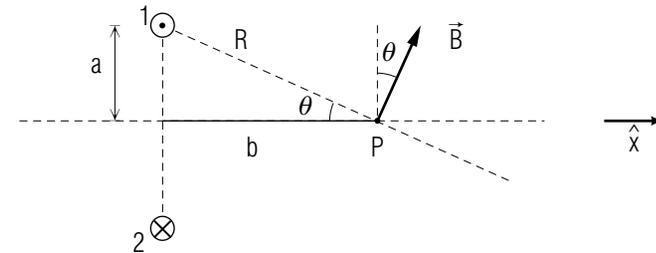
Part e: Note that  $\sin \theta = a/R$ . Furthermore,  $R$  is related to  $a$  and  $b$  by the Pythagorean theorem.

**s-19** (Suggestion [s-16]): The sum of the lengths of all the short pieces of the loop is just equal to the circumference of the loop. What is this circumference if the radius is  $a$ ?

**s-20** (Text problem F-5): Review text problem (E-3) of Unit 426 which discusses how to find the magnetic force on such a coil. Then use this knowledge, together with the result specifying the magnetic field due to the long current-carrying wire. (Note that the magnetic forces on the vertical sides of the coil are opposite and thus do not contribute to the total force.)

**s-21** (Text problem F-4): The diagram shows a cross-sectional view of the loop and the point  $P$ . It also indicates the magnetic field  $\vec{B}_1$  produced at  $P$  by the short wire piece 1 of length  $\ell_1$ . (a) What is the magnitude of  $\vec{B}_1$ ? Express your answer in terms of the current  $I$  in the

loop, the length  $\ell_1$ , the distance  $R$  of the wire piece 1 from  $P$ , and the angle  $\theta$  which the line from this wire piece to  $P$  makes with the direction  $\hat{x}$  along the axis of the loop. (b) What is the numerical component of  $\vec{B}_1$  along  $\hat{x}$ ? (c) What is the numerical component along  $\hat{x}$  of the total magnetic field at  $P$ , obtained by adding the components of the fields due to all small parts of the loop? (d) What is the numerical component, perpendicular to the axis of the loop, of the total magnetic field due to all parts of the loop? (e) Use the preceding results to find the magnitude of the total magnetic field at  $P$ , expressing  $R$  and the angle  $\theta$  in terms of the given quantities  $a$  and  $b$ . (Answer: 61) (Note: For additional help, see [s-18].)



**s-22** (Tutorial frame [f-4]): The compass needle is aligned along the direction of the resultant magnetic field, which is the vector sum of  $\vec{B}_c$  and  $\vec{B}_e$ . By drawing a vector diagram, find a relation between the angle  $\theta$  and the magnitudes of these fields. Then express this relation in terms of the current  $I$ . (Answer: 60)

**s-23** (Tutorial frame [f-2]): Part b: Compare the magnetic field lines produced by the earth with those produced by an equivalent coil wound around the periphery of the earth (parallel to its equator). In what sense should the current in this coil flow to produce the magnetic field lines indicated in the diagram? What then would be the direction of the magnetic moment of this coil?

## ANSWERS TO PROBLEMS

1. a. along  $\hat{y}$ , opposite  $\hat{x}$ , opposite  $\hat{y}$ , along  $\hat{x}$   
 b. opposite  $\hat{y}$ , along  $\hat{x}$ , along  $\hat{y}$ , opposite  $\hat{x}$   
 c. unchanged  
 d.  $k_e|q|/R^2$  at all four points
2.  $P_1$ : g,  $P_2$ : g,  $P_3$ : h,  $P_4$ : c,  $P_5$ : f,  $P_6$ : g
3. a. out of paper for each  
 b. out of paper; c. along  
 d. out of paper at  $P_1$ , into paper at  $P_2$
4. a. into paper, 0, out of paper, 0  
 b. out of paper, 0, into paper, 0  
 c. 5 times larger, d. 0
5. a. along  $\hat{x}$   
 b. force is zero; c. no
6. a. out of paper at  $P_1$ , into paper at  $P_2$   
 b. same
7. a. Fig. B-4a, into paper  
 b. Fig. B-4b, positive  
 c. perpendicular, parallel
8. a.  $\vec{F}_{\text{tot}} = 0$   
 b. opposite to  $\hat{x}$ , attracted  
 c. along  $\hat{x}$ , attracted  
 d. along  $\hat{y}$ , repelled
9. a. opposite to  $\hat{y}$   
 b. along  $\hat{y}$   
 c. yes, d. decrease
10. a.  $\vec{F}_{\text{tot}}$  would be opposite  
 b. 3 times larger, 3 times larger, 9 times larger
11. a. attracted, attracted, repelled  
 b. repelled, repelled, attracted

12. a.  $-\hat{y}$  for 1; 0 for 2;  $\hat{y}$  for 3  
 b.  $-\hat{y}$  for 1;  $\hat{y}$  for 2;  $-\hat{y}$  for 3
13. along  $\hat{x}$ , opposite  $\hat{x}$ , along  $\hat{x}$
14. a. into paper on  $R$  side, out of paper on  $L$  side  
 b. into paper inside of  $U$ , out of paper outside of  $U$ .
15. a.  $\hat{y}$ ,  $-\hat{x}$ ,  $-\hat{y}$ ,  $\hat{x}$   
 b.  $\hat{y}$ ,  $-\hat{x}$ ,  $-\hat{y}$ ,  $\hat{x}$   
 c.  $\hat{x}$ ,  $-\hat{x}$ ,  $\hat{x}$
16. a. opposite  $\hat{x}$   
 b.  $I$ , into paper on top, and out of paper at bottom
17. a. both along  $\hat{x}$ ; b. attract
18. a. yes, attracted; b. yes, attracted  
 c. attractive magnetic force pulls iron fragment out
19. a. out of paper; b. out of paper  
 c. out of paper; d. radially outward  
 e. same answer; f. expand; g. surrounded by casing
21. The weight rises because the spring is compressed
22.  $B = 2\pi k_m I/a$
23.  $B$  is  $\sqrt{2} \approx 1.41$  times larger
24. a.  $2k_m NwI^2/D$ ; b.  $\sqrt{DF/(2k_m Nw)}$   
 c. 10 ampere; d. 300 coulomb
25.  $B$  remains unchanged. ( $B$  does *not* depend on number of turns.)
27.  $2\pi k_m I a^2 / (a^2 + b^2)^{3/2}$
51. a. Magnetic force on iron filings: attracted; brass and aluminum remain.  
 b. same answers
52. a. into paper at  $P_0$ , out of paper at  $P_1$   
 b. 0 at  $P_0$ , opposite to  $\hat{x}$  at  $P_1$
53. a. opposite  $\hat{x}$ , opposite  $\hat{y}$ , along  $\hat{x}$ , along  $\hat{y}$

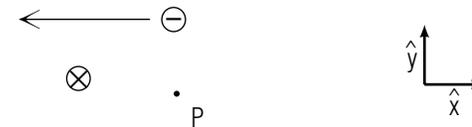
- b. equal
54. a.  $A/A_0$ ; b.  $\vec{M} = N\vec{M}_0$
55. a.  $\vec{B} = 0$   
b. out of paper at  $P_1$ , into paper at  $P_2$
56. Magnet rotates because its magnetic moment tends to remain aligned along magnetic field.
57. out at right, into at left
59.  $B_1 = k_m \ell I / a^2$
60.  $B_c = B_e \tan \theta$
61. a.  $k_m I \ell_1 / R^2$ ; b.  $k_m I \ell_1 \sin \theta / R^2$   
c.  $2\pi k_m I a \sin \theta / R^2$ ; d. 0  
e.  $2\pi k_m I a^2 / (a^2 + b^2)^{3/2}$
62.  $I = (aB_e / 2\pi N k_m) \tan \theta$
63. a. toward north; b. north to south
64. a.  $8 \times 10^{-17}$  newton opposite  $\hat{x}$   
b.  $8 \times 10^{-17}$  newton outward from wire
65. a. 0.07 gauss; b. yes

## MODEL EXAM

### INSTRUCTIONS

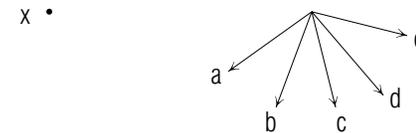
To specify directions (if no explicit instructions are given) use the unit vectors  $\hat{x}$  and  $\hat{y}$  given in the diagrams, and “into the paper” or “out of the paper,” as necessary. The directions opposite to  $\hat{x}$  and  $\hat{y}$  can be designated by  $-\hat{x}$  and  $-\hat{y}$ , respectively.

1. **Magnetic field due to a moving charged particle.** The following diagram shows a moving charged particle, and a nearby point at which the direction of the magnetic field due to this particle is to be found.



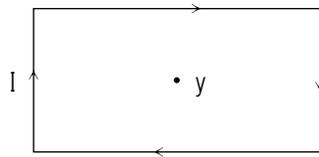
What is the direction of the magnetic field at  $P$ ?

2. **Direction of motion to produce a given magnetic field.** The following diagram shows several possible directions that can be taken by a particle of fixed charge and speed.



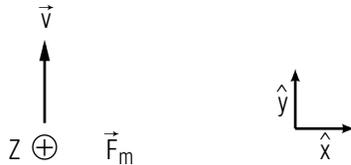
Which of the arrows in the diagram represents the velocity which would lead to the magnetic field of smallest magnitude at the point  $X$ ?

3. **Magnetic field direction near a rectangular loop.** A single rectangular loop of wire, carrying current in a clockwise sense, is shown in this diagram:



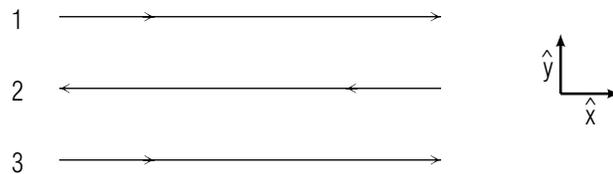
What is the direction of the magnetic field  $\vec{B}$  due to this loop at the point  $Y$ ?

4. **Direction of magnetic field line at a point in space.** A positively-charged particle, moving with a velocity  $\vec{v}$  at a point  $Z$  in space, has a magnetic force  $\vec{F}_m$  exerted on it, as shown in this diagram:



What is the direction of the magnetic field line at the point  $Z$ ?

5. **Magnetic interaction between current-carrying wires.** The three wires shown in the following diagram carry currents of the same magnitudes in the senses shown.



What is the direction of the force exerted on wire 3 due to its interaction with wires 1 and 2?

**Brief Answers:**

1. into the paper

2. e  
 3. into the paper  
 4.  $\hat{x}$   
 5.  $-\hat{y}$  (or opposite to  $\hat{y}$ )