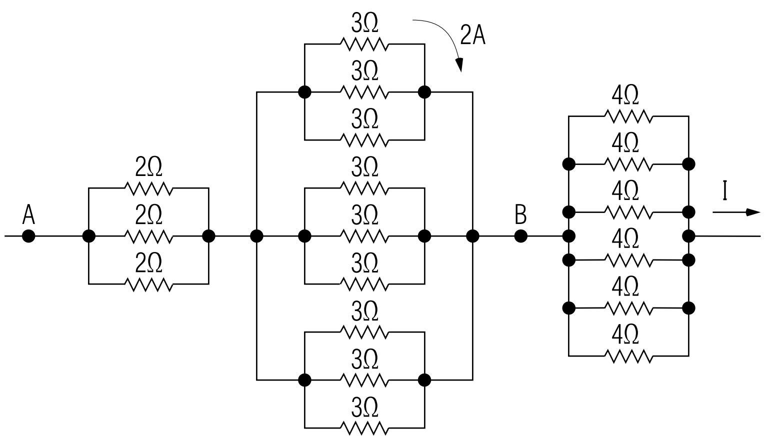
MISN-0-424

RESISTORS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

RESISTORS

by

F. Reif, G. Brackett and J. Larkin

CONTENTS

Title: **Resistors**

Author: F. Reif and J. Larkin, Dept. of Physics, Univ. of Calif., Berkeley.

Version: 5/1/2002 Evaluation: Stage 0

Length: 1 hr; 56 pages

Input Skills:

1. Vocabulary: electric current, two-terminal system, series connection, parallel connection, electric resistance (MISN-0-423).
2. State the two principles of circuit analysis (MISN-0-423).

Output Skills (Knowledge):

K1. Vocabulary: conductivity, mobility, resistor, resistivity, semiconductors.

K2. State Ohm's law for a resistor.

K3. State the relationship between conductivity and resistivity.

K4. Describe the atomic origin of resistivity.

K5. Explain how resistors can be used as transducers.

Output Skills (Problem Solving):

S1. Given some of the currents, resistances, and potential differences in a system of connected resistors, determine the unknown currents, resistances, and potential differences by systematically applying the two principles of circuit analysis and Ohm's law.

S2. Given the resistances of two resistors connected in parallel or in series, compare the potential drops across, or currents through, each resistor.

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

PROJECT STAFF

Andrew Schnepp	Webmaster
Eugene Kales	Graphics
Peter Signell	Project Director

ADVISORY COMMITTEE

D. Alan Bromley	Yale University
E. Leonard Jossem	The Ohio State University
A. A. Strassenburg	S. U. N. Y., Stony Brook

Views expressed in a module are those of the module author(s) and are not necessarily those of other project participants.

© 2002, Peter Signell for Project PHYSNET, Physics-Astronomy Bldg., Mich. State Univ., E. Lansing, MI 48824; (517) 355-3784. For our liberal use policies see:

`http://www.physnet.org/home/modules/license.html`.

MISN-0-424

RESISTORS

Abstract:
The preceding unit has provided us with the necessary principles needed to discuss electric currents flowing in many different kinds of systems. In the present unit we shall examine particularly common and simple systems (such as copper wires) in which steady currents flow merely as a result of coulomb electric forces. In the next unit we shall then turn our attention to more complex systems (such as batteries) in which steady currents flow as the result of chemical or other processes.

SECT.

A RESISTORS AND OHM'S LAW

► *Def. of resistor*

We want to examine particularly simple two-terminal systems called electric "resistors," and by:

Def. | **Resistor**: A dissipative two-terminal system in which steady currents flow solely as a result of work done by coulomb forces. | (A-1)

Such a resistor is conventionally indicated in a circuit diagram by the graphic symbol shown in Fig. A-1 (i.e., by a zig-zag line between two terminals). For example, such a resistor might simply be a metal wire whose two ends are the terminals of the resistor.

► *Ohm's law*

According to Def. (A-1), *no* non-coulomb work is done on a charged particle moving through a resistor. In other words, the emf $\mathcal{E}$ of a resistor is zero. Accordingly, in all ordinary cases where the current through a resistor is not too large, the relation Relation (F-4) of Unit 423 implies that the current I flowing through a resistor from its terminal a to its terminal b is related to the potential drop from a to b so that

$$RI = V \qquad \text{(A-2)}$$

This relation, first discovered by Ohm, is called Ohm's law. Here the constant R (independent of I or V) is positive or zero and is called the "resistance" of the resistor. Its reciprocal $1/R$ is called the "conductance" of the resistor. According to Eq. (A-2), the resistance $R = V/I$ is thus a quantity which describes the relation between the current I through a resistor and the potential drop V between the terminals of this resistor. [As pointed out in Relation (F-6) of Unit 423, the unit of resistance is volt/ampere = ohm.]

► *Utility of Ohm's law*

Suppose that we know the resistance R of a resistor (e.g., from measurements of one particular current and corresponding potential drop

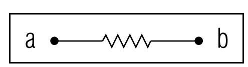


Fig. A-1: Circuit symbol for a resistor.

through this resistor). Then we can use this knowledge of the resistance to find, for *any* current I through the resistor, the corresponding potential drop $V = RI$ between the terminals of the resistor. Conversely, we can also find, for *any* potential drop V between the terminals of the resistor, the corresponding current $I = V/R$ through the resistor.

DISCUSSION

► *Sense of current*

According to Eq. (A-2), the potential drop across a resistor from a to b has the same sign as the current flowing through the resistor from a to b. Thus the potential drop is positive "along the sense of the current" (i.e., going between terminals along that sense along which positive current flows). The statement implies this conclusion:

> Positive current flows through a resistor from the terminal at the higher potential to the terminal at the lower potential. (A-3)

This conclusion is a direct consequence of the fact that, if charged particles move through a resistor despite the dissipation of their energy into random internal energy, positive coulomb electric work must be done on these particles (i.e., their coulomb potential energy must decrease). For example, if the potential V_a at terminal a is larger than the potential V_b at terminal b, mobile positively charged particles move from a to b since positive coulomb work is then done on them (corresponding to a *decrease* in their coulomb potential energy).

The conclusion, Rule (A-3), is also apparent by considering the forces on the charged particles in the simple case of a straight wire with ends a and b. If V_a is larger than V_b, the electric field inside the wire is then directed from a to b. Hence the electric force on positively charged particles is also directed from a to b so that such mobile particles move through the wire from a to b. *

> * The negatively charged electrons in a *metal* wire would move through the wire in the opposite direction (since the direction of the force on them is opposite), but would give rise to an electric current in the same sense as moving positively charged particles.

► *Magnitude of current*

According to Ohm's law $RI = V$, the current I through a resistor is proportional to the potential drop V between the terminals of the resistor. (For example, if this potential drop is 3 times as large, the current through the resistor is also 3 times as large.) In particular, the current I through a resistor is zero if the potential drop V between its terminals is zero. (Indeed, if $V = 0$, the electric field inside the resistor is zero. Hence no net electric force acts on the mobile charged particles in the resistor. Thus these particles cannot be kept moving while repeatedly colliding with all the atoms in the resistor).

By Ohm's law, a given potential drop V maintained between the terminals of a resistor produces through this resistor a current $I = V/R$. If the resistance R of the resistor is large, the current I is then small. Conversely, if the resistance R is small, the current I is large.

ANALOGY TO FLOW OF A FLUID

The steady flow of electric current through a resistor is analogous to the steady flow of a fluid in a horizontal pipe. In the latter case the fluid (or mass) current I flowing through the pipe is related to the pressure drop p between the ends of the pipe so that $RI = p$ where the constant R is the flow resistance characterizing the pipe.

Understanding $RI = V$ (Cap. 1)

A-1 *Relating quantities:* The two terminals of a light bulb have potentials $V_a = 90$ volt and $V_b = 0$ volt (because they are connected to the terminals of a battery). (a) What is the magnitude of the current through the lighted bulb if its resistance is 120 ohm? Is the sense of this current from a to b, or from b to a? (b) Suppose the light bulb is connected to the battery so that $V_a = 0$ volt and $V_b = 90$ volt. What then are the magnitude and sense of the current through the lighted bulb? *(Answer: 4)*

A-2 *Properties:* What algebraic symbols usually represent the quantities resistance, current, and potential drop? What are the units and the possible signs of each of these quantities? *(Answer: 8)*

A-3 *Meaning of V:* (a) Fig. A-2 shows two resistors connected in series so that a current of magnitude 0.30 ampere flows through each. These resistors have resistances $R_1 = 10\,\Omega$ and $R_2 = 5.0\,\Omega$. What

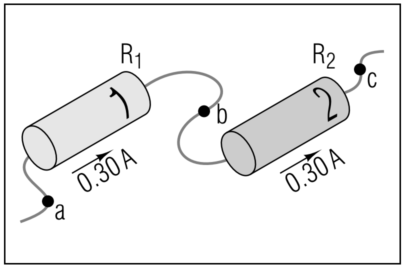


Fig. A-2.

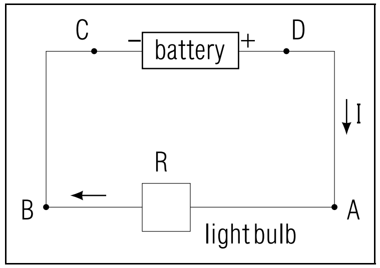


Fig. A-3.

are the potential drops V_{ab} and V_{bc} across each of these resistors? (b) *Organization of relations:* What is the potential drop V_{ac} across the system of two resistors? *(Answer: 3) (Suggestion: [s-3])*

A-4 *Interpretation of R:* If a particular resistor remains in the same condition (i.e., its temperature and arrangement of atoms remain the same), which of the following quantities must also remain the same: (a) The current I through the resistor. (b) The potential drop V across the resistor. (c) The resistance $R = V/I$ of the resistor. *(Answer: 1)*

A-5 *Dependence of V on R:* Suppose that the resistors shown in Fig. A-2 have resistances related by $R_1 = 3R_2$. Because the resistors are connected in series, the same current I flows through each. Is the potential drop V_1 across resistor 1 equal to 3, 1, or 1/3 times the potential drop V_2 across resistor 2? *(Answer: 6)*

A-6 *Organization of relations:* In different systems, the work per unit charge w is given by different expressions. Fig. A-3 shows a circuit diagram for a battery (having emf $\mathcal{E}$ and resistance r) connected to a lightbulb (which has a resistance R when lighted). The potential drop from A to B through the bulb is V. (a) Which of the expressions R_1, r_1, or V equals the total work per unit charge w done on a particle moving from A to B through the light bulb? (b) Which expression equals the work per unit charge w_b done on a particle imagined to move from A to B through the battery? *(Answer: 10) (Suggestion: [s-5]) (Practice: [p-1])*

SECT.

B COMBINATIONS OF RESISTORS

Resistors can be connected together in various ways so that their combination forms again a two-terminal system. This combined system is also a resistor (since only coulomb work is done on charged particles in the system). How then is the resistance R of the combined system related to the resistances of the individual resistors in the system?

RESISTORS IN SERIES

► *R of combination*

Suppose that two resistors, with resistances R_1 and R_2, are connected "in series," as discussed in text section E of Unit 423 and illustrated in Fig. B-1. Then the *same* steady current I flows through each resistor (and thus also through the combined system) since the steady current flowing out of one resistor must be equal to the steady current flowing into the next resistor. But the potential drop V across the combined system (i.e., from terminal a to terminal d in Fig. B-1) is just the sum of the potential drops across each of the individual resistors, i.e.,

$$V = V_1 + V_2 \qquad \text{(B-1)}$$

To find the resistance $R = V/I$ of the combined system, we need only divide both sides of Eq. (B-1) by the common current I through the resistors. Thus we get

$$\frac{V}{I} = \frac{V_1}{I} + \frac{V_2}{I}$$

or

$$\boxed{R = R_1 + R_2} \qquad \text{(B-2)}$$

where R_1 and R_2 are the resistances of the individual resistors. Thus we arrive at the following conclusion, valid for any number of resistors connected in series:

> The resistance of resistors connected in series is the sum of their individual resistances. (B-3)

► *Magnitude of R*

According to Rule (B-3), the resistance R of the combined system is *larger* than the resistance of any individual resistor. (This conclusion

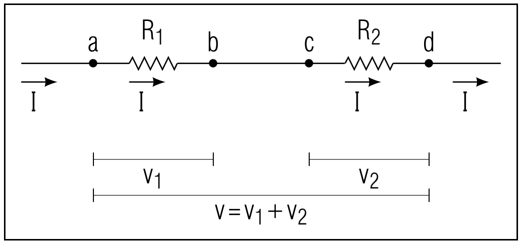


Fig. B-1: Resistors connected in series.

makes sense since it is more difficult for the current to flow successively through a series of resistors than through a single resistor alone.)

► *Relation between V_1 and V_2*

When resistors are connected in series, as indicated in Fig. B-1 the same current I flows through each. Hence the potential drops through these resistors are respectively $V_1 = R_1 I$ and $V_2 = R_2 I$. Accordingly,

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} \tag{B-4}$$

Thus the potential drop through the resistor with the *larger* resistance is proportionately *larger* than that through the resistor with the smaller resistance.

► *Identical resistors*

Suppose that N resistors, each having the same resistance R_1, are connected in series. By Rule (B-3), the resistance R of the combination is then merely the sum of N resistances, each equal to R_1. Thus

$$R = N R_1 \tag{B-5}$$

In short, the resistance of the combined system is simply N times larger than that of a single resistor.

RESISTORS IN PARALLEL

Suppose that two resistors, with resistances R_1 and R_2, are connected "in parallel," as discussed in text section E of Unit 423 and illustrated in Fig. B-2. Then there is the *same* potential drop across each resistor (and also across the combined system) since the systems are connected together so that their corresponding terminals have the same potential. But the steady current I flowing into the combined system at e in Fig. B-2 flows partly through each of the individual resistors (before flowing out of the other terminal f of the combined system). Hence the current I through the combined system is the *sum* of the currents I_1 and

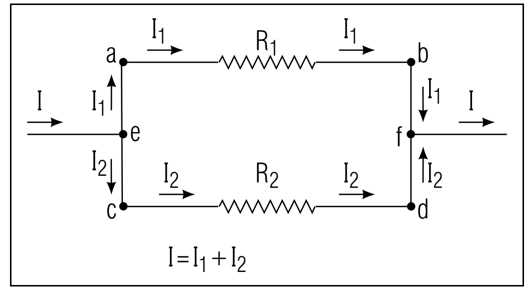


Fig. B-2: Resistors connected in parallel.

I_2 flowing through each of the individual resistors, i.e.,

$$I = I_1 + I_2 \tag{B-6}$$

To find the resistance $R = V/I$ of the combined system (or equivalently its conductance $1/R = I/V$), we need then only divide both sides of Eq. (B-6) by the common potential drop V. Thus we get

$$\frac{I}{V} = \frac{I_1}{V} + \frac{I_2}{V}$$

or

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}} \tag{B-7}$$

where $1/R_1$ and $1/R_2$ are the conductances of the individual resistors. Thus we arrive at the following conclusion, valid for any number of resistors connected in parallel:

The conductance of resistors connected in parallel is the sum of their individual conductances.	(B-8)

► *Magnitude of R*

According to Rule (B-8), the conductance $1/R$ of the combined system is *larger* than the conductance of any of the individual resistors. Correspondingly, the reciprocals of these quantities are related so that the resistance R of the combined system is *smaller* than the resistance of any of the individual resistors. (This conclusion makes sense since it is easier for the current to flow through several alternate paths in the combined system than through only the single path through one resistor.)

► *Relation between I_1 and I_2*

When resistors are connected in parallel, as indicated in Fig. B-2, the same potential drop exists across both. Hence the currents I_1 and I_2 through these resistors are respectively such that $V = R_1 I_1$ and $V = R_2 I_2$. Accordingly,

$$R_1 I_1 = R_2 I_2 \qquad \text{(B-9)}$$

Thus the current through the resistor with the larger resistance is *smaller* than that through the resistor with the smaller resistance.

► *Identical resistors*

Suppose that N resistors, each having the same resistance R_1, are connected in parallel. By Eq. (B-10), the conductance $1/R$ of the combined system is then merely the sum of N conductances, each equal to $1/R_1$. Thus

$$\frac{1}{R} = N\left(\frac{1}{R_1}\right)$$

or

$$R = \frac{R_1}{N} \qquad \text{(B-10)}$$

Hence the resistance of the combined system is simply N times *smaller* than that of a single resistor.

Systematically Relating Currents and Potentials (Cap. 2)

B-1 *Resistors in series:* Suppose the two resistors with the resistances R_1 and R_2 listed in Table B-1 are connected in series and then to the terminals of a battery such that a current of 0.10 ampere flows through the battery (and through each resistor). (a) Describe this situation by drawing a circuit diagram showing: the two resistors connected to the terminals a (positive) and b (negative) of the battery; an arrow indicating the sense of the current through the resistors. (b) What are the potential drops V_1 and V_2 across each resistor (in the direction of current flow)? (c) What is the potential drop V_{ab} across the system of the two resistors? *(Answer: 5)*

B-2 *Resistors in parallel:* Two resistors with the resistances listed in Table B-1 are connected in parallel and then to the terminals of a 12 volt battery. (a) Describe this situation by drawing a circuit diagram which shows: labeled symbols for the resistors; the battery with terminals labeled by a (positive) and b (negative); arrows and the symbols I_1 and I_2 indicating the sense and magnitude of the current through each resistor. (b) Apply $V = RI$ to find the magnitude of the current through each resistor. (c) Apply $I_{\rm in} = I_{\rm out}$ to find the magnitude I of the current through the battery. *(Answer: 2) (Suggestion: [s-7])*

$R_1 = 20$ ohm	
$R_2 = 40$ ohm	

Table B-1

Series	*Current*	*Potential drop*
R_1	0.10 A	2.0 volt
R_2	0.10 A	4.0 volt
Parallel		
R_1	0.60 A	12 volt
R_2	0.30 A	12 volt

Table B-2

B-3 *Resistance of a system:* Table B-2 summarizes the results of problems B-1 and B-2 by listing (for each of the individual resistors) the magnitudes of the current through that resistor and the potential drop across it when these resistors are connected in series (problem B-1) and in parallel (problem B-2). (a) For each of the two systems (series or parallel), what is the magnitude of the current I flowing into the system at a (and out at b)? What is the potential drop V_{ab} across the system? (b) Use the relation $V = RI$ to find the resistance of each of these systems. *(Answer: 9) More practice for this Capability: [p-2], [p-3]*

Relating Resistances for Parallel and Series Connections (Cap. 3)

B-4 Apply Eqs. (B-2) and (B-7) to find resistances for systems composed of the two resistors with the resistances listed in Table B-1 (a) connected in series and (b) connected in parallel. (c) Are your answers consistent with your answers for problem B-3? *(Answer: 17)*

B-5 If you have 5 identical copper wires, each having a resistance of 2.0 ohm, what are the resistances of the systems made of these wires (a) connected in series, and (b) connected in parallel? (c) Suppose these systems are each connected to the terminals of a 6.0 volt battery. What is the current into (and out of) each system? *(Answer: 13) (Sug-*

gestion: [s-1])

B-6 A single lighted light bulb is connected to the terminals of a battery. A second bulb can be connected either in series or in parallel with the first bulb. Is the resistance of the system made of the two connected bulbs larger or smaller than the resistance of the original bulb, (a) if the two are in series, and (b) if the two are in parallel? *(Answer: 19) (Suggestion: [s-14]) (Practice: [p-4])*

Comparing *I* and *V* for Two Resistors (Cap. 4)

B-7 A resistor 1 has a resistance which is larger than the resistance of a resistor 2. (a) If the two resistors are connected in series, is the current through resistor 1 larger, smaller, or the same in magnitude as the current through resistor 2? Is the potential drop across resistor 1 larger, smaller, or the same in magnitude as the potential drop across resistor 2? (b) Answer the preceding questions for the two resistors connected in parallel. *(Answer: 29) (Suggestion: [s-11])*

SECT. C EXAMPLES ILLUSTRATING RESISTOR COMBINATIONS

To illustrate some applications involving combinations of resistors, we shall discuss two examples, the first one quantitative and the second one qualitative.

Example C-1: Battery connected to two lamps

A storage battery maintains a fixed potential drop of 12 volt from its terminal a to its terminal b. As indicated in the circuit diagram in Fig. C-1a, this battery is connected by two wires, each having a resistance of 0.3 ohm, to two lamps having resistances $R_1 = 4$ ohm and $R_2 = 6$ ohm. What then is the current I flowing through the battery? Also, what are the currents I_1 and I_2 flowing through the two lamps?

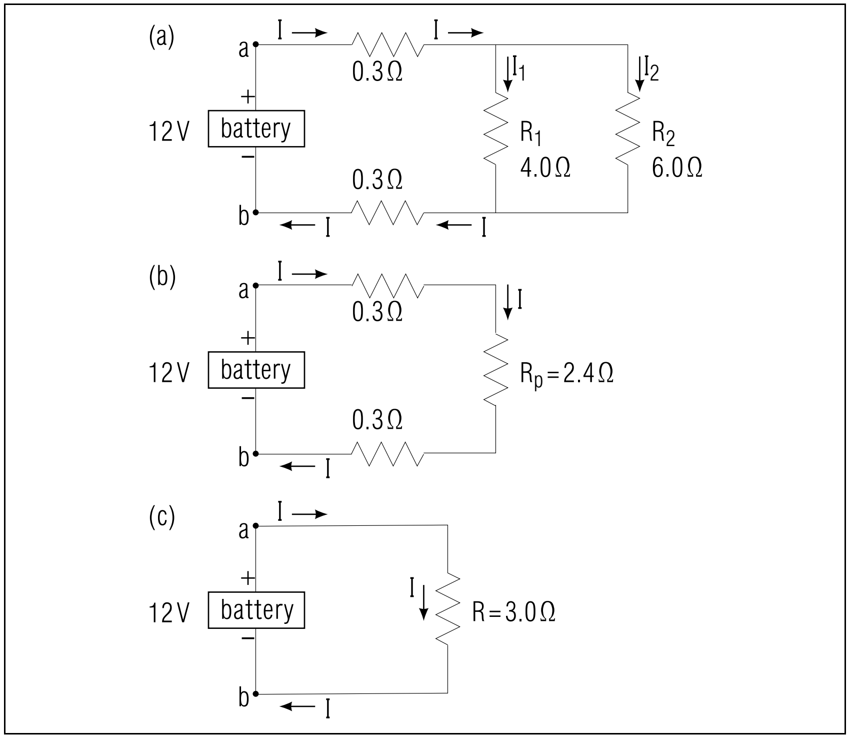


Fig. C-1: Circuit representing a battery connected by wires to two lamps. (a) Original circuit. (b), (c) Circuits equivalent to the original circuit.

► *Equivalent circuit*

The two lamps in Fig. C-1a are connected in parallel. Hence the resistance R_p of the parallel combination of these lamps is, by Eq. (B-7), such that

$$\frac{1}{R_p} = \frac{1}{4.0\,\Omega} + \frac{1}{6.0\,\Omega} = \frac{6.0\,\Omega + 4.0\,\Omega}{24.0\,\Omega^2} = \frac{10.0\,\Omega}{24.0\,\Omega^2} = \frac{1}{2.4\,\Omega}$$

Hence $R_p = 2.4\,\Omega$ and the circuit in Fig. C-1a is simply equivalent to that in Fig. C-1b. But the three resistances indicated in this circuit are seen to be connected in series. The resistance R of their combination is then, by Rule (B-3), simply the sum $R = 0.3\,\Omega + 2.4\,\Omega + 0.3\,\Omega = 3.0\,\Omega$. Thus the circuit in Fig. C-1b is equivalent to that in Fig. C-1c.

► *Current through battery*

In this last circuit, equivalent to that in Fig. C-1a, the current flowing through the resistance $R = 3.0\,\Omega$ is the same as the current I through the battery. *

> * To avoid excessive verbiage, one often uses the simple expression "resistance R" to mean, more precisely, "resistor with resistance R."

Hence the potential drop RI from a to b across this resistance must be equal to the known potential drop of 12 volt from a to b across the battery. Thus

$$(3.0\,\Omega)I = 12\,\text{volt}$$

or

$$I = 4.0\,\text{ampere} \tag{C-1}$$

► *Current through lamps*

In Fig. C-1a, the potential drop across each of the two lamps connected in parallel is the same. Hence $R_1 I_1 = R_2 I_2$ so that

$$(4.0\,\Omega)I_1 = (6.0\,\Omega)I_2$$

or

$$I_1 = 1.5 I_2 \tag{C-2}$$

But the steady current I flowing out of the battery must be equal to the sum of the steady currents flowing into these two lamps. Thus

$$I = I_1 + I_2 = 1.5 I_2 + I_2 = 2.5 I_2 \tag{C-3}$$

Using the value of I found in Eq. (C-1), we then conclude that $I_2 = I/2.5 = (4.0\,\text{ampere})/2.5 = 1.6\,\text{ampere}$. By Eq. (C-2), the current through the other lamp is then $I_1 = (1.5)(1.6\,\text{ampere}) = 2.4\,\text{ampere}$.

Example C-2: Skin resistance and electrocution

A person, repairing an appliance connected to an electric outlet, touches one point a on the appliance with one hand and another point b with the other hand. A potential difference of 110 volt exists between these points. What is likely to happen to this person?

► *Current through person*

Because of the potential difference $V = 110$ volt between a and b, a current I flows through the person from a to b. This current is equal

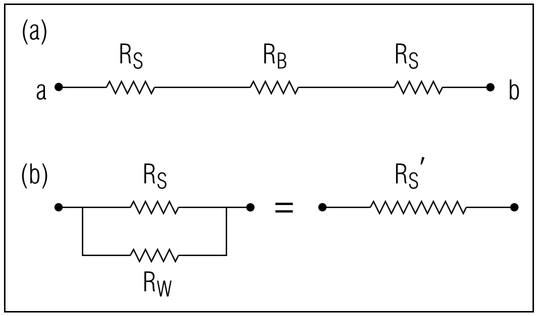


Fig. C-2: (a) Resistances encountered by a current flowing through a person. (b) Resistance representing one layer of wet skin.

to $I = V/R$, where R is the resistance of the person between the points a and b. The magnitude of the current I depends thus on the magnitude of this resistance. In particular, if the current I is large enough to cause fibrillation of the heart, the person may be killed.

► *Dry Skin*

The current I must flow successively through the skin of the hand at a, then through the body of the person, and finally through the skin of the hand at b. Suppose that the resistance of each layer of skin is R_S and that the resistance of the intervening body is R_B. Then the current I must flow successively through the three resistances indicated in Fig. C-2a. The resistance R between a and b is then that of three resistances in series, i.e., R is equal to the sum $R = R_S + R_B + R_S = 2R_S + R_B$. If the skin is dry, the resistance R_S of the skin is quite large, much larger than the resistance R_B of the inside body (which consists mostly of water containing ions). Hence the entire resistance R is also quite large and the current $I = V/R$ may correspondingly be small enough so that the person avoids being electrocuted.

► *Wet skin*

Suppose, however, that the person's skin is wet. Then the current through the skin of each hand can flow from one side of the skin through the other not only through the skin tissue itself, but also through the water contained in the pores of the skin. In other words, as illustrated in Fig. C-2b, the current can now also flow through another path (the water of resistance R_W) in parallel with the original path (of resistance R_S) directly through the dry skin. But since the resistance R_W of the water is smaller than the resistance R_S of the dry skin, the resistance R'_S of the wet skin (i.e., of the resistances of Fig. C-2b in parallel) is smaller than R_W and thus considerably smaller than the resistance of dry skin. As a result, the total resistance R between a and b (given by Fig. C-1a, with

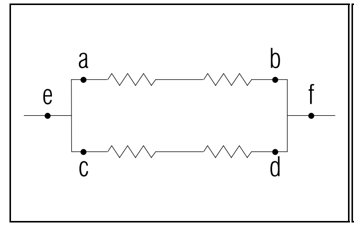


Fig. C-3.

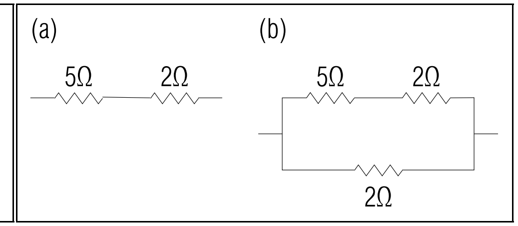


Fig. C-4.

the small resistance R'_S of wet skin replacing the much larger resistance R_S of dry skin) is now smaller than when the skin is dry. Hence the current $I = V/R$ produced by the same potential difference V can now be large enough to kill the person.

Relating Resistances for Parallel and Series Connections (Cap. 3)

C-1 Figure C-3 shows a system composed of four identical resistors each having a resistance of 50 ohm. (a) What is the resistance of each of the two two-terminal systems with terminals a and b and with terminals c and d? (b) What is the resistance of the entire system with terminals e and f? (c) Is this resistance larger than, smaller than, or equal to the resistance of one of the individual resistors? *(Answer: 11) (Suggestion: [s-2])*

C-2 (a) A 5 ohm resistor is connected in series to a 2 ohm resistor (Fig. C-4a). Is the resistance of the combined system larger or smaller than the original 5 ohm? (b) The system described in part (a) is now connected in parallel to another 2 ohm resistor (Fig. C-4b). Is the resistance of this three-resistor system larger or smaller than the resistance of the two-resistor system described in part (a)? *(Answer: 25) (Suggestion: [s-4])*

C-3 Two lamps having resistances of 200 ohm and 300 ohm are connected in parallel (because both are plugged into the same wall outlet). (a) Does the system of the two lamps have a resistance which is smaller than 200 ohm, between 200 and 300 ohm, or larger than 300 ohm? (b) Suppose a 100 ohm resistor could be connected in series with the system described in part (a). Would the addition of this resistor increase or decrease the resistance of the combined system? *(Answer: 30) (Suggestion: [s-4])*

SECT.

D RESISTIVITY

How does the resistance between the ends of a homogeneous uniform rod (or wire) depend on its geometric properties, such as its length and cross-sectional area?

► *Comparison with small rod*

To answer this question, consider any such rod of length L and uniform cross-sectional area A. We can then compare the resistance R of this rod with the resistance R_0 of some standard rod, made of the same material, having a small length L_0 and a small cross-sectional area A_0. (See Fig. D-1a.) To achieve this comparison, we need only imagine that the original rod is built up of many of the small standard rods. We can do this by first placing a certain number N of these standard rods end-to-end (as illustrated in Fig. D-1b) so as to make up a thin rod having a cross-sectional area A_0 and the length L of the original rod. The required number N of standard rods is then such that $NL_0 = L$ or $N = L/L_0$. Then we can place a certain number N' of these thin rods side-by-side (as illustrated in Fig. D-1c) so as to make up the original rod with the desired cross-section of area A. The required number N' of such thin rods is then such that $N'A_0 = A$ or $N' = A/A_0$.

► *Comparing resistances*

Let us then compare the resistance R_0 of the standard rod with the resistance R of the original rod. The N standard rods placed end-to-end to form the thin rod in Fig. D-1b, constitute N identical resistors, each of resistance R_0, connected in series. By Rule (B-5), the resistance of this thin rod is then NR_0. Such N' thin rods, placed side-by-side to form the original rod in Fig. D-1c, constitute N' identical resistors, each of resistance NR_0, connected in parallel. *

* Strictly speaking, the thin rods would be connected in parallel if their ends were joined together without their side surfaces being in contact. But the net current through such a side surface is zero, even if the side surfaces touch. We assume (as is verified by observations) that the current flowing along the rod is unaffected by whether the side surfaces of the thin rods touch each other along their entire lengths or not.

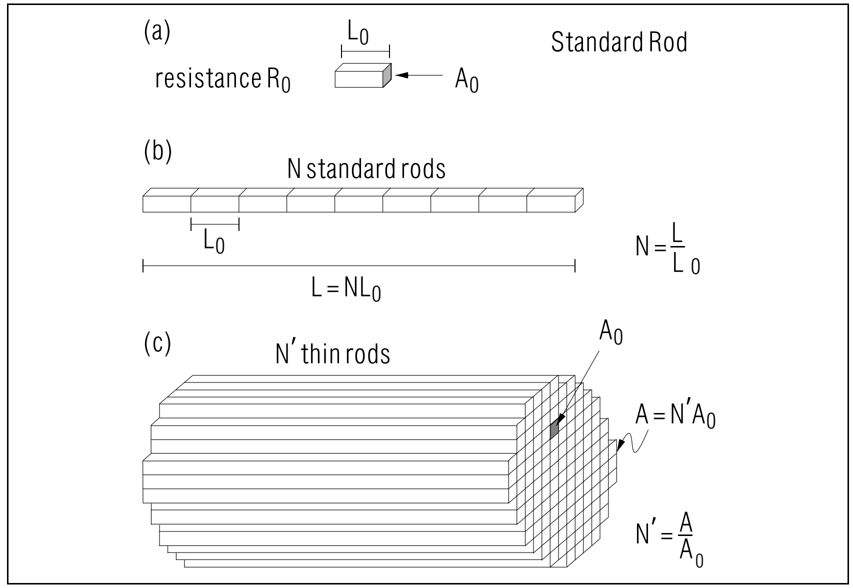


Fig. D-1: Combining small standard rods to make other rods.

By Eq. (B-10), the resistance R of the original rod is then N' smaller than that of a single thin rod, i.e.,

$$R = \frac{NR_0}{N'} = \frac{\left(\frac{L}{L_O}\right) R_0}{\left(\frac{A}{A_0}\right)} = \left(\frac{A_0 R_0}{L_0}\right) \frac{L}{A} \qquad \text{(D-1)}$$

or

$$\boxed{R = \rho \frac{L}{A}} \qquad \text{(D-2)}$$

where the quantity ρ involves only the quantities L_0, A_0, and R_0 characterizing the small standard rod.

► *Dependence of R on L and A*

The relation (D-2) applies to any rod of the specified material since any such rod can be compared with the small standard rod of this material. Thus Eq. (D-2) implies that the resistance R of any such rod is directly proportional to its length L, and inversely proportional to its cross-sectional area A. For example, if the length L of a rod of given cross-sectional area were 3 times as large, its resistance would also be 3 times as large. But if the cross-sectional area A of a rod of given length were 3 times as large, its resistance R would be 3 times as *small*.

► *Resistivity*

According to Eq. (D-2),

$$R\frac{A}{L} = \rho \qquad \text{(D-3)}$$

But the quantity ρ depends only on the characteristics of the small standard rod of the given material. Hence the quantity RA/L on the left side of Eq. (D-3) must have the *same* value for *any* homogeneous rod of the given material. In other words, the resistance R of any rod depends on its geometric properties (i.e., its length L and cross-sectional area A). But the quantity RA/L for any rod does *not* depend on its geometric properties, but only on the material of the rod. This quantity $RA/L = \rho$ describes thus a property characteristic of the given material and is called the electric "resistivity" of the material.

Def. **Resistivity**: $\rho = R\dfrac{A}{L}$ (D-4)

According to this definition, the resistivity of a material is the quantity obtained by multiplying the resistance R of *any* homogeneous rod of this material by the cross-sectional area A of the rod and then dividing by the length L of the rod. The measured resistivities of some common materials are listed in Table D-1. Note that the resistivities of metals, which are good conductors, are very much lower than those of other materials.

► *Finding R*

If one knows the resistivity of a material (e.g., from measurements of the resistance of one particular rod of this material), one can then use Eq. (D-2) to find the resistance of *any* homogeneous rod (or wire) of this material.

► *Conductivity*

The "conductivity" σ of a material is defined as the reciprocal of its resistivity.

Def. **Conductivity**: $\sigma = \dfrac{1}{\rho}$ (D-5)

Material	Resistivity ρ (ohm meter)
silver	1.6×10^{-8}
copper	1.7×10^{-8}
aluminum	2.7×10^{-8}
tungsten	5.6×10^{-8}
nickel	7.8×10^{-8}
iron	1.0×10^{-7}
constantan (Cu-Ni alloy)	4.5×10^{-7}
carbon (amorphous)	3.5×10^{-5}
sea water	2×10^{-1}
germanium	5×10^{-1}
water (distilled)	5×10^{3}
glass	$\approx 10^{12}$
rubber	$\approx 10^{15}$

Table D-1: Resistivities of some materials at room temperature. resistivity, table5

D-1 A 1.0 meter length of No. 10 gauge brass wire has a cross-sectional area of $8.2 \times 10^{-7}\,\text{m}^2$ and a resistance of $8.5 \times 10^{-2}\,\Omega$. What is the resistivity of brass? *(Answer: 12)*

D-2 A piece of zinc wire has a length of 4.0 meter, a cross-sectional area of 8.0×10^{-7} meter^2, and a resistance of $0.30\,\Omega$. What is the resistance of a zinc wire with length 8.0 meter and cross-sectional area 2.0×10^{-7} meter^2? *(Answer: 16) (Suggestion: [s-9])*

D-3 Consider two wires made of the same substance. For each of the following situations, is the resistance of wire B equal to 3, 1/3, 9, or 1/9 times the resistance of wire A? (a) The wires have the same cross-sectional area, but B is 3 times as long as A. (b) The wires have the same length, but B has a diameter three times as large as the diameter of A. (c) B is three times as long as A and the diameter of B is three times as large as the diameter of A. *(Answer: 14) (Suggestion: [s-6])*
More practice for this Capability: [p-5]

SECT.

E ATOMIC EXPLANATION OF RESISTIVITY

Let us now look more closely at how the resistivity of a material is related to its atomic properties.

CONDUCTIVITY AND MOTION OF CHARGED PARTICLES

► *Average velocity*

The resistivity ρ (or equivalently the conductivity $\sigma = 1/\rho$) of a material is related to themotion of the mobile charged particles in this material. As already mentioned in text section B of Unit 423, when the electric field $\vec{E}$ inside the material is zero, the mobile charged particles in this material merely move in completely random directions as they repeatedly collide with the atoms in the material. Hence the *average* velocity $\vec{v}$ of these charged particles is zero, as indicated in Fig. E-1a. (The material is then in equilibrium as discussed in Unit 421.) On the other hand, suppose that a non-zero electric field $\vec{E}$ exists inside the material. Then the mobile charged particles are accelerated along the direction of the electric force produced by the electric field. Although the velocity thus acquired by the charged particles never becomes very large (since the motion of these particles is repeatedly interrupted by collisions with the atoms in the material),the charged particles now drift with some small *average* velocity $\vec{v}$ in the direction of the electric force produced by the electric field $\vec{E}$, as indicated in Fig. E-1b. (This velocity $\vec{v}$ along the electric force has thus a direction along $\vec{E}$ if the particles are positively charged, and a direction opposite to $\vec{E}$ if they are negatively charged.)

► *Mobility*

The average velocity $\vec{v}$ of the mobile charged particles is thus zero if the electric field $\vec{E}$ is zero, but increases in magnitude as the electric field $\vec{E}$ is increased. Indeed, as long as $\vec{E}$ is not too large, one expects that $\vec{v}$ should be simply proportional to $\vec{E}$. This proportionality is expressed by writing

$$\vec{v} = \mu \vec{E} \qquad \text{(E-1)}$$

where the constant μ (independent of $\vec{v}$ or $\vec{E}$) is called the “mobility” of the charged particles in the material.

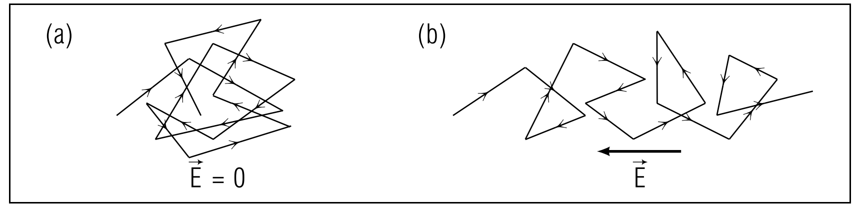


Fig. E-1: Path of a mobile electron inside a material.

► *Conductivity*

Consider a rod of given length and cross-sectional area. Then the conductivity σ of this rod is large (or its resistivity $\rho = 1/\sigma$ is small) if a given potential difference between the ends of the rod (i.e., a given electric field inside the rod) produces a large current in the rod. On the other hand, from an atomic point of view, such a large current is produced if the material contains many mobile charged particles and if these move with a large average velocity. Thus we see that the conductivity of a material is large if the material contains a large number of mobile charged particles per unit volume, and if the magnitude of the mobility of these charged particles is large (i.e., if these particles acquire a large average velocity as a result of a given electric field.)

TEMPERATURE DEPENDENCE OF THE CONDUCTIVITY

► *Effect on mobility*

Let us consider materials, such as metals, in which the mobile charged particles are electrons. In a pure material of this kind, the motion of the electrons is constantly interrupted by collisions with the vibrating atoms in the material. As the temperature is increased, the random vibration of the atoms in the material increases correspondingly, and thus the atomic collisions become more effective in interrupting the electron motion. Hence such an increase in temperature *decreases* the mobility of the electrons (i.e., reduces the average electron velocity produced by a given electric field).

► *Metals*

In the case of a metal, a certain *fixed* number of electrons are free to move throughout the metal. As the temperature of a metal is increased, the only effect is then to reduce the mobility of these electrons and thus to decrease the conductivity of the metal. Hence an increase in the temperature of a metal has the effect of *decreasing* its conductivity σ (or correspondingly *increasing* its resistivity $\rho = 1/\sigma$).

► *Semiconductors*

In the case of "semiconductors" (e.g., substances such as germanium or silicon) the mobility of the mobile electrons also decreases with increasing temperature. On the other hand, the number of mobile electrons in such a semi-conductor is *not* fixed, but increases rapidly with increasing temperature (because the increased random energy liberates electrons ordinarily bound to atoms and allows these electrons to move throughout the entire material). This large increase in the number of mobile electrons thus results in a larger conductivity despite the somewhat smaller electron mobility. Hence an increase in the temperature of a semi-conductor has the effect of appreciably *increasing* its conductivity σ (or correspondingly *decreasing* its resistivity $\rho = 1/\sigma$).

Knowing About Resistivity and Particle Motion

E-1 On the atomic level, a pure metal consists of a relatively regular array of atoms. When a second substance is mixed with the pure metal (e.g., to form a metal alloy), this regular array is disrupted by the differing atoms. Because of these irregularities, electrons moving in an alloy experience more collisions and thus have a smaller average "drift velocity" than electrons moving in a pure metal under the influence of the same electric field.

Two wires of identical size and shape are made of pure copper and of a copper alloy containing small amounts of nickel and manganese. (a) If the same potential drop is produced between the ends of these wires, through which wire is the current larger? (b) Which wire has the larger conductivity? Which has the larger resistivity? *(Answer: 18)*

Knowing About Metals and Semiconductors

E-2 The potential drop across each of two resistors remains constant because each is connected to the terminals of a battery. The two resistors are each heated, and the current through each is graphed as a function of the temperature T. Which of the two graphs in Fig. E-2 shows the current I through a metal and which shows the current I through a semi-conductor (as a function of temperature T)? *(Answer: 20)*

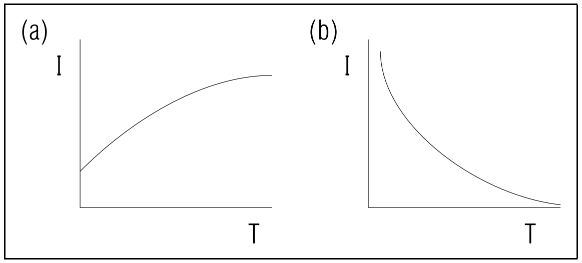


Fig. E-2.

SECT.

F RESISTORS AS TRANSDUCERS

It is quite easy to measure changes in the resistance of a resistor. (For example, if the current I through such a resistor is kept constant, one needs only to use a voltmeter to measure changes in the potential difference between the terminals of the resistor.) Hence resistors are frequently used as transducers to convert information about various interesting quantities into easily measured electrical form.

► *Temperature measurement*

As discussed at the end of last section, the resistivity of a material depends on its temperature. Hence one can use measurements of the resistance of a resistor to obtain corresponding information about the temperature of this resistor. [The resistor may be just a metal wire, or it may be a special resistor (called a "thermistor") made of a semiconducting material whose resistivity changes rapidly with small changes of temperature.] A resistor used in this way constitutes a very convenient thermometer because it can be made small and because it provides information about temperature in an electrical form. Such information can thus be transmitted through wires to remote locations, can be continuously displayed on a recording meter, or can be used to activate various kinds of electronic equipment. For example, one can use such a "resistance thermometer" to monitor continuously the body temperature of a patient in the intensive-care unit of a hospital.

► *Strain measurement*

If a thin wire is stretched, its length increases and its cross-sectional area correspondingly decreases. According to Eq. (D-2), both of these effects result in increasing the electric resistance of the wire. Accordingly, by measuring the resistance of a thin wire, one can measure the "strain" (i.e., relative elongation) of this wire and thus also the strain of any object to which the wire is glued. A thin wire used in this way is called a "strain gauge." For example, such a strain gauge fastened to a muscle (such as the heart muscle) can be used to study the contractions of this muscle.

► *Counting particles*

As a last example, we shall discuss how a measurement of resistance can be used to measure both the number and the size of particles (such as blood cells or bacteria) in an ionic solution. The principle of the method is illustrated in Fig. F-1 which shows the solution flowing from a tube

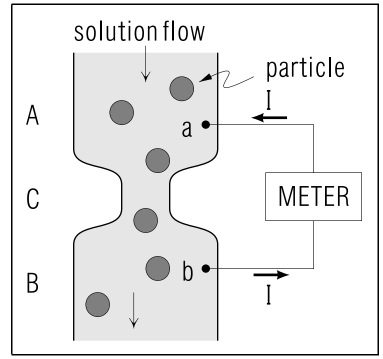


Fig. F-1: Electric measurement of number and sizes of particles in a solution.

A to a tube B through a very narrow constriction C. The resistance R between some point a in the tube A and some point b in the tube B can be readily measured (i.e., by measuring the potential difference between a and b when some constant current is made to flow through the solution from a to b). The resistance R is approximately equal to the resistance of the solution in the constriction. *

* The resistance R is really the sum of the resistances of the solutions in the tubes A, C, and B since these are in series. But since the tube C has such a very small cross-section, its resistance is much larger than that of the wider tubes.

Suppose now that a particle (of resistivity much larger than that of the surrounding solution) enters the narrow tube constituting the constriction. Then the current can only flow through the thin channel of solution left in the remaining unobstructed part of the constriction, i.e., the resistance R of the constriction is increased. Indeed, this resistance is larger if the size of the particle is larger so that it obstructs a larger part of the constriction.

Thus we see that, whenever a particle passes through the constriction, the measured resistance R increases. Furthermore, the amount by which this resistance increases indicates the size of the particle passing through the constriction.

The method which we have described is used in an instrument called the "Coulter counter," named in honor of its inventor. For example, such a Coulter counter is commonly used to measure quickly and automatically the number of blood cells in a sample of blood (i.e., to make a "blood count").

Knowing About Resistors As Transducers

F-1 *Measurement of moisture content:* Adding water to a solid substance ordinarily increases its conductivity because charged ions dissolved in the water are mobile. A variety of instruments for measuring moisture content consist essentially of two terminals which are maintained at a constant potential difference, and which can be placed on, or inserted into, a substance. The current through the substance (between the terminals) can then be measured and used to assess moisture content. Would each of the following changes result in an increase or a decrease in the current between the terminals of such an instrument? (a) The moisture content decreases in wood (being "cured" for building a piano). (b) Due to nervous perspiration, the surface moisture of the skin increases for a person undergoing a lie-detector test. (c) The moisture in a layer of rock decreases due to drainage into a new crevice below. *(Answer: 28)*

SECT.

G SUMMARY

DEFINITIONS

resistor; Def. (A-1)

resistivity; Def. (D-4)

conductivity; Def. (D-5)

mobility; Eq. (E-1)

IMPORTANT RESULTS

Ohm's law for a resistor: Def. (A-1)

$RI = V$ (if I is not too large)

Resistance of resistors in series: Eq. (B-2), Rule (B-3)

$R = R_1 + R_2 + \ldots$

Resistance of resistors in parallel: Eq. (B-7), Rule (B-8)

$1/R = 1/R_1 + 1/R_2 + \ldots$

Resistance of homogeneous uniform rod or wire: Eq. (D-2)

$R = \rho L/A$

Relation between conductivity and resistivity: Def. (D-5)

$\sigma = 1/\rho$

USEFUL KNOWLEDGE

Atomic origin of resistivity (Sec. E)

Resistors used as transducers (Sec. F)

NEW CAPABILITIES

(1) Understand the relation:

$RI = V$ between the resistance of a resistor, the current through it, and the potential drop across its terminals (Sec. A, [p-1]).

(2) Relate the currents, resistances, and potential drops for connected resistors by systematically applying $I_{\rm in} = I_{\rm out}$ and V_{ab} = sum of potential drops along any path from a to b (Sec. B, [p-2], [p-3]).

(3) Relate the resistances of individual resistors to the resistance of a system of these resistors connected in series or in parallel (Sects. B and C, [p-4]).

(4) Use the resistances of two resistors connected in parallel or in series to compare the potential drops across, or currents through, each resistor (Sec. B).

(5) Relate the resistance of a uniform rod to the dimensions of this rod and to the resistivity of the material of which it is made (Sec. D, [p-5]).

Relating Descriptions of Resistive Circuits (Cap. 1,3,4)

G-1 (a) If an overhead copper transmission wire has a diameter of 2.6 cm and so has a cross-sectional area of $5.3\,\text{cm}^2$, what is the resistance of $50\,\text{km} = 5.0 \times 10^4$ meter of this wire? (b) If a 300 A current flows through this wire, what must be the potential drop across its ends? *(Answer: 24)*

G-2 Which of the following actions would reduce the work per unit charge required to produce a given current through a copper transmission wire? (a) Replacing the wire by a copper wire with a larger diameter. (b) Replacing the copper wire by an aluminum wire with the same diameter. (c) Reducing the wire's temperature by surrounding it with liquid nitrogen. *(Answer: 27)*

G-3 What are the resistances of a 3.0 ohm resistor and a 6.0 ohm resistor (a) connected in series and (b) connected in parallel? What is the current through a 12 volt battery when each of the following systems is connected to its terminals? (c) a 3.0 ohm resistor. (d) A 3.0 ohm resistor and a 6.0 ohm resistor connected in series. (e) A 3.0 ohm resistor and a 6.0 ohm resistor connected in parallel. *(Answer: 21)*

Note: Tutorial section G provides further practice in relating descriptions of resistive circuits.

SECT. H PROBLEMS

H-1 Suppose you have a supply of 2 ohm resistors. How can you connect them so as to make a resistor having resistances of (a) 1 ohm, (b) 4 ohm, and (c) 3 ohm? *(Answer: 23)*

H-2 A person (whose body has a resistance of about 500 ohm) is in electrical contact with the ground, and then touches a wire. (a) Suppose this wire has a large potential (10,000 volt) relative to the ground but carries a small current (3.0 ampere) before the touching occurs. What is the resulting current through the person's body? (b) Suppose instead the person touches a wire which has a low potential (10 volt) relative to the ground but which carries a large current (100 ampere). What then is the current through the person's body? (c) Why does one never see signs saying "Danger - High Amperage," although "Danger—High Voltage" signs are common? *(Answer: 22)*

H-3 A charge of 2.0×10^{-3} C is transferred between the plates of the large (2.0×10^{-6} farad) capacitor in a defibrillator. (a) What is the corresponding potential difference between the plates of the capacitor? (b) The resistance of a person's body is about 150 ohm (between large metal terminals placed on the chest). What is the magnitude of the current passing through a patient's body immediately after the defribrillator terminals are placed on the chest? *(Answer: 26)*

Note: Tutorial section H includes additional problems.

TUTORIAL FOR G

SYSTEMATICALLY RELATING CURRENTS AND POTENTIALS

g-1 *RESISTORS IN PARALLEL:* A battery produces a constant potential difference between its terminals. A resistor is connected to the terminals of this battery, so that the same current flows through the battery and through the resistor. If a second resistor is then connected in parallel to the first, does the current through the battery increase or decrease? Systematically solve this problem by: (a) Drawing circuit diagrams of the two situations. (b) Saying whether the potential drop across and thus the current through the original resistor increases, decreases, or remains the same when the second resistor is connected. (c) Relating the current through the battery to the current through the two resistors. *(Answer: 55) (Suggestion: s-13)*

g-2 *RESISTORS IN SERIES:* Suppose that in the situation described in frame [g-1], the second resistor is connected in *series* to the first, and the two are then reconnected to the terminals of the battery. Use the three steps outlined in frame [g-1] to determine whether the addition of the second resistor in series causes the current through the battery to increase or decrease. *(Answer: 52) (Suggestion: s-10)*

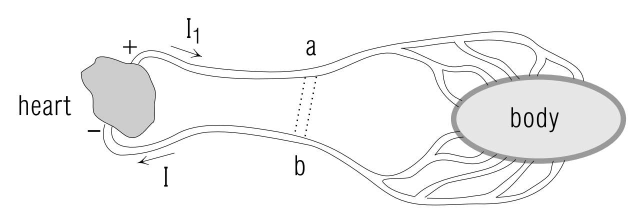
TUTORIAL FOR H

ADDITIONAL PROBLEMS

h-1 *THE UTILITY OF FUSES:* Large currents can be dangerous because the random internal energy (or heat) produced can cause fires. A fuse is a safety device consisting essentially of a wire which melts if a sufficiently large current flows through it. Fuses are connected in series with many electrical devices (e.g., an oven). Then, if a defect in the oven causes a dangerously large current to flow, the fuse simply melts, disconnecting the oven.

Suppose a defect in the oven causes its resistance suddenly to decrease. The potential drop across the system, including the oven and fuse, remains the same because the terminals of this system are connected to a wall outlet. (a) Do the potential drop across the fuse and the current through the fuse increase, decrease, or remain the same in magnitude as a result of this defect? (b) Suppose a (totally unrealistic) fuse is designed such that if the fuse is stupidly connected to the oven in parallel (instead of in series) this fuse does not melt when the oven operates normally. The terminals of the oven and this fuse are connected together in parallel, and then the terminals of this combined system are connected to the wall outlet. If the resistance of the oven now decreases, does the current through the fuse increase, decrease or remain the same in magnitude? Does the fuse melt? If it did melt, would it disconnect the oven? *(Answer: 57)*

h-2 *RELATING FLUID CURRENTS AND PRESSURES:* The following diagram shows a very simplified drawing of the circulation of the human body.



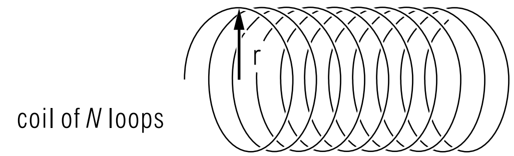
Blood (at a high pressure) flows out of the heart, through various blood vessels in the body, and then returns to the heart (at a lower pressure). The heart responds to pressure sensors so that it maintains a constant pressure difference between its "terminals" (the vessels carrying blood to and from the heart). Injury or disease can cause an abnormal connection or "shunt" between major blood vessels. [See dotted lines in the drawing. Blood can flow directly from a to b, although some blood still flows through the body.] Let us apply our principles of circuit analysis to see the medical effects of such a shunt. (a) If the pressure drop from a to b remains normal when the shunt appears, does the blood current from a to b through the body increase, decrease, or remain the same? (b) The resistance of the shunt to fluid flow is small compared with the resistance of the body. Is the current through the shunt large or small compared with the current through the body? (c) When the shunt appears, does the current through the heart increase, decrease, or remain the same in magnitude? (d) Is a shunt likely to produce overload of the heart muscle or to produce gangrene of the limbs (due to decreased blood flow)? *(Answer: 59)*

h-3 *CONSTRUCTION OF FUSES:* The wire through which current flows in a fuse should satisfy the following requirements: (1) It should melt when a sufficiently large current flows through it. (2) It should have a low resistance so that the potential drop $V = RI$ across the fuse does not greatly decrease the potential drop across the system to which it is connected. Should an effective fuse wire be long and thin, long and thick, short and thin, or short and thick? *(Answer: 63)*

h-4 *SAFETY IN WORKING WITH LARGE POTENTIAL DIFFERENCES:* When two resistors are connected in parallel, a larger current flows through the resistor with smaller resistance. More generally, when current can flow along a variety of alternate paths (e.g., through various parts of a person's body), the largest current flows along the shortest, most direct, path (which has the smallest resistance). Apply these comments to answer these questions: (a) Why does grounding the metal case of an electrical object ensure that very little current flows through a person touching the case, even though a large current flows from the case through the wire connected to the ground? (b) When working with very large potential differences, or when touching a household switch with damp hands, a good rule is to use only one hand, keeping the other raised or in one's pocket. Why does following this rule decrease the danger of electrocution? *(Answer: 58)*

h-5 *RESISTANCE OF A WOUND-WIRE RESISTOR:* Resistors are commonly made of thin wire wound as indicated in the following

diagram. Such resistors have a precisely determined resistance, which is relatively independent of changes in temperature.



Write an expression for the resistance R of such a resistor in terms of the resistivity ρ of the substance of which the wire is made, the cross-sectional area A of the wire, and the radius r and number N of the wire loops. *(Answer: 62)*

h-6 *RESISTANCE OF DRAWN WIRE:* Thick wire can be "drawn" through a small hole to make a longer thinner wire. Suppose a wire is drawn out to form a new wire which is four times as long as the original one (but which has the same resistivity). (a) Which of the following quantities has the same value for the old and new wires: cross-sectional area, diameter, volume? (b) Is the new cross-sectional area 1/2, 1/4, or 1/16 as large as the original cross-sectional area? (c) Is the new resistance 2, 4, 8, or 16 times as large as the original resistance? *(Answer: 67)*

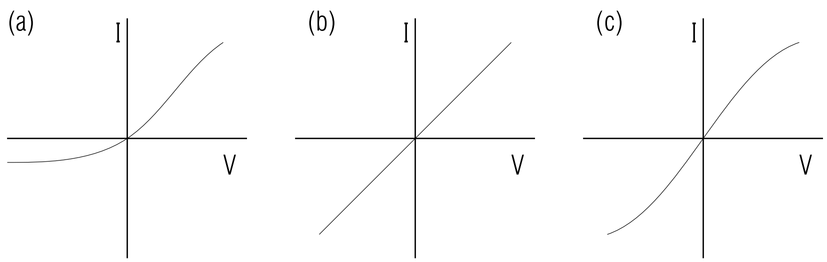
h-7 *VARIOUS RELATIONS BETWEEN CURRENT AND POTENTIAL DROP:* In this unit we have discussed resistors, two-terminal systems for which Ohm's law, $V = RI$ describes the relation between the current I through the system and the potential drop V across its terminals. Although for any two-terminal system, the quantities V and I are related, the relation between them is generally *not* Ohm's law.

When the potential drop V across a light bulb is small, a small current flows through the bulb, and the temperature is low. Under these conditions, a certain change in V produces a corresponding change in I, and the rate of change $\Delta I/\Delta V$ has a certain value. When V is larger, I is also larger, the temperature is higher, and the mobile electrons experience more collisions. Thus the *same* change in V now produces a *smaller* change in I, and the rate of change $\Delta I/\Delta V$ is smaller than for smaller values of V.

A "rectifier" is a device which allows only current with one chosen sense to flow through it. Thus if current with alternating senses is available, a rectifier can be used to provide current which flows in one sense, and so can be used, for example, to charge a battery or capacitor.

One kind of "rectifier" consists of two terminals, a and b, in an evacuated glass tube. Terminal b is hot and so can emit electrons which are then acquired by a. Terminal a is cool and cannot emit appreciable numbers of electrons. Thus when the potential drop V_{ab} is positive, an appreciable current flows (because negatively charged particles move from b to a). But when V_{ab} is negative, then a very small current flows (because very few electrons are emitted by a).

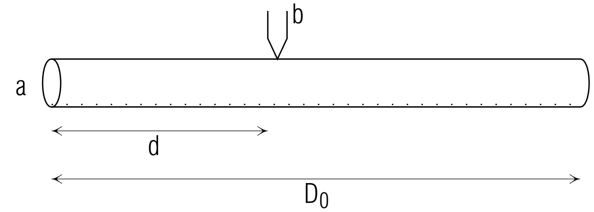
Which of the following graphs shows the relation between I and V for a resistor, which shows this relation for a light bulb, and which for a rectifier? *(Answer: 53)*



h-8 *A TRANSDUCER FOR MEASURING SMALL PRESSURES:* A Pirani gauge measures very low pressures (in nearly evacuated containers) in the following way. A wire with a current flowing through it extends into the container. The resistance of this metal wire depends on its temperature. However, this temperature depends on how much heat (random internal energy) is transferred to the surrounding molecules. When more molecules are available near the wire, larger amounts of random energy are transferred from the wire to these molecules and the temperature is lower. Suppose the potential drop V between the ends of the wire remains the same, but the current I through the wire decreases. Does this decrease indicate a decrease or an increase in the pressure in the container? *(Answer: 65)*

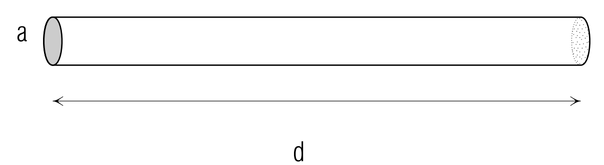
h-9 *RHEOSTATS:* A rheostat is a device which controls the amount of current flowing through a two-terminal system to which it is connected. For example, rheostats are used as "dimmer" switches controlling the brightness of lamps. They are also used in some electric stoves to control

the temperatures of the "burners." The following diagram indicates how a rheostat works:



One end of a resistive wire is attached to the terminal a. A second terminal b is free to slide along the wire, making electrical contact at a variety of points. (a) Describe the path of charged particles flowing into this system at a and out at b. (b) Write an expression for the resistance of this system in terms of the resistance R_0 and length D_0 of the entire resistor, and the distance d between the terminals a and b. (c) Suppose the rheostat is connected in series with a lamp, and the combined system is connected to a wall socket (which provides a constant potential difference between its terminals). Does moving the terminal b to the left increase or decrease the current through the lamp? *(Answer: 68)*

h-10 *MOTION OF CHARGED PARTICLES IN RESISTORS:* We look here in detail at how and why charged particles (electrons) move when there is a potential difference between the terminals of a resistor (e.g., a piece of metal wire).



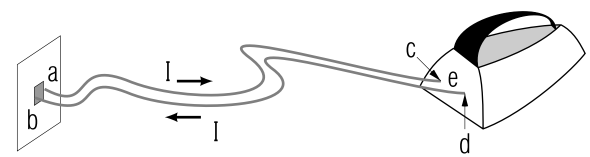
(a) The surfaces a and b in the preceding drawing have a potential difference of V. When current flows through a uniform wire, a small amount of charge accumulates at the edges of the wire so as to produce inside the wire a total field which is nearly constant in magnitude and directed along the wire. What is the magnitude of the electric field inside the wires shown in the preceding drawing?

(b) According to text problem (B-2) of Unit 423, electrons in a wire move with an average "drift velocity" of only a few millimeter/sec. Why does an electric light appear to come on immediately after a switch on the wall is pressed? *(Answer: 66)*

PRACTICE PROBLEMS

p-1 *UNDERSTANDING RI = V: DEPENDENCE (CAP. 1):* The potential drop V across a resistor is 1.5 volt and the current I through the resistor is 0.30 ampere. (a) What is the resistance R of the resistor? (b) Now suppose that the potential drop V is twice as large as before. Which of the following quantities have values different from those described above: I, V, V/I, R? Find the new value for each quantity whose value has changed. *(Answer: 54) (Suggestion: Review text problems A-1 through A-6.)*

p-2 *RELATING CURRENTS AND POTENTIALS (CAP. 2):* When in operation, the iron shown in the following drawing can be considered as a resistor (with very large resistance) connected to a wall outlet by wires (each with a small resistance).



At the base of the iron the insulation on the two wires cracks or wears away, so that these two wires are in electrical contact. Thus current can now flow directly between the points c and d. [Such a direct contact between wires is called a "short circuit."] Between the terminals at the wall outlet, the (average) potential drop V_{ab} remains the same.

(a) Draw a circuit diagram of the two situations described in this problem. (b) Write an equation relating the potential drops V_{ab} across the terminals at the wall outlet, the potential drops V_{ac} and V_{db} along the wires, and the potential drop V_{cd} across the iron. (c) When the short circuit occurs, do V_{ab} and V_{cd} increase, decrease, or remain the same? What are the corresponding changes in V_{ac} and V_{db}? What is the corresponding change in the current I through the wires? (d) What is the value of the potential drop V_{cd} after the short circuit has established contact between the points c and d? What is then the current through the iron (which has terminals c and d)? *(Answer: 56) (Suggestion: Review text problems B-1, B-2, and B-3.)*

p-3 *RELATING CURRENTS AND POTENTIALS (CAP. 2):* Two headlights for a car are connected in parallel and then to the terminals of a 12 volt battery. (The terminals are connected with wires of negligible resistance.) If one headlight burns out (so that no current flows through it), does the current through the remaining headlight increase, decrease, or remain the same? *(Answer: 64) (Suggestion: Review text problems B-1, B-2, and B-3.)*

p-4 *RELATING RESISTANCES FOR PARALLEL AND SERIES (CAP. 3):* Two resistors have resistances 10 ohm and 20 ohm when in operation. (a) What are the resistances of the systems consisting of the two resistors connected in series and in parallel? Suppose a system composed of one or more of these resistors is connected to the terminals of a 1.5 volt battery. Find the current flowing into (or out of) this system if it consists of (b) the 10 ohm resistor, (c) the 20 ohm resistor; (d) the two resistors connected in series; (e) the two resistors connected in parallel. *(Answer: 60) (Suggestion: Review text problems B-4, B-5, and B-6.)*

p-5 *RELATING RESISTANCE TO RESISTIVITY (CAP. 5):* Commonly the easiest and most accurate method for finding the thickness of a thin metal film is to measure its resistance, and then use the known resistivity of the metal to calculate the thickness of the film.

A film of silver (resistivity 1.5×10^{-8} ohm meter) has a length of 1.0 cm, a width of 0.20 cm, and a resistance (between its ends) of $0.25\,\Omega$. What is the thickness of the film? *(Answer: 61) (Suggestion: Review text problems D-1, D-2, and D-3.)*

SUGGESTIONS

s-1 *(Text problem B-5):* (c) The definition of resistance is $R = V/I$, where V is the potential drop from one terminal to the other of *any* resistor, and I is the current into one terminal of that resistor (or out of the other). A combination of resistors connected in series or parallel is itself a resistor.

s-2 *(Text problem C-1):* (b) The complete 4-resistor system is just a system composed of *two* identical resistors connected in parallel. Each of these two resistors has the resistance you found in part (a).

s-3 *(Text Problem A-3):* Recall that the potential drop along an entire path is equal to the sum of the potential drops along the segments making up that path.

s-4 *(Text problems C-2 and C-3):* Whenever two resistors are connected in series, the resistance of the combined system is larger than the resistance of either individual resistor because $R = R_1 + R_2$.

Whenever two resistors are connected in parallel, the resistance of the combined system is smaller than the resistance of either individual resistor. The reason is that for a given potential drop, more current can flow through the two resistors than through either of them individually.

s-5 *(Text problem A-6):* For *any* two-terminal system, the current I is equal to $1/R$ times the *total work per unit charge*, w (for small enough values of I). However, *only* for resistors, the total work per unit charge is just equal to the electric work per unit charge (or potential drop) V. [In systems more complex than resistors, other, non-electric work is done on the moving particles.]

Always:	For resistors:
$RI = w = V + \mathcal{E}$	$RI = w = V$

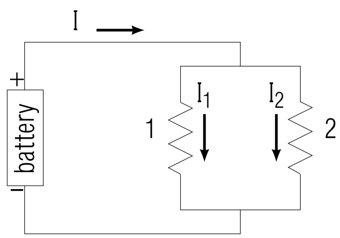
s-6 *(Text problem D-3):* According to equation (D-2), $R = \rho L/A$, where ρ is the same for both wires. Thus if one wire has a length 5 times as large as the length of another with the same area A, then its resistance must also be 5 times as large as that of the other.

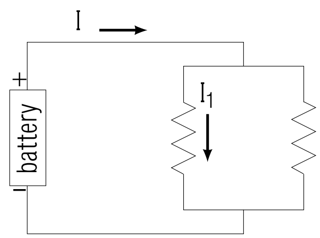
Don't forget that if the diameter of one wire B is three times the diameter of wire A, then the *area* of wire B is *nine* times the area of wire A.

s-7 *(Text problem B-2):* (a) When two-terminal systems are connected in parallel, corresponding terminals are connected together. Thus *each* terminal of resistor 1 should be connected to one corresponding terminal of resistor 2. Recall that current flows through a resistor in the sense in which potential decreases (because, as the charged particles move, their potential energy decreases at the expense of increasing random internal energy of the resistor).

(c) Find a small region surrounding a junction of wires such that the current I (from the battery) flows *into* this region and the currents I_1 and I_2 flow *out of* this region and towards the battery.

s-8 *(Practice problem [p-3]):* Your solution to this problem should include circuit diagrams equivalent to:





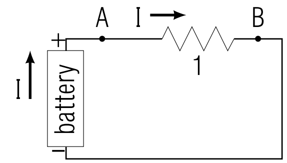
Notice that the potential drop across headlight 1 remains 12 volts both before and after headlight 2 burns out. Thus according to $V = RI$, the current through this headlight must remain the same.

s-9 *(Text problem D-2):* Since both wires are made of the same material, they both have the same resistivity ρ. Thus:

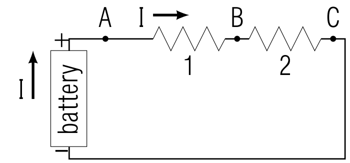
$$\rho = \frac{R_1 A_1}{L_1} = \frac{R_2 A_2}{L_2}$$

where R_1, A_1, and L_1 are the resistance, cross-sectional area, and length of the first zinc wire and R_2, L_2, and A_2 are the resistance, length and cross-sectional area for the second zinc wire.

s-10 *(Tutorial frame [g-2]):* According to the following circuit diagram, the potential drop across the battery equals the potential drop across resistor 1 (before resistor 2 is added) and equals the potential drop across *both* resistors (after resistor 2 is added).



before



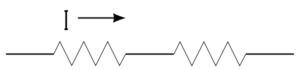
after

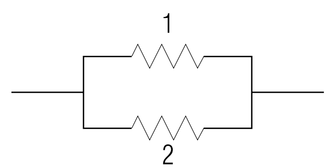
$$V_{battery} = V_{AB} \text{ (before)}$$

$$V_{battery} = V_{AB} + V_{BC} \text{ (after)}$$

Therefore, because the potential drop across the battery remains the same, the potential drop V_{AB} across resistor 1 must be smaller after resistor 2 is added than before.

s-11 *(Text problem B-7):* Circuit diagrams like the following almost always help in solving problems involving circuits.





(a) If the two resistors are connected in series, then the same current flows directly from one resistor into the other. But then, according to $V = IR$, the resistor with larger resistance must have a larger potential drop across its terminals.

(b) If the two resistors are connected in parallel, then their terminals are connected to the same points, and so the potential drops across the resistors are equal. But then, according to $V = IR$, the current must be smaller through the resistor with larger resistance.

s-12 *(Text problem A-7):* For each resistor in each system, your diagram should show the graphic symbol for a resistor, shown in Fig. A-1, labeled by an algebraic symbol (e.g., R) for the resistance of that resistor. To indicate a battery, you can simply use a rectangle with positive and negative terminals indicated by + and −. In the next unit we shall introduce a more generally used symbol for a battery.]

Recall that a circuit diagram simply uses lines to indicate how various terminals in the system are connected, but that such a diagram need not show how various things are arranged in space.

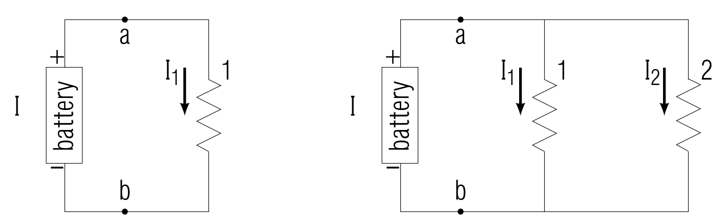
Your diagrams may not be identical to those shown in the answers. However, they are correct if they show labeled symbols for each resistor and battery, and if all terminals are connected to other terminals as shown in the answers. In addition, it is useful to indicate and label on a circuit diagram any points (e.g., A, B) which appear in the original drawing of the circuit.

(Note: text section D of Unit 423 (especially Tutorial D of Unit 423) provides a complete discussion of how to make circuit diagrams.)

s-13 *(Tutorial frame [g-11]):* (a) A description, including a circuit diagram (large enough to label), is essential to solving almost any circuit problem. Such a diagram usually includes arrows and algebraic symbols indicating the senses and magnitudes of all currents. The following diagrams show the circuits described in problem C-1. Notice that through each resistor, the current has a sense from the higher (positive) potential at a towards the lower (negative) potential at b.

On each diagram, draw an arrow indicating the sense of the current I through the battery.

►



(b) Both before and after resistor 2 is added, the potential drop V is equal to the potential drop across the battery *and* is equal to the potential

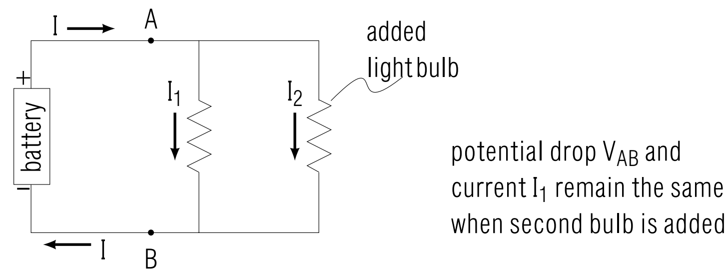
drop (along the other path from a to b) through the resistor 1. Thus the potential drop across resistor 1 remains the same when resistor 2 is added.

(c) After resistor 2 is added,the particles flowing through the battery divide, with some flowing through each resistor. Thus, according to $I_{\rm in} = I_{\rm out}$, $I = I_1 + I_2$.

(Answer: 51) (Note: For further help in determining current senses, review tutorial frame [c-3].)

s-14 *(Text problem B-6):* You can answer these questions most easily by simply applying text equations (B-2) and (B-7) and perhaps by reviewing the qualitative discussion associated with these equations.

When the bulbs are connected in parallel, the physical reason for the change in resistance is the following. When a second bulb is connected in parallel with the first, the potential drop across the first bulb remains the same (because it is still connected to the terminals of the battery). Hence the current I_1 through the first bulb remains the same, but an additional current I_2 now flows through the second bulb.

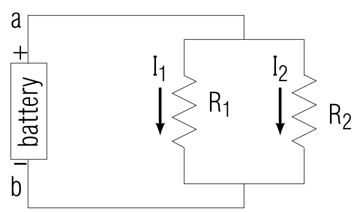


The potential drop V_{AB} across the combined system is therefore the same as the potential drop across the single bulb. But the current I into (and out of) the combined system is *larger* than the original current through the single bulb. Therefore the resistance $R = V/I$ of the combined system is *smaller* than the resistance of the single bulb.

[In the special case where the resistance of the second light bulb is *very* large, almost no current flows through this additional bulb, so that the resistance of the combined system is nearly the same as the resistance of the original system.]

ANSWERS TO PROBLEMS

1. (c)
2. a. diagram equivalent to:



   b. $I = 0.60$ ampere, $I_2 = 0.30$ ampere

   c. $I = 0.90$ ampere

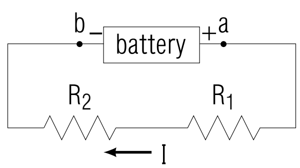
3. a. $V_{ab} = 3.0$ volt, $V_{bc} = 1.5$ volt

   b. $V_{ac} = 4.5$ volt

4. a. 0.75 ampere, A to B

   b. 0.75 ampere, B to A

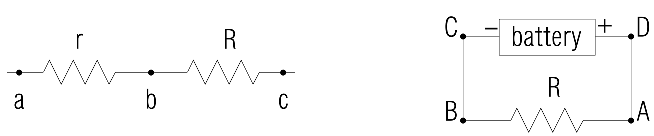
5. a. diagram equivalent to:



   b. $V_1 = 2.0$ volt, $V_2 = 4.0$ volt

   c. $V_{ab} = 6.0$ volt

6. $V_1 = 3V_2$
7. diagrams equivalent to:



8.

R, r	ohm (Ω)	+, 0
I, I_1	ampere (A)	+, −, 0
V, V_1	volt (V)	+, −, 0

9. a. series: $I = 0.10$ ampere, $V_{ab} = 6.0$ volt. parallel: $I = 0.90$ ampere, $V_{ab} = 12$ volt

   b. series: $R = V/I = 60$ ohm. parallel: $R = 13$ ohm

10. (a) $w_l = RI = V$; (b) $w_b = rI$
11. a. 100 ohm

    b. 50 ohm

    c. equal

12. 7.0×10^{-8} ohm meter
13. a. 10 ohm

    b. 0.40 ohm

    c. series: 0.60 ampere, parallel: 15 ampere

14. a. 3

    b. 1/9

    c. 1/3

15. a. D

    b. yes, because A and D, and B and C are connected by lines, they have the same potentials.

16. 2.4 ohm
17. a. $R = R_1 + R_2 = 60$ ohm

    b. $1/R = (1/20\,\Omega) + (1/40\,\Omega)$, $R = 13$ ohm

    c. yes

18. a. copper

    b. copper, alloy

19. a. larger

    b. smaller

20. a. semi-conductor

    b. metal

21. a. $9.0\,\Omega$

b. $2.0\,\Omega$

c. 4.0 A

d. 1.3 A

e. 6.0 A

22. a. 20 ampere

b. 0.02 ampere

c. Potential difference determines the current flowing through objects in contact with the wires.

23. Connect 2 ohm resistors in these ways:

a. 2 in parallel

b. 2 in series

c. 1 in series with a system of 2 in parallel

24. a. 1.6 ohm

b. 4.8×10^2 volt

25. a. larger

b. smaller

26. a. 1.0×10^3 volt

b. 6.7 ampere

27. (a) and (c) decrease resistance and so decreases $w = IR$

28. a. decrease

b. increase

c. decrease

29. a. same, larger

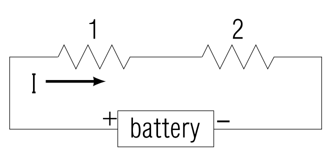
b. smaller, same

30. a. smaller than 200 ohm

b. increase

51. In each circuit, sense is from B to A (through the battery).

52. a.



b. V_{AB} across resistor 1 is smaller after than before. I (through resistor 1) is smaller after than before.

c. I (through both resistors and battery) is smaller after than before.

53. a. rectifier

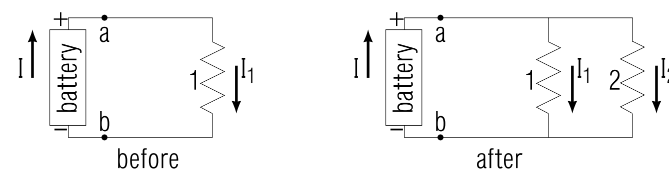
b. resistor

c. light bulb

54. a. 5.0 ohm

b. $I = 0.60$ ampere, $V = 3.0$ volt

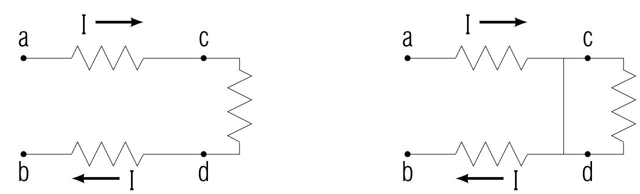
55. a.



b. V_{ab} across battery and resistor 1 remains the same. Therefore, I_1 remains the same.

c. Since $I = I_1$ *before* and $I = I_1 + I_2$ *after*, I is larger after second resistor is added.

56. a.



b. $V_{ab} = V_{ac} + V_{cd} + V_{db}$

c. V_{ab} remains the same, V_{cd} decreases (becomes zero), V_{ac} and V_{db} increase, I increases

d. $V_{cd} = 0$, current through iron is zero.

57. a. both increase

b. both remain the same, no, no

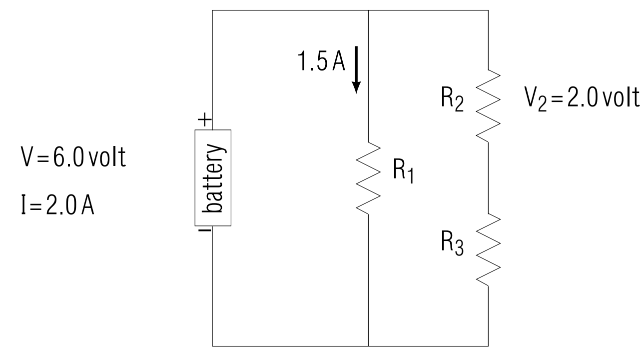
58. a. Resistance of wire is much less than resistance of person.

b. Avoids large current flowing from one hand to the other directly through heart.

59. a. same
    b. large
    c. increases (by large amount)
    d. overload
60. a. series: 30 ohm, parallel: 6.7 ohm
    b. 0.15 ampere
    c. 0.075 ampere
    d. 0.050 ampere
    e. (0.22 or 0.23) ampere
61. 3.0×10^{-7} meter $= 3.0 \times 10^{-5}$ cm
62. $R = 2\pi N\rho r/a$
63. thin so it will melt; short so it has low resistance and does not greatly reduce the current in the circuit
64. remains the same (if wrong, see [s-8])
65. decrease [Larger resistance indicates higher temperature and lower pressure.]
66. a. $E = V/d$
    b. Almost immediately the electric field is established throughout the wire, and so electrons move in bulb.
67. a. volume
    b. 1/4
    c. 16
68. a. from a directly to b (without flowing through the part of the resistor beyond b)
    b. $R = dR_0/D_0$
    c. increase

MODEL EXAM

1. **Relating resistances, currents, and potential drops.** The following diagram shows a circuit, with values indicated for some of the resistances, currents, and potential drops describing this circuit.



   a. What is the value of the resistance R_1?
   b. What is the potential drop across the resistor labeled R_3?
   c. What is the magnitude of the current through the resistor labeled R_3?

2. **Comparing currents and potential drops for connected resistors.** There is a potential drop $V_{ab} = 6.0$ volt across the two-terminal system shown in this diagram:

   a. If the potential is larger at a than at b, is the sense of the current I_1 from a to b or from b to a?
   b. Through which resistor is the current larger in magnitude, or is the current the same through both resistors?
   c. Across which resistor is the potential drop larger in magnitude, or is it the same across both resistors?
   d. What is the resistance of the complete two-terminal system?

3. **Resistivity of a seawater sample.** A sample of seawater is contained in a glass tube of length 0.1 meter and cross-sectional area 25×10^{-6} meter^2. The seawater completely fills the tube, and has a resistance of 800 ohm between the ends of the tube.

   What is the resistivity of the seawater in the sample?

Brief Answers:

1. a. 4.0 ohm

   b. 4.0 volt

   c. 0.5 ampere

2. a. a to b

   b. same

   c. R_2, the 9 ohm resistor

   d. 12.0 ohm

3. 0.2 ohm meter