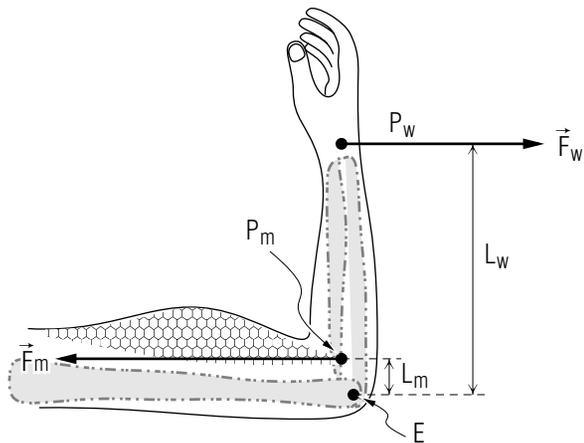


ENERGY OF A SYSTEM OF PARTICLES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ENERGY OF A SYSTEM OF PARTICLES

by

F. Reif, G. Brackett and J. Larkin

CONTENTS

- A. Kinetic Energy and Work
- B. Potential Energy
- C. Conservation of Energy
- D. Energy Transformations
- E. Macroscopic and Internal Energy
- F. Direction of Energy-Transformation Processes
- G. Macroscopic Equilibrium
- H. Summary
- I. Problems

Title: **Energy of a System of Particles**

Author: F. Reif, G. Brackett, and J. Larkin, Department of Physics,
University of California, Berkeley.

Version: 4/30/2002 Evaluation: Stage 0

Length: 1 hr; 60 pages

Input Skills:

1. Vocabulary: potential energy, energy (MISN-0-415).
2. State the principle of conservation of energy (MISN-0-415).

Output Skills (Knowledge):

- K1. Vocabulary: kinetic energy of a system, work done on a system, potential energy of a system, energy of a system, macroscopic equilibrium of a system, macroscopic equilibrium of a rigid body, torque.
- K2. Relate the energies and work for a system.
- K3. State the principle of conservation of energy for a system in terms of; (a) kinetic and potential energy; (b) macroscopic and internal energy.
- K4. Describe how energy transformations proceed.

Output Skills (Problem Solving):

- S1. Apply the principle of conservation of energy to an isolated system of atomic particles.
- S2. Given the masses, speeds, and/or potential energies of particles in a system, calculate: (a) its kinetic energy; (b) its potential energy; (c) its energy.
- S3. Given a description of the macroscopic motion of an isolated system, describe changes in its macroscopic potential and kinetic energies, its macroscopic energy, and its internal energy.
- S4. Describe the variation of internal and macroscopic energies of dissipative and non-dissipative isolated systems, and the final macroscopic equilibrium state of isolated dissipative systems.

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

PROJECT STAFF

Andrew Schnepf	Webmaster
Eugene Kales	Graphics
Peter Signell	Project Director

ADVISORY COMMITTEE

D. Alan Bromley	Yale University
E. Leonard Jossem	The Ohio State University
A. A. Strassenburg	S. U. N. Y., Stony Brook

Views expressed in a module are those of the module author(s) and are not necessarily those of other project participants.

© 2002, Peter Signell for Project PHYSNET, Physics-Astronomy Bldg., Mich. State Univ., E. Lansing, MI 48824; (517) 355-3784. For our liberal use policies see:

<http://www.physnet.org/home/modules/license.html>.

MISN-0-416

ENERGY OF A SYSTEM OF PARTICLES

- A. Kinetic Energy and Work
- B. Potential Energy
- C. Conservation of Energy
- D. Energy Transformations
- E. Macroscopic and Internal Energy
- F. Direction of Energy-Transformation Processes
- G. Macroscopic Equilibrium
- H. Summary
- I. Problems

Abstract:

In the preceding units we discussed energy and the conservation of energy in the simple case of a system consisting of a single particle. We shall now generalize these ideas to systems consisting of many particles. Thus we shall be able to apply energy arguments to many practical situations and to deal with systems as complex as biological organisms.

SECT.

A KINETIC ENERGY AND WORK

Consider a system consisting of several interacting particles labeled 1, 2, 3, (For example, the system might consist of the sun and the planets.) Then the change dK of the kinetic energy of each particle during a small enough displacement is related to the work δW done on this particle by all forces so that

$$dK_1 = \delta W_1, \quad dK_2 = \delta W_2, \quad \dots \quad (\text{A-1})$$

By adding corresponding sides of these equations for all particles in the system we obtain

$$dK_1 + dK_2 + \dots = \delta W_1 + \delta W_2 + \dots \quad (\text{A-2})$$

But since the sum of the changes on the left side is equal to the change in their sum, we can write

$$d(K_1 + K_2 + \dots) = \delta W_1 + \delta W_2 + \dots \quad (\text{A-3})$$

Let us introduce the convenient abbreviations

$$K = K_1 + K_2 + \dots \quad (\text{A-4})$$

and

$$\delta W = \delta W_1 + \delta W_2 + \dots \quad (\text{A-5})$$

Then we can write the relation (A-3) in the simple form

$$\boxed{dK = \delta W \text{ (by all forces)}} \quad (\text{A-6})$$

Note that δW is the sum of the works done by *all* forces acting on each particle in the system.

The quantity K in Eq. (A-4) is called the “kinetic energy of the system,” i.e.,

Def.	Kinetic energy of a system: The sum of the kinetic energies of all the particles in the system.	(A-7)
------	--	-------

The quantity δW in Eq. (A-5) is called the “work done on the system in a small displacement.” Here we have used this general definition:

Def.	Work done on a system: The work done on a system by specified forces is the sum of the works done by these forces on <i>all</i> particles in the system.	(A-8)
------	---	-------

The relation (A-6) then states that, in any small displacement of the particles, the change in the kinetic energy of the system is equal to the work done on the system by *all* forces.

By Definition (A-2) of Unit 413, the “state” of a system is specified by the positions and velocities of *all* the particles in the system. Consider then a system which goes from some initial state *a* (specified by the initial positions and velocities of all the particles) to some final state *b* (specified by the final positions and velocities of all these particles). By applying Eq. (A-6) to all the successive small displacements of the particles leading from state *a* to state *b* and then adding the results, we obtain (as in text section B of Unit 414) the result

$$\boxed{K_b - K_a = W_{ab} \text{ (by all forces)}} \quad (\text{A-9})$$

where $K_b - K_a$ is the total change in the kinetic energy of the system and where W_{ab} is the work done on the system by *all* forces during the *entire* process whereby the system goes from state *a* to state *b*.

Note that the relations (A-6) and (A-9) look exactly like those for a single particle, but their interpretation is more general since all quantities now refer to *all* the particles in the system.

Example A-1: System consisting of two particles

Consider a system consisting of two balls which collide with each other. The kinetic energy of this system is the *sum* of the kinetic energies of each of the balls. The change in the kinetic energy of the system is then equal to the work done on the system, i.e., to the sum of the works done on each ball by the other during the collision, plus the sum of the works done on the balls by all other forces (such as gravitational forces).

WORK DONE BY MUTUAL FORCES

Consider a system of two interacting particles 1 and 2. Then the mutual forces $\vec{F}_{1,2}$ and $\vec{F}_{2,1}$ exerted on each particle by the other have equal magnitudes but opposite directions. Suppose that both particles are displaced by the *same* small amount $d\vec{r}_1$ (from P_1 to P'_1 and from P_2 to P'_2 in Fig. A-1). Then the works δW_1 and δW_2 done on these particles by the mutual forces must have equal magnitudes but opposite signs. Hence the total work $\delta W = \delta W_1 + \delta W_2$ done on the system by these mutual forces is zero.

Suppose now that the small displacements of the particles are *different*. Then the displacement $d\vec{r}_2$ of particle 2 (from P_2 to P''_2 in Fig. A-1)

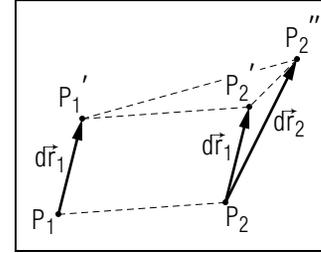


Fig. A-1: Work done by mutual forces in the displacement of two particles.

can be regarded as consisting of two successive displacements: a displacement from P_2 to P'_2 equal to the displacement $d\vec{r}_1$ of particle 1, plus a remaining displacement from P'_2 to P''_2 . But in the first of these displacements, when both particles are displaced by the same amount (so that their *relative* position remains unchanged), the work done on the system of *both* particles is zero. Hence the work done on the system consists merely of the work done on particle 2 during the displacement from P'_2 to P''_2 when the *relative* position of the particles changes.

This conclusion for two particles is equally true for any number of particles and may be summarized:

$$\boxed{\text{The work done on a system of particles due to their mutual interaction depends only on changes in the relative positions of these particles.}} \quad (\text{A-10})$$

In particular, it does not matter whether this change in relative positions occurs while one of the particles remains fixed or not. But, as we have seen in Unit 415, the work done on a particle by a central force due to another fixed particle depends only on the change in *distance* between these particles. Since the mutual forces between atomic particles are central forces, Rule (A-10) implies this conclusion:

$$\boxed{\text{The work done on a system due to the mutual interaction between its atomic particles depends only on changes in the distances between the particles.}} \quad (\text{A-11})$$

For example, since the distance between any two particles in a *rigid* object remains always unchanged, the work done by the mutual forces in a rigid object is *zero*. *

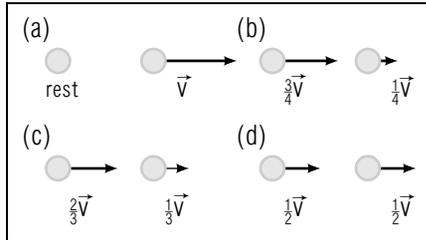


Fig. A-2.

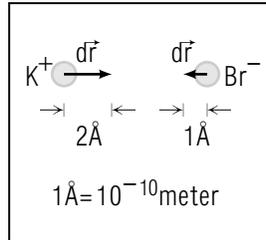


Fig. A-3.

displacements? (*Answer: 102*) (*Suggestion: [s-7]*)

* In the case of a string of constant length, or in the case of an incompressible liquid, the *average* distance between adjacent atoms interacting with each other remains constant. Hence the work done by the mutual forces between atomic particles is then also zero.

Finding the Kinetic Energy of a System (Cap. 2a)

A-1 Figure A-2 shows four possible states of an isolated system consisting of two helium atoms, each of mass m . The system has the same momentum $\vec{P} = m\vec{v}$ in each state. Write an expression (in terms of m and v) for the system's kinetic energies K_a , K_b , K_c , and K_d in these states. Does the system have the same kinetic energy in each state? (*Answer: 106*) (*Suggestion: [s-1]*)

Knowing About Work and Kinetic Energy for Systems

A-2 A positively-charged potassium ion (K^+) and a negatively-charged bromide ion (Br^-) form an isolated system as they approach each other to form a potassium bromide (KBr) molecule (Fig. A-3). (a) Suppose that the potassium ion, of mass 7.0×10^{-26} kg, has an initial velocity of 800 m/s toward the right, while the bromide ion, of mass 14×10^{-26} kg, has an initial velocity of 400 m/s toward the left. What is the initial kinetic energy of this system? (b) These ions then move through the small displacements illustrated in Fig. A-3. (These displacements are exaggerated for clarity.) If each ion exerts on the other an attractive electric force of magnitude 1×10^{-11} N, what is the small work δW done on the system by all forces during this motion? (c) What is the final kinetic energy of the system after the ions have moved through those

SECT.

B

 POTENTIAL ENERGY

Suppose that the work done by certain forces on every particle in a system is independent of the process (i.e., independent of its path or the speed with which it moves). Then the work W_{ab} done by these forces on the entire system of several such particles must also be independent of the process whereby the system goes from any state a to another state b . In this case the work W_{ab} can depend only on the initial and final positions of all the particles in the system. An argument completely identical to that of text section A of Unit 415 then implies that the work W_{ab} can be written as the difference

$$\boxed{W_{ab} = U_a - U_b} \quad (\text{B-1})$$

where the number U is called the “potential energy” of the system due to the specified forces. The potential energy U_a in a state a depends only on the positions of the particles in this state and is defined by

$$\boxed{U_a = W_{as}} \quad (\text{B-2})$$

This definition can be stated:

Def.	<p>Potential energy of a system : The potential energy, due to specified forces, of a system in a state \mathbf{a} is the work done by these forces on the system when it goes from the state a to some specified standard state s.</p>	(B-3)
------	---	-------

The standard state s of an isolated system of particles is ordinarily chosen to be that where all particles are at rest so far apart from each other that the interaction between them is negligible. By Eq. (B-2), $U_s = W_{ss} = 0$. Thus the potential energy of a system in its standard state is zero.

The relations (B-1) and (B-2) look again exactly the same as those for a single particle, but their interpretation is more general since the potential energy of a *system* depends on the positions of *all* the particles in the system.

POTENTIAL ENERGY AND INDIVIDUAL INTERACTIONS

Suppose that we want to find the potential energy of an isolated system due to the mutual interaction between its particles. How could we

find this energy from what we learned in text section D of Unit 415 about the potential energy of a particle interacting with another fixed particle?

Consider first an isolated system consisting of only two interacting particles 1 and 2. Then the system can be brought from some state \mathbf{a} to its standard state s , where the particles are far apart, by simply moving particle 1 far away while leaving particle 2 fixed. The work done (which is just the potential energy U of the *system*) is then just equal to the potential energy U_{12} of particle 1 due to particle 2. Thus

$$U = U_{12} \quad (\text{B-4})$$

As we have seen in Unit 415, the potential energy U_{12} depends only on the distance between the interacting particles. Thus the same work is done whether particle 1 is moved far away while particle 2 remains fixed, or vice versa. Hence $U_{12} = U_{21}$ and this energy can be simply called the “potential energy of interaction between particles 1 and 2.”

Consider now a more complex system consisting of three particles 1, 2, and 3. (See Fig. B-1) Then the system can be brought from a state \mathbf{a} to the standard state s , where all particles are far apart, by first moving particle 1 far from all the other particles, as shown in Fig. B-1b. The work done in this process (where 1 is acted on jointly by forces due to both 2 and 3) is then the sum of the works done on 1 by 2 and done on 1 by 3. Hence this work is equal to $U_{12} + U_{13}$, where U_{12} is the potential energy of 1 due to 2 (i.e., the work done on 1 in moving to its standard position far from 2) and where U_{13} is the potential energy of 1 due to 3. Now the particle 2 can be moved to its standard position far from both 3 and 1, as shown in Fig. B-1c. In this process, particle 2 is acted on only by the force due to 3. Hence the work done on 2 is merely equal to the potential energy U_{23} of 2 due to 3. Note that the system is now in its standard state where all its particles are at rest far from each other. The total work required to bring the system to this standard state (i.e., the potential energy U of the entire system) is thus merely the sum of all the works done in separating the particles, i.e.,

$$U = U_{12} + U_{13} + U_{23} \quad (\text{B-5})$$

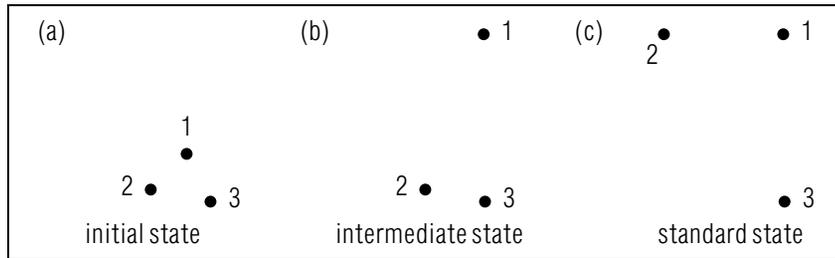


Fig. B-1: Work done in bringing an isolated system of three particles from some initial state to the standard state.

The preceding conclusion can be stated in these general terms:

The potential energy of an isolated system of particles is the sum of the potential energies between all interacting *pairs* of particles. (B-6)

Since the potential energy of a pair of particles depends only on the distance between them, the potential energy U of a system depends thus only on the *distances* between all the interacting particles in the system.

Finding the Potential Energy of a System (Cap. 2b)

B-1 (a) What is the Coulomb potential energy of a system consisting of two protons separated by a distance of 1.0×10^{-15} meter, a typical separation within a nucleus? Use the values 1.6×10^{-19} C for the charge of a proton, where C is the abbreviation for a coulomb of charge and $k_e = 9 \times 10^9$ N m²/C². (b) In a beryllium nucleus, four protons are arranged in space so that each proton is separated from the others by the distance described in part (a). (See Fig. B-2.) What is the Coulomb potential energy of the system consisting of these four protons? (*Answer: 104*) (*Suggestion: [s-8]*)

Knowing About Work and Potential Energy for Systems

B-2 A lithium nucleus consists of three protons, each separated from the others by the distance specified in part (a) of problem B-1. (a) In the initial state a of a “fusion” reaction, a fourth proton is very far from the lithium nucleus. What is the initial Coulomb potential energy U_a of the system consisting of all four protons? (b) The distant proton

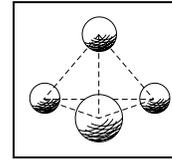


Fig. B-2.

then approaches the lithium nucleus, and the strong nuclear force binds the four protons together to form a beryllium nucleus. In this final state b , the system consisting of the four protons is arranged as described in part (b) of problem B-1. What is the final Coulomb potential energy U_b of this system? (c) What is the work W_{ab} done on this system by the Coulomb electric forces due to the protons as the system goes from the state a to the state b ? (*Answer: 101*)

SECT.

C

 CONSERVATION OF ENERGY

The change in the kinetic energy of a system between any two states is always given by $K_b - K_a = W_{ab}$ where W_{ab} is the work done on the system by all forces. If the work done by all forces which do work is independent of the process, then $W_{ab} = U_a - U_b$ where U is the potential energy of the system due to all forces. By combining these relations we then obtain (analogously to the results obtained in text section E of Unit 415)

$$K_b - K_a = W_{ab} = U_a - U_b \quad (\text{C-1})$$

Hence it follows that

$$K_b + U_b = K_a + U_a \quad (\text{C-2})$$

Let us then introduce this definition:

$$\text{Def. } \left| \text{Energy of a system: } E = K + U \right| \quad (\text{C-3})$$

Then Eq. (C-2) is equivalent to the statement that

$$E = \text{constant} \quad (\text{C-4})$$

Thus we arrive at this general principle:

Conservation of energy: If the work done on a system by all forces which do work is independent of the process, the energy of the system due to these forces remains constant. (C-5)

The kinetic energy K of the system depends on the speeds of all the particles in the system. The potential energy U of the system depends on the distances between all the interacting particles in the system. In the course of time, both K and U change. But the principle of conservation of energy implies that their sum, the energy $E = K + U$, remains unchanged.

From a detailed point of view, every system consists ultimately of atomic particles which interact by fundamental forces. Since the work done by such fundamental forces is independent of the process, the principle of conservation of energy, Rule (C-5), implies that, if such a system is *isolated* (so that the energy of interaction between all its particles is the energy due to *all* forces), the energy of the system must remain constant.

Thus Rule (C-5) implies this important conclusion:

The energy of any *isolated* system of atomic particles remains constant. (C-6)

The importance of the principle of conservation of energy is its great generality. Thus the statements in Rule (C-5) and Rule (C-6) apply to *any* system, even if it is as complex as an animal.

Example C-1: Gravitational interaction between two particles

Consider an isolated system consisting of two particles 1 and 2 interacting by gravitational forces. (For example, consider the double-star system "Sirius" which consists of two neighboring stars revolving around each other, one having a mass about twice as large as the other.) Then the kinetic energy of this system is $K = K_1 + K_2 = (1/2)m_1v_1^2 + (1/2)m_2v_2^2$ where v_1 and v_2 are the speeds of the particles. The potential energy of the system is simply the gravitational potential energy of interaction between the two particles so that $U = -Gm_1m_2/R$ where R is the distance between the particles. The conservation of energy then implies that the particles always move so that

$$E = (K_1 + K_2) + U = \text{constant} \quad (\text{C-7})$$

Suppose that the mass m_2 is very much larger than the mass m_1 as would be the case if particle 1 were the earth and particle 2 were the sun), Since the acceleration of particle 2 is then negligibly small, its velocity v_2 (and thus also its kinetic energy K_2) remains then nearly constant. Hence Eq. (C-7) reduces simply to the relation $E = K_1 + U = \text{constant}$, previously encountered in Unit 415 where we considered a single moving particle interacting with other fixed particles.

Finding the Energy of a System (Cap. 2c)

C-1 Two protons having the same mass m and charge q collide head-on in the earth's upper atmosphere. During the collision, the protons form an isolated system, and they interact by the Coulomb electric force alone. For each of the following states of this system, write an expression for the system's energy E due to all interactions. (a) The protons are so far apart that their interaction is negligible, and are moving toward each other with the same speed v_0 . (b) The protons have a smaller

separation R , and each is moving toward the other with a smaller speed v . (c) The protons have their smallest separation R' and both protons are at rest. (*Answer: 108*) (*Suggestion: [p-1]*)

Understanding Energy Conservation for Isolated Systems (Cap. 1)

C-2 *Example:* For the system described in problem C-1, suppose the initial speed of each proton is $v_0 = 1 \times 10^3$ m/s. The mass of each proton is $m = 2 \times 10^{-27}$ kg. (a) What is the value of the initial energy E of this system in the state described in part (a) of problem C-1? (b) What is the value of this energy E when the system is in the states described in parts (b) and (c) of problem C-1? (*Answer: 103*)

C-3 *Applicability:* An isolated helium atom forms a system consisting of a nucleus and two electrons. Does the principle of conservation of energy apply to this system? Why or why not? (*Answer: 110*)

C-4 *Meaning of E :* Consider the helium atom described in problem C-3. In this system, the nucleus has a speed v_n and a mass m_n and the two electrons (labeled 1 and 2) have speeds v_1 and v_2 and the same mass m_e . Let us call $U_{1,n}$ and $U_{2,n}$ the Coulomb potential energies due to the interaction of each electron with the nucleus, and $U_{1,2}$ the Coulomb potential energy due to the interaction between the two electrons. Write an expression for the conserved energy E of this system. (*Answer: 105*)

C-5 *Dependence:* (a) A carbon atom and an oxygen atom, which are isolated from other particles, combine to form a carbon monoxide molecule. The potential energy due to the interaction of these atoms becomes smaller during this reaction. What happens to the kinetic energy of the system consisting of the carbon and oxygen atoms? (This result is typical of so-called “exothermic” reactions, such as those occurring as fuels are burned.) (b) In a typical collision between two molecules in a gas, the colliding molecules form an isolated system. If such a collision is “elastic,” this system’s potential energy due to all interactions has the same value before and after the collision. However, the kinetic energy of each individual molecule rarely has the same value before and after the collision. Does the kinetic energy of the system consisting of both molecules have the same value before and after the collision? (*Answer: 112*)

SECT.

D ENERGY TRANSFORMATIONS

According to Rule (C-6), the energy of any isolated system remains constant if this energy is regarded as that due to all the atomic particles in the system. But this energy consists of the sum of the kinetic energies of all these particles and of the sum of the potential energies of interaction between all these particles. These individual energies may change although their sum (i.e., the energy of the entire isolated system) remains unchanged. Thus the energy of an isolated system may be converted into various forms despite the fact that its value remains unchanged. For example, the energy of an isolated system might be transformed from potential energy to kinetic energy, or vice versa. Alternatively, there might be a transfer of energy between parts of an isolated system, so that the energy of some particles in the system increases while the energy of other particles in the system decreases.

Any system can always be considered as part of a larger system which is isolated. Hence any process in nature can usefully be analyzed by examining the energy transformations occurring in some isolated system whose energy remains constant. As a simple example, suppose that one wishes to accelerate a trailer on a horizontal road. Then the trailer can be considered as part of an isolated system consisting of the trailer, the truck connected to it, and the earth. The energy of this isolated system remains constant. But energy can be transferred from the truck to the trailer so as to increase the kinetic energy of the trailer and thus result in its acceleration.

Many very important processes can be analyzed from the point of view of energy transformations. For example, the following problem is fundamental to the technological processes in an industrial society: What mechanisms can be used to transform the energy contained in various substances (such as coal, oil, or uranium) into the large-scale gravitational potential energy of lifted weights or the kinetic energy of moving vehicles and machinery? Similarly, the following question is fundamental to all biological processes: What chemical reactions are used to transform the energy contained in foodstuffs, or obtained from sunlight, into the potential energy of assembled polymers (such as proteins or nucleic acids) or into the kinetic energy of moving limbs? (The energy contained in oil or in foodstuffs is largely just the potential energy due to the electric interaction between the atomic particles constituting the molecules in these

substances.)

The principle of conservation of energy is one of the most important scientific principles, contributing deeply to our understanding of nature and having many far-reaching practical implications. In particular, since the solar system is nearly isolated, the energy in it can be transformed but not created. Hence the energy available to us constitutes a valuable and potentially scarce resource of crucial importance to the survival of mankind.

The concept of energy is used extensively in all of the sciences as well as in daily life. Various units of energy, other than the SI unit “joule,” are used in different contexts for historical reasons. For example, in chemistry or biology, one often encounters the unit “calorie” which is nowadays *defined* by this relation:*

$$\boxed{1 \text{ calorie} = 4.184 \text{ joule}} \quad (\text{D-1})$$

* The Calorie (or “large” calorie), commonly used as a unit for expressing the energy content of foods, is actually a kilocalorie = 1000 calorie.

Example D-1: Energy transformations and efficiency

Consider a gasoline-powered crane which lifts a steel beam some height above the ground. Then the crane with its attached gasoline motor, the beam, the earth, and the surrounding air constitute an isolated system whose energy E must remain unchanged. *

* Although this system interacts also with the sun, no work is done on the system by the sun. Hence the system can be considered as isolated with respect to energy transfer to its surroundings.

When the beam is lifted, the gravitational potential energy of every atom in the beam increases because of gravitational interaction with the earth. This energy is ultimately provided by the gasoline. Indeed, as the gasoline is burned in the motor, the energy of the atoms in gasoline molecules decreases as these atoms recombined to form simpler molecules. Some of this energy is transformed into the increased gravitational potential energy of the atoms in the lifted beam. The rest of this energy is transformed into increased energy of random motion of the atoms in the motor and in the air (so that the motor and the air are observed to get “warmer.”)

In the preceding energy-transformation process, the increase of gravitational potential energy of the beam is the “desired energy output.” The energy provided by decreasing the energy of atoms in the gasoline is the “energy input.” The “efficiency” of the energy-transformation process can then be conveniently described by the ratio

$$\text{efficiency} = \frac{\text{desired energy output}}{\text{energy input}} \quad (\text{D-2})$$

Ideally, we would want all of the energy input to be transformed into the desired output of gravitational potential energy of the beam. Then the efficiency of the energy-transformation process would be 1. But ordinarily some of the energy input is unavoidably transformed into the unwanted increase of energy of random motion of atoms in the motor and the air. Then the desired energy output is smaller than the energy input and the efficiency of the energy transformation process is correspondingly smaller than 1. (The typical efficiency of the crane might be only about 0.3.)

Example D-2: Jumping heights of animals

As a simple example of a biological energy-transformation process, consider an animal of mass M which jumps up from the ground. Then the energy of the isolated system consisting of the animal and the earth remains constant. When the animal is momentarily at rest at its maximum height y above the ground, the potential energy of the system has been increased by Mgy , the gravitational potential energy of interaction of the animal with the earth. *

* Since the gravitational potential energy of every atom of mass m in the animal has been increased by mgy , the gravitational potential energy of the whole animal has been increased by $m_1gy + m_2gy + \dots = (m_1 + m_2 + \dots)gy = Mgy$ where M is the mass of the entire animal.

This increase in potential energy is obtained from the decrease E_m of the energy originally stored in the muscles of the animal before its jump. Thus the energy transformation process can be summarized by the relation $Mgy = E_m$ so that

$$y = \frac{E_m}{Mg} \quad (\text{D-3})$$

This result leads to an interesting conclusion. Suppose that we considered another animal B having a similar construction but a very different size than the original animal A . For example, the volume of B

might be 8 times as large as that of A . Then the energy E_m stored in the muscles of B would be 8 times as large as the energy stored in the muscles of A (since B has 8 times as many muscle molecules in its larger muscle volume). But the mass M of B would also be 8 times as large as the mass of A . Hence the ratio E_m/M , and thus also the height y in Eq. (D-3), would be the same for B as for A . In other words, Eq. (D-3) implies the following (intuitively unexpected) conclusion: *All animals of similar construction, irrespective of their size, can jump to the same maximum height.*

Experimental data are in accord with this conclusion. For example, the observed maximum jump height of a kangaroo is about 2.7 meter, that of a kangaroo rat about 2.4 meter, despite the fact that a kangaroo rat (which is about the size of a rabbit) is much smaller than a kangaroo so that its jump height is very much larger than its own size. *

* R. M. Alexander, *Animal Mechanics*, (University of Washington Press, Seattle, 1968), p. 29.

Knowing About Efficiency of Energy Transformations

D-1 (a) Using as energy input the energy stored in one gallon of gasoline (about 1×10^8 J), a standard American car can travel 15 mile along a level road at a constant speed of 60 mph. During this motion, the desired energy output of the car, which is almost entirely used to overcome air resistance, is about 1×10^7 J. What is the efficiency of this car? (b) To lose some weight, a man decides to take up cycling. In this exercise, the efficiency of a human is about 20 percent. What must the man's energy output be in order to consume 1 pound of body fat, so that he uses its stored energy of 4.1×10^3 kilocalorie as energy input? Express your answer in terms of joule. (c) In every hour of cycling, the man's energy output is 4.0×10^5 J. How many hours must he cycle to lose a pound of fat? Would you say that exercise is an easy way to lose weight? (*Answer: 107*)

SECT.

E MACROSCOPIC AND INTERNAL ENERGY

In our preceding discussion of the principle of conservation of energy we have considered every system from a detailed atomic (or "microscopic") point of view by focusing attention on all the atomic particles constituting this system. But we are usually quite content to describe a system from a "macroscopic" (i.e., from a "large-scale") point of view which considers only the gross measurable features of the system without being interested in atomic details. Let us then examine the connection between such a macroscopic description and our previous detailed energy arguments.

A "macroscopic particle" is any object (such as a golf ball or a chip of metal) which is, from a macroscopic point of view, small enough so that its position can be adequately described by that of a single point, but which is large enough so that it consists of very many atoms. (For example, a golf ball consists of about 10^{25} atoms.) The observable large-scale motion of such a macroscopic particle corresponds then to the collective *average* motion of its atoms. In addition, the atoms also move back and forth *relative* to their average positions, although this small-scale motion *averages* to zero and is usually not directly observable.

A complex large-scale system (such as a spring or a bicycle) can be considered as consisting of many macroscopic particles, (i.e., of many small enough parts). Correspondingly, the "macroscopic energy" E_{mac} of the system is the energy associated with the positions and velocities of these macroscopic particles (i.e., with the *average* positions and average velocities of the atoms in them). *

* For example, the macroscopic energy of a spring consists of the kinetic energy associated with the average velocities of all the macroscopic particles in the spring and of the macroscopic potential energy associated with their average positions. (This macroscopic potential energy changes if the spring is deformed so that the average distance between its macroscopic particles is changed.)

In addition, there is the energy associated with the relative motion of the very many atoms within each of the macroscopic particles in the system. This energy is called the "internal energy" E_{int} and it can be quite large, even if the macroscopic energy of the system is zero (e.g., even if the

system is a coin lying at rest on the ground so that both its macroscopic kinetic energy and gravitational potential energy are zero). Thus the entire energy E of the system consists both of the macroscopic energy E_{mac} associated with the average motion of its macroscopic particles and of the internal energy E_{int} associated with the relative motion of the atoms within these particles. Accordingly, we can write:

$$\boxed{E = E_{\text{mac}} + E_{\text{int}}.} \quad (\text{E-1})$$

NATURE OF THE INTERNAL ENERGY

Let us look somewhat more closely at the internal energy E_{int} associated with the relative positions and velocities of the individual atoms within the macroscopic particles in the system. Part of this internal energy is the “structural energy” E_{str} associated with the potential energy of interaction between nearby atoms at their average positions. [For example, this structural energy, which is ultimately due to the Coulomb potential energy between the atomic particles, is responsible for holding atoms together in structures such as molecules or solids (so that external work would have to be done to tear the atoms apart).] In addition, the atoms vibrate rapidly back and forth in random directions about their average positions. Hence the atoms have also “random internal” energy E_{ran} consisting of the kinetic energy due to the randomly changing velocities of the atoms and of the potential energy due to the resulting randomly changing separations between nearby atoms. (This random energy is closely related to the observable property called “temperature,” an increase in random energy corresponding to an increase in temperature.) Thus the internal energy is partly structural and partly due to random motion, so that we can write

$$E_{\text{int}} = E_{\text{str}} + E_{\text{ran}} \quad (\text{E-2})$$

The conservation of energy implies that the energy E of any isolated system remains constant. But this does *not* imply that either the macroscopic energy E_{mac} or the internal energy E_{int} of the system must separately remain constant, but only that their *sum* must remain constant.

For example, suppose that a high-speed bullet strikes a tree and becomes embedded in it. Then the macroscopic kinetic energy of the

bullet has been reduced to zero since the bullet has been brought to rest. But the energy E of the entire isolated system consisting of the bullet, the tree, and the earth must have remained unchanged. Hence the internal energy E_{int} of the system must have increased by an amount equal in magnitude to the decrease in macroscopic kinetic energy of the bullet. In other words, some of the energy of the system has been transformed from macroscopic energy into internal energy, while leaving the energy of the entire system constant.

What has happened in detail? As the bullet slammed into the tree, the interaction between the atoms in the bullet with the atoms in the tree caused the atoms in the bullet to lose their average horizontal velocity and to increase the speeds of the atoms in the tree. As a result of the continuing interaction between the moving atoms in the bullet and the tree, all these atoms finally ended up with slightly larger speeds of random motion than before. The resulting increase in the random internal energy of the system is observable since the bullet and the wood in the tree feel warmer (i.e., have a higher “temperature”) than before the impact of the bullet.

Knowing About the Relation Of E_{ran} to Temperature

E-1 When a man lifts a hammer above his head in preparation for hitting a nail, the contracting muscles in his arm become slightly warmer (i.e., their temperatures increase slightly). During this process, does the random internal energy of these muscles increase, decrease, or remain the same? (*Answer: 116*)

Using a System’s Macroscopic Motion to Describe Changes In Its Macroscopic and Internal Energies (Cap. 3)

In these and later problems, we shall refer to an isolated system S_0 consisting of the earth, the air, and the macroscopic objects, including people, described in the problem. The energy $E = E_{\text{mac}} + E_{\text{int}}$ of this system is then conserved. For simplicity, we shall include in the macroscopic energy E_{mac} only the macroscopic gravitational potential energy U_{mac} due to the interaction of macroscopic particles with the earth.

E-2 A man pounds a nail into a board with a hammer. For each of the following parts of this process, describe what happens to the macroscopic kinetic energy K_{mac} the macroscopic potential energy U_{mac} ,

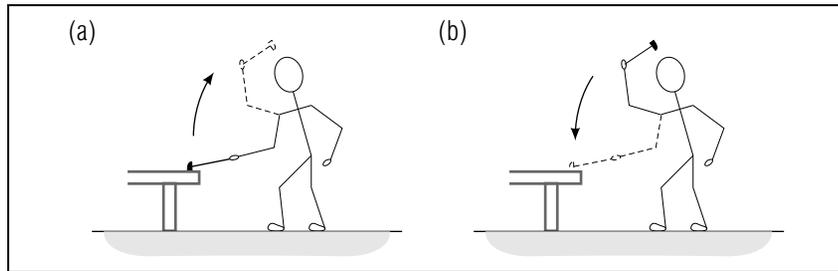


Fig. E-1.

the macroscopic energy $E_{\text{mac}} = K_{\text{mac}} + U_{\text{mac}}$, and the internal energy E_{int} of the system S_0 . In other words, state whether the final values of these energies are larger than, smaller than, or the same as their initial values. (a) The man begins by raising the hammer from an initial position on the nail to a final position over his head (Fig. E-1a). At the beginning and end of this motion, every macroscopic particle in the system S_0 is at rest. (b) The man then brings the hammer forcefully down to strike the nail, driving it into the board (Fig. E-1b). At the beginning and end of this motion, every macroscopic particle in S_0 is at rest. (*Answer: 114*) (*Suggestion: [s-9]*)

E-3 In a “demolition derby” (a race won by the driver who keeps his car running the longest despite frequent collisions with other cars), two cars collide head-on on the level track and come to rest, steaming. Describe what happens to the macroscopic and internal energies of the system S_0 during this collision. (*Answer: 109*) (*Suggestion: [s-5]*)

E-4 Suppose an object slides some distance along a horizontal surface. (a) If the frictional force on the object due to the surface is negligible, what happens to the object’s speed during this motion? What happens to the macroscopic and internal energies of the system S_0 ? (b) Answer the preceding questions if the frictional force on the object is *not* negligible. (*Answer: 113*) (*Practice: [p-2]*)

SECT.

F DIRECTION OF ENERGY-TRANSFORMATION PROCESSES

In the example of the bullet slamming into the tree, we encountered a process where macroscopic energy is converted into random internal energy. Let us now look at this process more closely and ask whether the opposite process can occur, i.e., whether random internal energy can be transformed into macroscopic energy.

Suppose that interactions between atoms (e.g., collisions between them) make it possible to transform the macroscopic energy of an isolated system into random internal energy or vice versa. What can we then expect to happen? The macroscopic energy of the system is associated with the macroscopically observable motion of the system (e.g., with the macroscopic motion of the bullet in our example). In such a motion billions of atoms in the system move jointly in a very special way so that they all have the same non-zero average velocity. But, as these atoms interact with each other and with other atoms in the system, it is highly unlikely that this special motion will persist indefinitely. Instead, the continual interactions between atoms will cause them to move hither and yon in increasingly more random directions until finally they no longer move preferentially in any particular direction (i.e., until they finally move in completely random directions). During this process the macroscopic energy of the system becomes gradually converted into random internal energy of the system. This is the process which we observed when the bullet became embedded in the tree.

Let us now consider the opposite process where the random internal energy of an isolated system is converted into macroscopic energy. To produce this process, billions of atoms, which move initially hither and yon in completely random directions, would all have to start moving together with the same average velocity in the same direction. But it is exceedingly unlikely that billions of atoms spontaneously start moving in such a special way. (For example, if a bullet is initially at rest embedded in a tree, it is *exceedingly* unlikely, although in principle not impossible, that the bullet suddenly flies out of the tree with high speed because all of its atoms start spontaneously moving in the same direction.) In other words, it is exceedingly unlikely that the atoms in an isolated macroscopic system start moving in such a special way that the random internal energy of the system is converted into macroscopic energy.

The preceding comments show that it is much more likely that the interacting atoms in a macroscopic system cease moving in special ways and start moving in more random ways, rather than doing the opposite. Correspondingly, it is much more likely that the macroscopic energy of an isolated system is transformed into random internal energy than vice versa. Hence the processes occurring in such a system have a preferred direction, as expressed by this conclusion:

The random internal energy of an isolated macroscopic system tends to increase at the expense of its macroscopic energy. (F-1)

As the random internal energy of an isolated macroscopic system keeps on increasing, it ultimately reaches the maximum possible value which it can attain. (Since the total energy of the system remains unchanged, its macroscopic energy reaches then correspondingly its minimum possible value. Thus its macroscopic kinetic energy is then zero and its macroscopic potential energy is minimum.) After this the macroscopic system no longer tends to change and is said to be in “equilibrium” (i.e., in a situation where it does not tend to change). *

* The important conclusion in Rule (F-1) and the approach to equilibrium will be discussed much more fully and carefully in the last part of this book.

Note that our conclusion, Rule (F-1), does not specify how *rapidly* the random internal energy of the system tends to increase, i.e., how long it takes for the system to reach equilibrium. Depending on the nature of the interactions between the atoms in the system, the time to reach equilibrium might be 10^{-6} second for one system or a century for another system.

DISSIPATION OF MACROSCOPIC ENERGY

The process whereby the macroscopic energy of a macroscopic system is converted into internal energy of random atomic motions within the system is called “dissipation” of macroscopic energy. Depending on how effectively and rapidly atomic processes bring about this energy dissipation, we can distinguish between two cases, that where the dissipation is negligible and that where it is appreciable:

(1) The transformation of macroscopic energy into random internal energy may proceed not at all or so slowly that it is of negligible importance during the time of interest to us. Then the dissipation of macroscopic energy is negligible. Hence the macroscopic energy of such a “non-dissipative” system remains constant and its internal energy remains also constant. (For example, when the interaction between a baseball and the surrounding air is sufficiently small, the macroscopic energy of the baseball remains constant.)

(2) The transformation of macroscopic energy into random internal energy may proceed sufficiently rapidly to be significant during the time of interest to us. Then the internal energy of such a “dissipative” system tends to increase at the expense of its macroscopic energy. This process continues until the system ultimately reaches its equilibrium situation where its internal energy attains its maximum possible value and its macroscopic energy attains correspondingly its minimum possible value.

Example F-1: Object sliding along a surface

Consider a book sliding along the horizontal surface of a table. The energy of the system consisting of the book, the table, and the earth must then remain constant. Since the atoms near the bottom surface of the book interact with the adjacent atoms at the surface of the table, there is the possibility of gradually converting some of the macroscopic kinetic energy of the book into the random internal energy of relative motion of the atoms in the system. The rate of this conversion process depends on the nature of the surfaces in contact. If the surfaces are very smooth, the rate may be negligible. Then there is no dissipation of macroscopic energy and the macroscopic energy of the sliding book remains constant. But the gravitational *potential* energy of the system remains unchanged as the book moves along the horizontal surface. Thus the macroscopic *kinetic* energy of the system (i.e., of the book) must remain constant. Hence the book continues sliding with constant speed.

On the other hand, suppose that the surfaces are rough enough so that the conversion of macroscopic into random internal energy proceeds at an appreciable rate. Then the random internal energy of the system increases gradually while its macroscopic energy decreases. Hence the macroscopic kinetic energy, and thus also the macroscopic speed, of the book gradually decreases. (This dissipation of macroscopic energy is the atomic explanation for the empirical “friction force” appearing in a macroscopic description.) Ultimately the book then comes to rest, i.e., it reaches the equilibrium situation where the internal energy of the system

is maximum and its macroscopic energy is minimum (i.e., its macroscopic kinetic energy is zero). (The increase in the internal energy of random motion of the atoms in the system becomes apparent by a slight increase in the temperature of the book and table surface.)

Example F-2: Collision between objects

Consider a ball bouncing off the floor. During a collision of the ball with the floor, the atoms in the ball and in the floor interact and thus provide the possibility for the conversion of macroscopic into random internal energy. The extent of this energy conversion depends on the nature of the colliding objects. If there is no dissipation of macroscopic energy, the collision is said to be “elastic.” Then the ball keeps on bouncing indefinitely with constant macroscopic energy. (A good rubber ball on a hard floor might approach this situation.) But ordinarily there is appreciable dissipation of macroscopic energy (so that the collision is “inelastic”). In this case the ball bounces up and down with decreasing macroscopic energy until it finally comes to rest on the floor. The isolated system consisting of the ball and the earth has then reached its final equilibrium situation where its internal energy is maximum (and its macroscopic energy is minimum).

An extreme example of an inelastic collision is that of a ball made of putty which sticks to the floor upon impact. Then all the macroscopic kinetic energy of the ball is immediately converted into internal energy as a result of the first collision with the floor.

Knowing About Dissipation of Macroscopic Energy

F-1 In which of the following situations is the macroscopic energy of the system S_0 being dissipated? (The system S_0 and its macroscopic energy are described in the problems for Sec. E.) (a) A car’s speed decreases as it travels down a hill toward a stop sign. (b) A parachutist and his parachute drift downward with constant speed. (c) A satellite travels with constant speed in its circular orbit around the earth. (d) Two boxcars roll with constant speed toward each other on a level track. (e) The boxcars couple together and come to rest. (f) A thrown ball moves vertically upward with negligible air friction, so that the increase in the ball’s gravitational potential energy exactly compensates for the decrease in its kinetic energy. (*Answer: 111*)

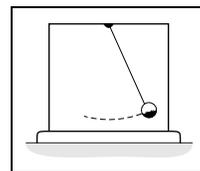


Fig. F-1.

Describing Changes In Macroscopic and Internal Energies for Dissipative and Non-Dissipative Systems (Cap. 4)

F-2 Consider a system S_0 consisting of the earth, the air, and a pendulum which swings from a frictionless pivot within an enclosure (Fig. F-1). If the air is pumped out of the enclosure, the system S_0 is non-dissipative. Under these conditions, the pendulum bob is initially released from rest at a position on the right side 1.0 cm above the lowest point on its arc, as shown in Fig. F-1. (a) As the pendulum swings 100 times, what happens to the random internal energy of the system S_0 ? What happens to the macroscopic energy of this system? (b) After its hundredth swing, the pendulum bob is momentarily at rest on the right side of the enclosure. At this time, is the final value of the system’s macroscopic kinetic energy larger than, smaller than, or the same as its initial value? Is the final value of the system’s macroscopic potential energy larger than, smaller than, or the same as its initial value? (c) *Review:* Is the final height of the pendulum bob larger than, equal to, or smaller than 1.0 cm? (*Answer: 118*)

F-3 Answer the questions (a), (b), and (c) in problem F-2 for the situation where air fills the pendulum’s enclosure. The system S_0 is then dissipative. (d) Describe the position and speed of the pendulum bob when this system is in its final macroscopic equilibrium state. (*Answer: 125*) (*Practice: [p-3]*)

NOTE:

Because the following section discusses material having many important applications, we recommend that you work through it.

SECT.

G MACROSCOPIC EQUILIBRIUM

As a final illustration of the utility of arguments based upon work and energy, let us consider a macroscopic system (such as a bridge or a skeleton) which we shall consider from a completely macroscopic point of view. Then we may ask under what conditions such a system will be in “macroscopic equilibrium” in this sense:

Def.	Macroscopic equilibrium of a system: A situation where every macroscopic particle of the system remains at rest. *	(G-1)
------	---	-------

* Note that a system may be in macroscopic equilibrium without being in equilibrium from an atomic point of view (e.g., internal energy might still be transferred from some atoms to other atoms in the system).

If the system is in macroscopic equilibrium so that each of its macroscopic particles remains at rest, the acceleration of each such particle, and thus also the *total* force on each such particle, must be zero. Imagine then any small displacement of the particles in the system, no matter how this displacement is produced. (Such an imaginary displacement is commonly called a “virtual” displacement.) Since the total force on every particle is zero, the work done on every particle in such a displacement must also be zero. Hence the total work δW done by all forces on the entire system must also be zero. Thus we arrive at this conclusion:

If a system is in macroscopic equilibrium, the total work done on the system in *any* small displacement must be zero. (G-2)

(This conclusion is called the “principle of virtual work.”)

The result, Rule (G-2), provides often the simplest starting point for discussing situations of macroscopic equilibrium because in many cases the work done by most forces is zero. Then the work δW done on the system by all forces can be easily calculated and the principle in Rule (G-2) can be readily applied. As an important application of these ideas, let us discuss the macroscopic equilibrium of *rigid* objects (such as bridges or bones).

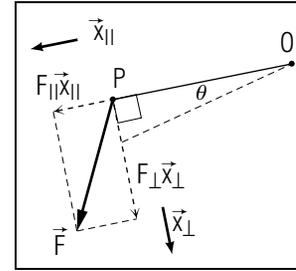


Fig.G-1: Small rotation of a rigid object around the point O . The unit vectors \hat{x}_{\parallel} and \hat{x}_{\perp} indicate directions parallel and perpendicular to the line OP .

MACROSCOPIC EQUILIBRIUM OF RIGID OBJECTS

If an object is rigid, the distance between all its constituent macroscopic particles remains always the same. Hence, as pointed out at the end of Sec. A, the work done on a rigid object by all the *mutual* forces between its particles is zero. Thus the total work done on a rigid object in any small displacement is merely the work done on this object by the *external* forces on the object due to particles outside the object.

Consider then any rigid object and suppose that an external force \vec{F} acts on a particle located at a point P in the object. (See Fig. G-1.) To apply the principle in Rule (G-2), imagine that a small displacement of this object is produced, as shown in Fig. G-1, by rotating the object counterclockwise by a small angle θ around an axis through some point O . (The axis is perpendicular to the plane of the paper in Fig. G-1.) If the point P is at a distance L from O , the resulting displacement of the particle at P has then a magnitude $L\theta$ (if θ is expressed in radians) * and a direction perpendicular to the line OP (in the counterclockwise direction indicated by \hat{x}_{\perp}).

* From Fig. G-1, $\theta = (\text{magnitude of displacement})/L$.

Suppose then that the external force \vec{F} is decomposed into its component vectors parallel and perpendicular to the line OP . Then the parallel component vector of \vec{F} does no work on the particle at P since it is perpendicular to the displacement of this particle. On the other hand, the perpendicular component vector of \vec{F} is parallel to the displacement of the particle at P and thus does on this particle the work $F_{\perp}(L\theta)$ if F_{\perp} denotes the numerical component of \vec{F} along the counterclockwise direction \hat{x}_{\perp} perpendicular to the line OP (i.e., parallel to the displacement of the point P).

When several external forces $\vec{F}_1, \vec{F}_2, \dots$ act on the rigid object on particles located at points at distances L_1, L_2, \dots from the point O , the work done on the object by all these forces acting jointly is simply the sum of the individual works done by these forces. Hence the work δW done on the rigid object in the small rotation indicated in Fig. G-1 is

$$\delta W = F_{1\perp} L_1 \theta + F_{2\perp} L_2 \theta + \dots \quad (\text{G-3})$$

But, if the object is to be in equilibrium, the principle in Rule (G-2) states that this work should be zero. Thus $\delta W = 0$ and Eq. (G-3) is equivalent to the relation

$$F_{1\perp} L_1 + F_{2\perp} L_2 + \dots = 0 \quad (\text{G-4})$$

For every force \vec{F} , the product $F_{\perp} L$ is called the “torque” due to this force in accordance with this definition:

Def.	<p>Torque: The torque exerted about an axis through a point O by a force \vec{F} applied at a point P is the product $F_{\perp} L$ where F_{\perp} is the numerical component of \vec{F} along a counterclockwise direction perpendicular to the line OP and where L is the distance from O to P.</p>	(G-5)
------	--	-------

With this definition, the condition Eq. (G-3) for the macroscopic equilibrium of a rigid object can be stated in words:

<p>If a rigid object is in macroscopic equilibrium, the sum of the torques exerted around any axis by all external forces on the object must be equal to zero.</p>	(G-6)
--	-------

Example G-1: Force exerted by the biceps muscle

To measure the force \vec{F}_m exerted by the biceps muscle, one can apply a known horizontal force \vec{F}_w to a person’s wrist while asking him to keep his forearm in the vertical position indicated in Fig. G-2. The forearm is then in macroscopic equilibrium while acted on by the horizontal force \vec{F}_m of the muscle (attached to the forearm bone at the point P_m) and the applied force \vec{F}_w (acting on the wrist at P_w). The forearm is free to pivot at the elbow E . The distance from E to P_m is $L = 5.0$ cm, while the distance from E to P_w is $L = 27.5$ cm. How then can the force \vec{F}_m exerted by the muscle be found from the known force \vec{F}_w ?

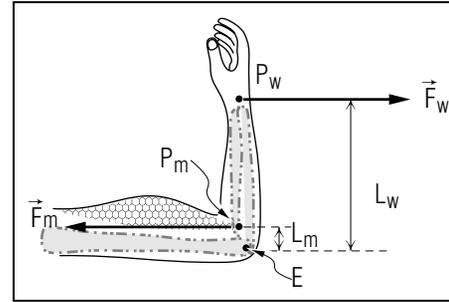


Fig. G-2: Macroscopic equilibrium of the forearm.

Since the forearm is in macroscopic equilibrium, we can use the condition in Rule (G-6) to relate the external forces acting on the forearm. These external forces are the force \vec{F}_m exerted by the muscle, the force \vec{F}_w applied to the wrist, the force \vec{F}_e exerted on the forearm by the elbow joint, and the gravitational force \vec{F}_g on the forearm due to the earth. (This gravitational force on all particles in the forearm is equivalent to a total gravitational force acting at the center of mass of the forearm.) To apply Rule (G-6), we shall calculate torques around an axis through the elbow joint E . Then the torque exerted by the force \vec{F}_e due to the elbow is zero since this force acts at zero distance from E .

The torque exerted by the downward gravitational force F_g is also zero since the numerical component of this force along a horizontal direction perpendicular to the vertical forearm is zero. The torque exerted by the force F_m due to the muscle is $F_m L_m$ since the numerical component of F_m along the counterclockwise direction perpendicular to the forearm is just equal to the magnitude F_m . The torque exerted by the force F_w applied to the wrist is $-F_w L_w$ since the numerical component of F_w along the counterclockwise direction perpendicular to the forearm is $-F_w$ (because \vec{F}_w is along a *clockwise* direction perpendicular to the forearm). Thus the condition in Rule (G-6) becomes

$$F_m L_m - F_w L_w = 0 \quad (\text{G-7})$$

Hence

$$F_m = F_w \frac{L_w}{L_m} \quad (\text{G-8})$$

Note that F_m must be much larger than F_w since L_m is much smaller than L_w .

It is found experimentally that a young man can maintain his forearm in equilibrium against a maximum force of magnitude $F_w = 375$ newton

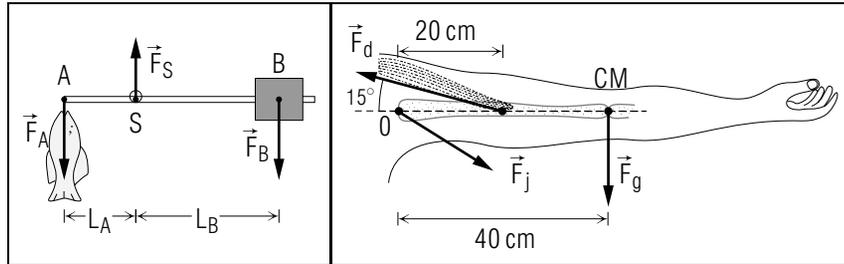


Fig. G-3.

Fig. G-4.

(i.e., 84 pound) applied to his wrist. Hence Eq. (G-8) shows that the maximum magnitude of the force which can be supplied by the biceps muscle is $(375 \text{ newton})(27.5 \text{ cm}/5.0 \text{ cm}) = 2060 \text{ newton}$ (i.e., about 460 pound). This is an impressively large force.

Now: Go to tutorial section G.

APPLYING THE TORQUE CONDITION FOR MACROSCOPIC EQUILIBRIUM

G-1 Figure G-3 shows a simple scale used throughout the world to measure the mass M of an object (such as the fish shown). The scale consists of a bar of negligible mass, which is free to rotate about a supporting ring passing through the bar at a point S , and a metal block of mass m which can be moved along the bar. The object is hung from a hook at the point A on one end of the bar, a distance L_A from the point S , and the block is moved along the bar until the bar is horizontal and in macroscopic equilibrium (i.e., at rest). In this situation, the block is located at a point B , a distance L_B from the point S . The block then exerts on the bar at the point B a downward force \vec{F}_B equal in magnitude to the block's weight, and the object exerts on the bar at the point A a downward force \vec{F}_A equal in magnitude to the object's weight. (a) To relate the object's mass M to the known mass m of the block, apply the equilibrium torque condition with an axis passing through the point S at which the supporting force \vec{F}_S is applied to the bar. Write an equation expressing M in terms of m and the known distances L_A and L_B . (b) Suppose that $m = 1 \text{ kg}$ and $L_A = 0.1 \text{ meter}$. What is the object's mass M if $L_B = (1/2)L_A = 0.05 \text{ meter}$, if $L_B = L_A = 0.1 \text{ meter}$, and if $L_B = 2L_A = 0.2 \text{ meter}$? (Answer: 122) (Suggestion: [s-3])

G-2 Because of the force \vec{F}_d exerted on the humerus (the bone in the upper arm) by the deltoid muscles attached to the shoulder, a

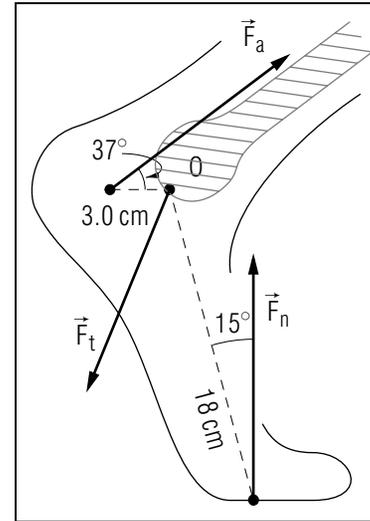


Fig. G-5.

man can keep his 15 kg arm outstretched horizontally (see Fig. G-4). In this situation, the shoulder joint exerts on the arm a force \vec{F}_j , and the earth exerts on the arm a gravitational force \vec{F}_g applied at the arm's center of mass (CM). What is the magnitude F_d of the force the deltoid muscles must exert to maintain the arm in macroscopic equilibrium? (Since the force \vec{F}_j is unknown, apply the equilibrium torque condition with an axis passing through the point O at which \vec{F}_j is applied.) (Answer: 115)

G-3 Figure G-5 shows the forces acting on the foot of an 80 kg man in a crouching position. The normal force \vec{F}_n exerted by the floor thus has a magnitude equal to half the man's weight. In this situation, what is the magnitude of the force \vec{F}_a exerted by the Achilles tendon and calf muscles? (Since the force \vec{F}_t exerted on the foot by the tibia is unknown, choose an axis passing through the point O at which this force is applied. You may neglect the gravitational force on the foot, since it is negligible compared with the other forces on the foot.) (Answer: 120) More practice for this Capability: [p-4], [p-5]

SECT.

H SUMMARY**DEFINITIONS**

kinetic energy of a system; Def. (A-7)

work done on a system; Def. (A-8)

potential energy of a system; Def. (B-3)

energy of a system; Def. (C-3)

macroscopic equilibrium of a system; Def. (G-1)

torque; Def. (G-5)

IMPORTANT RESULTS

Relation between energies and work for a system: Eq. (A-9), Eq. (B-1), Eq. (B-2)

$$K_b - K_a = W_{ab} \text{ (by all forces)}$$

$$U_a - U_b = W_{ab} \text{ (if work is independent of process)}$$

$$U_a = W_{as}, U_s = 0$$

Energies of a system and of its particles: Def. (A-7), Rule (B-6), Eq. (C-4)

$$K = K_1 + K_2 + \dots \text{ (all particles)}$$

$$U = U_{12} + U_{13} + U_{23} + \dots \text{ (all pairs of particles)}$$

$$E = K + U$$

Conservation of energy: Rule (C-5)

$$E = K + U = \text{constant if } U \text{ is due to all forces doing work}$$

Macroscopic and internal parts of the energy: Eq. (E-1)

$$E = E_{\text{mac}} + E_{\text{int}}$$

Direction of energy transformations: Rule (F-1)

Random internal energy tends to increase in an isolated system.

Definition of torque: Def. (G-5)

$$F_{\perp} L$$

Macroscopic equilibrium of a system: Rule (G-2), Rule (G-6)

$$\delta W = 0 \text{ for any displacement}$$

For rigid object, sum of torques exerted around any axis by all external forces is zero.

NEW CAPABILITIES

You should have acquired the ability to:

- (1) Understand the principle of conservation of energy for an isolated system of atomic particles. (Sec. C)
- (2) Use the masses, speeds, and potential energies of particles in a system to find:
 - (a) the kinetic energy of the system (Sec. A),
 - (b) the potential energy of the system (Sec. B),
 - (c) the energy of the system (Sec. C, [p-1]).
- (3) Use a description of the macroscopic motion of an isolated system to describe changes in its macroscopic potential and kinetic energies, its macroscopic energy, and its internal energy. (Sec. E, [p-2]).
- (4) Describe the variation of internal and macroscopic energies of dissipative and non-dissipative isolated systems, and the final macroscopic equilibrium state of isolated dissipative systems. (Sec. F, [p-3]).

Knowing About the Energy of Systems of Particles

H-1 Consider the system in which you live, which consists of you and the things with which you interact, including the earth, the air, and the food you eat. Assuming that we can regard this system as isolated, which of the following statements about its energy are true? (a) The energy of atomic particles in this system due to all interactions must remain constant. (b) The macroscopic energy of this system must remain constant. (c) The internal energy of this system must remain constant. (d) The internal energy of this system must increase. (e) Since this system is dissipative, its random internal energy is very unlikely to decrease. (*Answer: 124*)

Describing the Energies of Systems (Cap. 3, 4)

H-2 In Unit 415, we applied the principle of conservation of energy for single particles (e.g., macroscopic particles such as cars) interacting with fixed particles (e.g., the earth). The following example illustrates how this principle is related to the principle of conservation of energy for isolated systems of atomic particles, as discussed in this unit. Suppose that a car moves down a hill with its engine off and its gears in neutral, so that the structural energy stored in the car's fuel is not transformed

into macroscopic energy or random internal energy. For our purposes, the system S_0 consisting of the car, the earth, and the air is isolated, and we can regard the earth as fixed relative to an inertial reference frame. Since the earth's kinetic energy is thus zero, the macroscopic energy E_{mac} of the system S_0 is just the car's energy $E = 1/2mv^2 + mgy$ (i.e., the sum of the car's macroscopic kinetic and gravitational potential energies). (a) Suppose that the frictional forces on the car due to the air and the road are negligible, so that the system S_0 is non-dissipative. What happens to the random internal energy E_{ran} , the internal energy E_{int} , and the macroscopic energy E_{mac} of the system S_0 ? Are we correct in saying, as we did in Unit 415, that the car's energy E is conserved? (b) Suppose that these frictional forces are *not* negligible, so that the system S_0 is dissipative. What happens to the random internal energies E_{ran} , E_{int} , and E_{mac} in this case? Are we correct in saying, as we did in Unit 415, that the car's energy E is *not* conserved in this case? (*Answer: 126*)

SECT.

I PROBLEMS

I-1 *Conservation of momentum and energy in a ballistics experiment:* A bullet of mass 2.0 gram is fired horizontally with an initial speed of 500 m/s toward a 1.0 kg wood block at rest on a flat, smooth surface (Fig. I-1). The total external force on the system consisting of bullet and block is thus negligible. (a) What is the block's speed just after the bullet hits and becomes embedded in it? (b) If we choose a standard position at the level of the block's center, the macroscopic gravitational potential energy of this system is zero during the collision. What is the system's macroscopic energy E_{mac} just before and just after the collision? (c) Is E_{mac} conserved during the collision? If not, find the changes ΔE_{mac} and ΔE_{int} in the system's macroscopic and internal energies during the collision. (*Answer: 121*) (*Suggestion: [s-6]*)

I-2 *Distance of closest approach for two protons:* In a scattering experiment, a proton of mass m and charge q collides head-on with another proton initially at rest in a "target." When it is far from the target, the incident proton has a speed v , and the two protons form an isolated system during their collision. If the protons interact only by the Coulomb electric force, what is the distance R between them at the time during their collision when both protons have the same speed $v/2$? (This distance R is the "distance of closest approach," or the smallest distance between the protons during their collision.) (*Answer: 117*) (*Suggestion: [s-4]*)

I-3 *Scattering angle for colliding protons:* A glancing collision of two protons is illustrated in Fig. I-2. When the protons are far apart before the collision, proton 1 has a velocity \vec{V} and proton 2 is at rest (Fig. I-1a). When the protons are far apart after the collision, the two

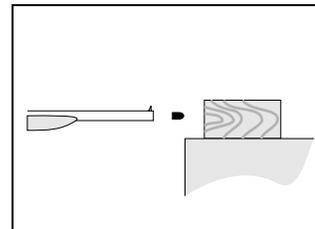


Fig. I-1.

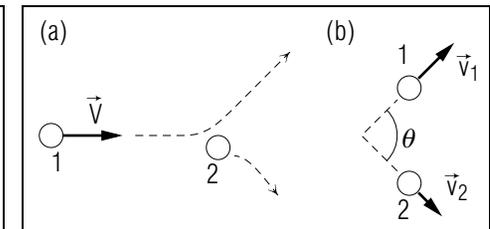


Fig. I-2.

protons have velocities \vec{v}_1 and \vec{v}_2 (Fig. I-2b). By applying the principles of conservation of momentum and conservation of energy to the isolated system consisting of the two protons, show that the angle θ between \vec{v}_1 and \vec{v}_2 is equal to 90° . (*Hint*: Use conservation of momentum to draw a vector diagram showing the triangle formed by \vec{V} , \vec{v}_1 , and \vec{v}_2 . Then use conservation of energy to show that the sides of this triangle are related by the Pythagorean Theorem.) (*Answer*: 123)

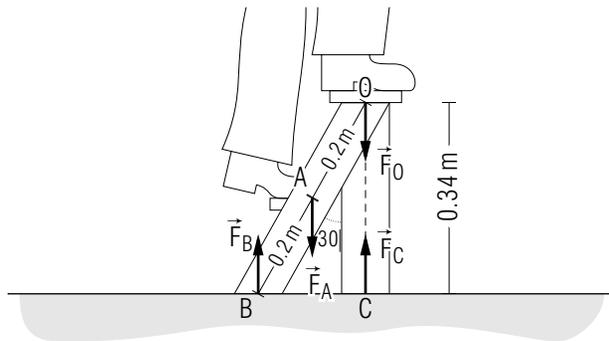
I-4 *Force exerted at the ankle joint*: The bones in a joint suffer very little wear even though they exert very large forces on each other, presumably because the region between the bones is very well lubricated. To illustrate the magnitudes of such forces, use your results in problem G-3 and the equation of motion to find the magnitude F_t of the force exerted by the tibia on the ankle joint in a crouching man. (*Answer*: 119) (*Suggestion*: [s-2])

TUTORIAL FOR G

APPLYING THE TORQUE CONDITION FOR MACROSCOPIC EQUILIBRIUM

g-1 *PURPOSE:* Whenever a rigid object is in macroscopic equilibrium, we can use the torque condition for equilibrium to relate external forces acting on the object. We shall illustrate a method for applying this relation systematically by following the basic steps of the problem-solving strategy outlined in text section D of Unit 409. In the process, we shall briefly illustrate how to find the torque exerted on an object by a force.

g-2 *A METHOD FOR APPLYING THE EQUILIBRIUM TORQUE CONDITION:* The following drawing shows a man standing on a light stepladder. The man's two feet exert a force $\vec{F}_O = 200\text{ N}$ downward at the point O and a force $\vec{F}_A = 600\text{ N}$ downward at the point A ,



and we wish to find the magnitudes of the upward forces \vec{F}_B and \vec{F}_C exerted on the stepladder by the floor at the points B and C . The remaining external force on the stepladder, the gravitational force, is negligible.

By applying the equation of motion to the stepladder, we can write that $\vec{F}_O + \vec{F}_A + \vec{F}_B + \vec{F}_C = \vec{0}$, since the stepladder is at rest, or in macroscopic equilibrium. Thus $\vec{F}_B + \vec{F}_C = -(\vec{F}_O + \vec{F}_A) = 800\text{ N}$ upward. To find the magnitudes of the individual forces \vec{F}_B and \vec{F}_C , however, we need another relation. Let us systematically apply the torque condition for macroscopic equilibrium in order to find this relation and the magnitudes F_B and F_C . To apply this relation, let us choose an axis passing through the point O at the hinge of the stepladder.

DESCRIPTION:

Sketch: We already have one. *Known:* A force $\vec{F}_O = 200\text{ N}$ downward is applied at O . A force $\vec{F}_A = 600\text{ N}$ downward is applied at A , a distance $L_A = 0.20$ meter from O . The upward force \vec{F}_B is applied at B , a distance $L_B = 0.40$ meter from O . The upward force \vec{F}_C is applied at C , a distance $L_C = 0.34$ meter from O . The stepladder leg makes an angle of 30° with the vertical, so that the forces \vec{F}_A and \vec{F}_B make an angle of 30° with the leg. From $\vec{F} = m\vec{a}$, $\vec{F}_B + \vec{F}_C = 800\text{ N}$ upward. *Desired:* magnitudes of the forces \vec{F}_B and \vec{F}_C .

PLANNING:

- Choose the system and the principle to be applied.
Since the stepladder is in macroscopic equilibrium, we shall apply the torque condition for macroscopic equilibrium to the stepladder, with an axis chosen through the point O .
- Express the principle in terms of symbols for known and desired information.

We first write the torque condition in the form

Sum of torques due to external forces = 0.

Next we must find the torque exerted around an axis through the point O by each of the external forces \vec{F}_O , \vec{F}_A , \vec{F}_B , and \vec{F}_C . Each torque is equal to the quantity $F_\perp L$, where L is the length of the straight line joining the point O and the point at which the force is applied, and F_\perp is the component (i.e., the numerical component) of this force along the counterclockwise direction perpendicular to this line.

Let us first find the forces which exert *zero* torque. Does any force exert zero torque because the distance $L = 0$? If so, which one(s)?

► _____

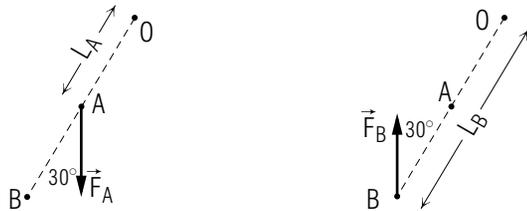
Does any force exert zero torque because its component $F_\perp = 0$? If so, which one(s)?

► _____

(Answer: 4)

Let us now find the torques exerted by the remaining forces \vec{F}_A and \vec{F}_B . On the following diagrams, construct the component vectors of \vec{F}_A and \vec{F}_B parallel and perpendicular to the line joining the point O to the points A and B , and indicate with a small arrow the counterclockwise direction perpendicular to this line. Using the diagrams, express the component F_{\perp} for each force, and the torque exerted by each force, in terms of symbols for known and desired information.

▶



\vec{F}_A : Component = _____

Torque = _____

\vec{F}_B : Component = _____

Torque = _____

Use your results to write the torque equation for macroscopic equilibrium in terms of symbols for known and desired information.

▶ _____

(Answer: 2)

IMPLEMENTATION:

(1) *Solve algebraically for desired quantities.*

Write an equation for the magnitude F_B in terms of symbols for known quantities.

▶ $F_B =$ _____

Write an equation for the magnitude F_C in terms of F_B and known quantities.

▶ $F_C =$ _____

(2) *Substitute known values and find the desired quantities.*

▶ $F_B =$ _____

$F_C =$ _____

CHECKING:

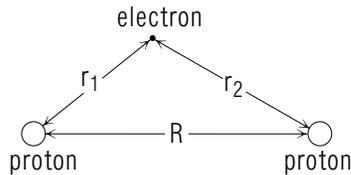
Check that the work is correct, that the results have the correct signs and units, and that the magnitudes are not unreasonably large or small.

The method we have illustrated is useful in applying the torque condition for macroscopic equilibrium to any problem. In particular, it should help you solve systematically the problems in text section G.

(Answer: 6) Now: Go to text problem G-1.

PRACTICE PROBLEMS

p-1 *FINDING THE ENERGY OF A SYSTEM (CAP. 2C)*: An isolated hydrogen molecular ion forms a system consisting of two protons, each of mass m_p and charge e , and a single electron, of mass m_e and charge $-e$, as shown in the following drawing. These particles interact only by the Coulomb electric force. The two protons are separated by a distance R and are at rest. The electron is separated from the two protons by distances r_1 and r_2 and has a speed v . Write an expression for the energy E of this system due to all interactions. (*Answer: 5*) (*Suggestion: Review text problem C-1.*)



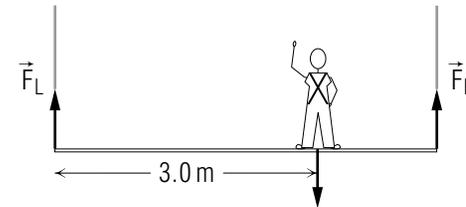
p-2 *USING A SYSTEM'S MOTION TO DESCRIBE ITS ENERGIES (CAP. 3)*: For each of the following motions of a car along a road, describe what happens to the macroscopic and internal energies of the system S_0 consisting of the car, the air, and the earth. (The system S_0 and its macroscopic energy are described in the problems for text section E.) The car drives (a) with increasing speed along a level road, (b) with constant speed along the level road, (c) with constant speed up a hill, (d) with constant speed down the hill, and finally (e) with decreasing speed along a level road. (*Answer: 1*) (*Suggestion: review text problems E-2 through E-4.*)

p-3 *DESCRIBING DISSIPATIVE AND NON-DISSIPATIVE SYSTEMS (CAP. 4)*: Consider a system S_0 consisting of the earth, the air, and a satellite which is initially in a circular orbit at a height h above the earth's surface. (a) If the initial height h is large enough, the interaction of the satellite with the earth's atmosphere (i.e., the air resistance on the satellite) is negligible, and the system S_0 is non-dissipative. What happens to the random internal energy and macroscopic energy of this system? Suppose the satellite has its original speed v after 1000 revolutions around the earth. At this time, are the values of the system's macroscopic kinetic energy and gravitational potential energy larger than, smaller than, or the same as their original values? Is the value of the satellite's

height above the earth's surface larger than, smaller than, or the same as its original value? (b) If the initial height h is not large enough, the satellite does interact appreciably with the atmosphere, and the system S_0 is dissipative. Answer the questions in part (a) for this case. Describe the satellite's motion in the final macroscopic equilibrium state of this system. (*Answer: 7*) (*Suggestion: review text problems F-2 and F-3.*)

More Difficult Practice Problems (Text Section G)

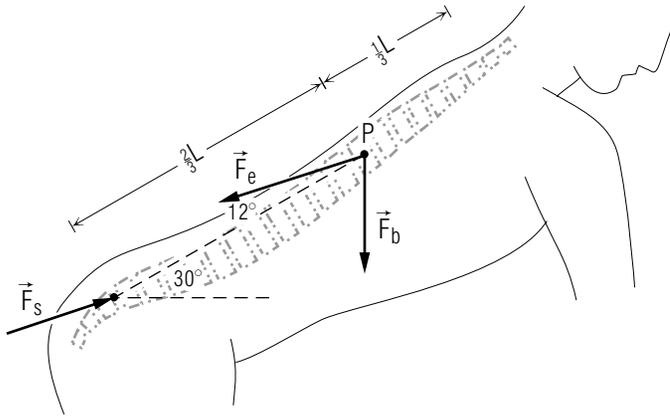
p-4 *APPLYING THE TORQUE CONDITION FOR MACROSCOPIC EQUILIBRIUM*: A painter of mass 80 kg stands 3.0 meter from the left end of a horizontal plank 4.0 meter long, which is supported at each end by vertical ropes. The painter thus exerts on the plank a downward force equal in magnitude to his weight, as shown in this drawing:



We neglect the gravitational force on the plank, so that the only other forces acting on it are the tension forces \vec{F}_L and \vec{F}_R exerted by the left-hand and right-hand ropes. (a) By applying the equation of motion, write an expression relating the magnitudes F_L and F_R to the man's weight. (b) By applying the torque condition for macroscopic equilibrium with an axis passing through either end of the plank, find one of the magnitudes F_L or F_R . Then find the other. (*Answer: 3*) (*Suggestion: review text problem G-1.*)

p-5 *APPLYING THE TORQUE CONDITION FOR MACROSCOPIC EQUILIBRIUM*: When a man bends over, he can maintain his upper body in equilibrium because of the force exerted by the erector spinae muscles (literally, the muscles used to "erect" or raise the spine). The combined action of these muscles can be described by a force \vec{F}_e applied to the vertebral column at a point P . This point is a distance of $(2/3)L$ from the sacrum (at the bottom of the spine), where L is the length of the vertebral column. When a man of weight W bends over at an angle

of 30° from the horizontal as shown in the following drawing, the force exerted on the vertebral column by his upper body can be described by a force \vec{F}_b of magnitude $0.60W$, which is also applied at the point P . The remaining force on the vertebral column is the force \vec{F}_s applied by the sacrum. In this situation, what is the magnitude F_e of the force applied by the erector spinae muscles? (Since the force F_s is unknown, apply the torque condition for macroscopic equilibrium with an axis chosen through the point at which F_s is applied. The gravitational force on the vertebral column can be neglected in this situation.) (Answer: 8) (Suggestion: review text problems G-2 and G-3.)



SUGGESTIONS

s-1 (Text problem A-1): Express the kinetic energy of each atom in terms of the symbols provided, and then add these individual kinetic energies to find the kinetic energy of the system.

s-2 (Text problem I-4): Express each of the known forces \vec{F}_n and \vec{F}_a in terms of a horizontal unit vector \hat{x} and a vertical unit vector \hat{y} . Using the equation of motion $\vec{F} = m\vec{a} = \vec{0}$ for the stationary foot, you can express the force \vec{F}_t in terms of \hat{x} and \hat{y} , and thus find its magnitude.

s-3 (Text problem G-1): Use the method outlined in tutorial frame [g-2] as a guide. Note that all forces applied to the bar are vertical, and so are perpendicular to the horizontal bar. Thus the component F_\perp of each force along a counterclockwise direction perpendicular to the bar is equal in magnitude to the magnitude of the force.

s-4 (Text problem I-2): Apply the principle of conservation of energy to the system consisting of both protons, since only this system is isolated. Write an expression for the energy of this system in the initial state where the protons are far apart, and in the final state where they are separated by a distance R . Since the system's energy is conserved, these two expressions must be equal. (You may want to review text problem C-1.)

s-5 (Text problem E-3): Use your work in text problem E-2 as a guide. First decide what happens to the macroscopic kinetic energy K_{mac} and macroscopic gravitational potential energy U_{mac} of the system S_0 . Then decide what happens to their sum, the macroscopic energy $E_{\text{mac}} = K_{\text{mac}} + U_{\text{mac}}$. Then, using conservation of energy, decide what happens to the internal energy E_{int} of the system.

s-6 (Text problem I-1): Since it is likely that macroscopic energy is not conserved in this collision, apply the principle of conservation of momentum to answer part (a). (You may want to review text section F of Unit 413.) Then use your result to find the macroscopic energy $E_{\text{mac}} = K_{\text{mac}} + U_{\text{mac}}$ of the system before the collision (when only the bullet is moving) and after the collision (when the bullet and block move together) Note that since $U_{\text{mac}} = 0$, $E_{\text{mac}} = K_{\text{mac}}$.

s-7 (Text problem A-2): Part (b): Use the relation $\delta W = F dr_F$ to find the small work done on each ion by the total force acting on it (i.e., by the

attractive electric force exerted by the other ion). Since each ion moves along this force, the component dr_F of the displacement of each ion is just the distance it moves. The small work δW done on the *system* by all forces is just the *sum* of these small works done on the two ions in the system.

Part (c): The change dK in the system's kinetic energy is equal to the small work δW done on the system by all forces. Thus the system's final kinetic energy is equal to its initial kinetic energy plus the small work δW found in part (b).

s-8 (*Text problem B-1*): Part (a): To find the Coulomb potential energy of this single pair of protons, use the result (derived in text section D of Unit 415) that the Coulomb potential energy due to the interaction of two particles of charge q_1 and q_2 separated by a distance R is

$$U_{1,2} = U_{2,1} = k_e \frac{q_1 q_2}{R}$$

Part (b): Each pair of protons in the beryllium nucleus is separated by the same distance as the pair of protons described in part (a), and so the Coulomb potential energy of each pair of protons is equal to the value found in part (a). The Coulomb potential energy of the system consisting of four protons is then the sum of this Coulomb potential energy for each *pair* of protons. Thus you need only count the number of pairs of protons in the nucleus, and multiply this number by the potential energy found in part (a). (I find it easy to count pairs by first drawing a diagram showing the particles as dots, and then connecting every dot to every other one by a line. The number of lines is then the number of pairs. For example, the following diagram shows that there are 10 pairs of particles in a system of 5 particles.)



s-9 (*Text problem E-2*): Let us work part (a) as an example. The system S_0 we are considering consists of the man, the hammer, the nail, the board, the earth, and the air. Since all macroscopic particles in the system S_0 are at rest when the hammer is resting on the nail and when the

hammer is above the man's head, the macroscopic kinetic energy K_{mac} of the system is zero at both times. Thus the final value of K_{mac} is the *same* as its initial value. (The kinetic energy K_{mac} increased as the man accelerated the hammer and his arm in raising them, but it decreased by an equal amount when he brought them to rest again.) Since every macroscopic particle in the hammer and arm moves to a final position higher than its initial one, the final value of the macroscopic gravitational potential energy U_{mac} of the system is *larger* than its initial value. (Since the other macroscopic particles in the system remained fixed, their contribution to U_{mac} remained unchanged.) By combining these results, we find that the system's macroscopic energy $E_{\text{mac}} = K_{\text{mac}} + U_{\text{mac}}$ has a final value *larger* than its initial value.

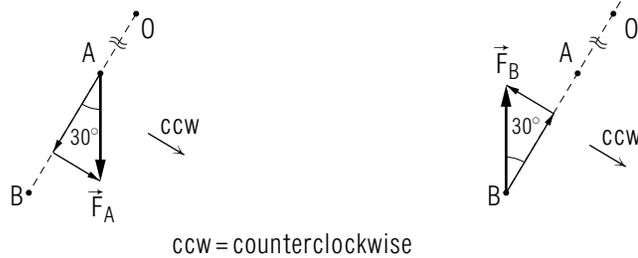
Since the energy $E = E_{\text{mac}} + E_{\text{int}}$ of the system S_0 must remain constant, it follows that the final value of the system's internal energy E_{int} must be *smaller* than its initial value. This is true despite the result (from text problem E-1) that the *random* internal energy of this system becomes larger as the man raises the hammer. The reason is that the *structural* internal energy E_{str} of the system becomes markedly smaller as chemical fuels are burned in contracting the man's muscles, so that $E_{\text{int}} = E_{\text{ran}} + E_{\text{str}}$ becomes smaller.

ANSWERS TO PROBLEMS

1.

Motion	Macroscopic energy	Internal energy
a	becomes larger	becomes smaller
b	remains same	remains same
c	becomes larger	becomes smaller
d	becomes smaller	becomes larger
e	becomes smaller	becomes larger

2.



\vec{F}_A : Component = $F_A \sin 30^\circ$, Torque = $F_A \sin 30^\circ L_A$.

\vec{F}_B : Component = $-F_B \sin 30^\circ$, Torque = $-F_B \sin 30^\circ L_B$.

$F_A \sin 30^\circ L_A - F_B \sin 30^\circ L_B = 0$, or $F_A L_A - F_B L_B = 0$.

3. a. $F_L + F_R = 800$ Nb. $F_L = 200$ N, $F_R = 600$ N4. Yes, \vec{F}_O . Yes, \vec{F}_C 5. $E = (1/2)mv^2 - k_e e^2/r_1 - k_e e^2/r_2 + k_e e^2/R$ 6. $F_B = F_A L_A / L_B$, $F_C = 800$ N $- F_B$, $F_B = 300$ N, $F_C = 500$ N7. a. E_{ran} and E_{mac} remain the same. K_{mac} and U_{mac} are the same. Height is the same.b. E_{ran} becomes larger, E_{mac} becomes smaller. K_{mac} is the same, U_{mac} is smaller. Height is smaller. Satellite at rest on the earth's surface (or all parts of it at rest on the earth's surface).8. $F_e = 2.5W$, two and a half times the man's weight!101. a. 6.9×10^{-13} Jb. 1.4×10^{-12} Jc. -7×10^{-13} J102. a. 3.3 or 3.4×10^{-20} J (either is acceptable)b. 3×10^{-21} Jc. 3.6 or 3.7×10^{-20} J103. a. 2×10^{-21} J

b. the same as in part a.

104. a. 2.3×10^{-13} Jb. $6(2.3 \times 10^{-13} \text{ J})$, or 1.4×10^{-12} J105. $E = (1/2)m_n v_n^2 + (1/2)m_e v_1^2 + (1/2)m_e v_2^2 + U_{1,n} + U_{2,n} + U_{1,2}$ 106. $K_a = (1/2)mv^2$, $K_b = (5/16)mv^2$, $K_c = (5/18)mv^2$, $K_d = (1/4)mv^2$.
No.

107. a. 0.1 or 10 percent

b. 3.4×10^6 J

c. 8.5 hours! I wouldn't.

108. a. $E = mv_0^2$ b. $E = mv^2 + k_e q^2 / R$ c. $E = k_e q^2 / R'$ 109. E_{mac} becomes smaller, E_{int} becomes larger

110. Yes, because it is an isolated system of atomic particles.

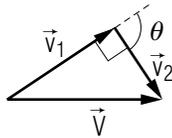
111. (a), (b), and (e)

112. a. It becomes larger.

b. Yes.

113. a. Speed remains the same. E_{mac} and E_{int} remain the same.b. Speed becomes smaller. E_{mac} becomes smaller, E_{int} becomes larger.114. a. K_{mac} : same, U_{mac} : larger, E_{mac} : larger, E_{int} : smallerb. K_{mac} : same, U_{mac} : smaller, E_{mac} : smaller, E_{int} : larger

115. $F_d = 1.1 \times 10^3 \text{ N}$
116. It increases.
117. $R = 4k_e q^2 / mv^2$
118. a. E_{ran} and E_{mac} both remain the same.
 b. K_{mac} is the same, U_{mac} is the same.
 c. equal to
119. $F_t = 1.4 \times 10^3 \text{ N}$
120. $F_a = 1.0 \times 10^3 \text{ N}$
121. a. 1.0 m/s
 b. Before: $E_{\text{mac}} = 2.5 \times 10^2 \text{ J}$. After: $E_{\text{mac}} = 0.5 \text{ J}$.
 c. No! $\Delta E_{\text{mac}} = -250 \text{ J}$, $\Delta E_{\text{int}} = +250 \text{ J}$
122. a. $M = m(L_B/L_A)$
 b. If $L_B = (1/2)L_A$, $M = (1/2)m = 0.5 \text{ kg}$. If $L_B = L_A$, $M = m = 1 \text{ kg}$. If $L_B = 2L_A$, $M = 2m = 2 \text{ kg}$.
123. From momentum conservation, $\vec{V} = \vec{v}_1 + \vec{v}_2$. From energy conservation, $V^2 = v_1^2 + v_2^2$. Since the triangle formed by the velocities has sides related by the Pythagorean theorem, it is a right triangle and $\theta = 90^\circ$



124. (a) and (e) are true.
125. a. E_{ran} becomes larger, E_{mac} becomes smaller.
 b. K_{mac} is the same, U_{mac} is smaller.
 c. smaller than
 d. The bob hangs vertically at rest.
126. a. E_{ran} , E_{int} , and E_{mac} remain the same. Yes.
 b. E_{ran} and E_{int} become larger, E_{mac} becomes smaller. Yes.

MODEL EXAM

GIVEN INFORMATION:

Potential energies due to the interaction of two particles:

Coulomb potential energy: $U = k_e q_1 q_2 / R$

Gravitational potential energy: $U = -Gm_1 m_2 / R$

1. **Motion of a diver.** Consider an isolated system S_0 consisting of a swimming pool, the air, the earth, and a diver who dives from the high diving board into the water. The diver strides out along the board, bounces once, dives into the water, swims to the side of the pool, and rests there. At the beginning and end of this motion, all macroscopic particles in the system S_0 can be considered to be at rest.

For the motion described, state whether the final values of the macroscopic and internal energies of the system S_0 are larger than, smaller than, or the same as the initial values of those energies. (Take the macroscopic potential energy of this system to be the gravitational potential energy due to the interaction of macroscopic particles with the earth.)

2. **Energy of a positronium atom.** A “positron” is an atomic particle having the same mass m_e as an electron but a positive charge $+e$ equal in magnitude to the negative charge $-e$ of the electron. Suppose that an electron and a positron interact by the Coulomb electric force, and briefly form an isolated system called a “positronium atom.” In this system, the electron and positron are separated by a distance D and move with the same speed v in a common orbit around the system’s center of mass.

- a. Using the symbols provided, write an expression for the energy of this system due to all interactions.
- b. After a brief interval, the electron and positron in this isolated system begin to move away from each other with decreasing speed. Does the energy E of this system increase, decrease, or remain constant?

Brief Answers:

1. Macroscopic energy: *smaller*
Internal energy: *larger*
2. a. $E = m_e v^2 - k_e e^2 / D$
b. remain constant

