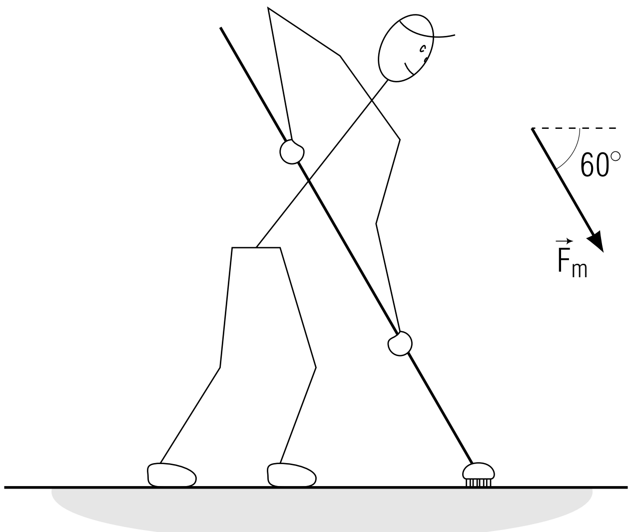
PHYSNET

MISN-0-414

KINETIC ENERGY AND WORK



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

KINETIC ENERGY AND WORK

by

F. Reif, G. Brackett and J. Larkin

CONTENTS

A. Small Displacements

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Title: **Kinetic Energy And Work**

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Input Skills:

1. Vocabulary: state (of a mechanical system) (MISN-0-413).
2. State the equation of motion for a particle (MISN-0-408).

Output Skills (Knowledge):

K1. Vocabulary: kinetic energy, work, joule, power, watt.

K2. State the relationship between work and kinetic energy.

K3. State the equation for the work done by a constant force.

K4. State the superposition principle for work.

K5. Write the defining equations for kinetic energy, work, and power.

Output Skills (Problem Solving):

S1. Solve problems using these relations for a single particle:

S2. Determine the sign of the work done on a particle by a force everywhere parallel or perpendicular to the particle's path.

S3. Solve problems relating the speed of a particle to the work done on it by all forces.

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Abstract:
Up to now we have used the theory of motion to study how the positions and velocities of particles change with time. But in many cases we are primarily interested in the relationship between the positions and velocities of particles, *irrespective* of any explicit mention of time. (For example, we might want to know what velocity a particle falling from rest attains after traveling a distance of 20 meter, but might not care about the time when the particle reaches this velocity.) By focusing attention directly on the relationship between the positions and velocities of particles, we shall be led to some extremely important concepts (such as the concept of "energy"). As usual, we shall discuss simple situations before proceeding to more complex ones. Thus we shall devote this unit and the next to discuss systems consisting of a single particle. We shall then use Unit 416 to extend our discussion to systems consisting of many moving particles.

SECT.

A SMALL DISPLACEMENTS

Throughout this unit and the next we shall consider a system consisting of a single moving particle which interacts with other particles fixed relative to an inertial frame. (For example, the particle might be a ball interacting with the earth, as illustrated in Fig. A-1a.) The equation of motion of the moving particle is then $m\vec{a} = \vec{F}$, where $\vec{a}$ is the acceleration of the particle of mass m and where $\vec{F}$ is the total force on the particle due to all the other particles.

To relate information about the particle's position and velocity, let us first use the definition of acceleration to express the equation of motion $m\vec{a} = \vec{F}$ in terms of the velocity $\vec{v}$ of the particle. Thus we can write

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

or

$$m\,d\vec{v} = \vec{F}\,dt \qquad \text{(A-1)}$$

Furthermore, the velocity $\vec{v}$ of the particle is related to its position vector $\vec{r}$ by the definition

$$\vec{v} = \frac{d\vec{r}}{dt}$$

or

$$\vec{v}\,dt = d\vec{r} \qquad \text{(A-2)}$$

We should now like to eliminate the time interval dt between these two equations in order to obtain a direct relationship between small changes in the position $\vec{r}$ and velocity $\vec{v}$ of the particle.

ARGUMENT RELATING POSITION AND VELOCITY

To avoid the complexities of vector equations, we shall consider the numerical components of all vectors along the direction of the total force $\vec{F}$. The equality of the vectors in Eq. (A-1) implies the corresponding equality of their numerical components along $\vec{F}$. Thus

$$m\,dv_F = F\,dt \qquad \text{(A-3)}$$

where v_F is the numerical component of $\vec{v}$ along $\vec{F}$ and where we have used the fact that the numerical component of $\vec{F}$ along its own direction

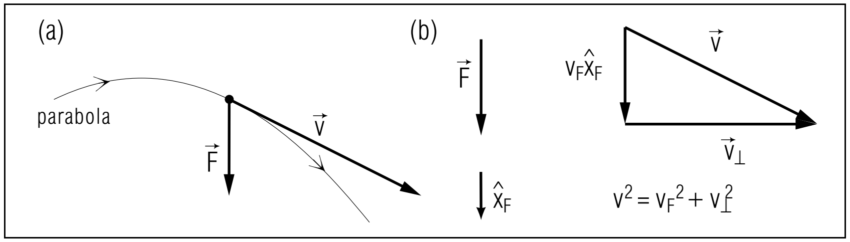


Fig. A-1: Motion of a particle acted on by a total force $\vec{F}$. (a) Velocity $\vec{v}$ of the particle at some instant, (b) Component vectors of $\vec{v}$ parallel and perpendicular to $\vec{F}$. [$\vec{v} = v_F\hat{x} + \vec{v}_\perp$ if $\hat{x}_F$ is a unit vector along $\vec{F}$.]

is just equal to the magnitude F of $\vec{F}$. Similarly, the equality of the vectors in Eq. (A-2) implies the corresponding equality of their numerical components along $\vec{F}$. Thus

$$v_F dt = dr_F \qquad \text{(A-4)}$$

where dr_F is the numerical component of the particle's displacement $d\vec{r}$ along $\vec{F}$. From Eq. (A-4) we conclude that $dt = dr_F/v_F$. By substituting this value into Eq. (A-3) we then find

$$m dv_F = F\frac{dr_F}{v_F}$$

or

$$m v_F dv_F = F dr_F \qquad \text{(A-5)}$$

Thus we have succeeded in eliminating the time interval dt and thereby to obtain a direct relation connecting velocity and position.

We can write Eq. (A-5) in a more convenient form. Thus we can express $v_F dv_F$ in terms of the change in the quantity v_F^2 since the relation in Relation (C-9) of Unit 404 tells us that $d(v_F^2) = 2v_F dv_F$. Hence $v_F dv_F = 1/2d(v_F^2)$ so that Eq. (A-5) can be written as

$$\frac{1}{2}md(v_F^2) = F dr_F \qquad \text{(A-6)}$$

Furthermore, v_F^2 can be expressed directly in terms of the magnitude v of the particle's velocity since $v^2 = v_F^2 + v_\perp^2$ where $v_\perp$ is the magnitude of the component vector $\vec{v}_\perp$ of $\vec{v}$ perpendicular to $\vec{F}$. (See Fig. A-1b.) Hence the relation between the corresponding changes of these quantities is

$$d(v^2) = d(v_F^2) + d(v_\perp^2) \qquad \text{(A-7)}$$

But since the acceleration $\vec{a}$ of the particle is parallel to the force $\vec{F}$, the velocity change of the particle perpendicular to $\vec{F}$ is zero. *

> * Indeed, Eq. (A-1) shows explicitly that the velocity change $d\vec{v}$ is *parallel* to $\vec{F}$.

Thus $d\vec{v}_\perp = 0$ so that $\vec{v}_\perp$, and hence also its magnitude $v_\perp$, remain unchanged. Therefore $d(v_\perp^2) = 0$ and Eq. (A-7) implies simply that

$$d(v^2) = d(v_F^2) \qquad \text{(A-8)}$$

Accordingly, the left side of Eq. (A-6) is equal to

$$\frac{1}{2}md(v_F^2) = \frac{1}{2}md(v^2) = d\left(\frac{1}{2}mv^2\right)$$

and Eq. (A-6) is equivalent to

$$\boxed{d\left(\frac{1}{2}mv^2\right) = F dr_F} \qquad \text{(A-9)}$$

This relation forms the basis of all subsequent discussion in this and the next two units.

DISCUSSION

As we have seen, the equation of motion, Eq. (A-1), and the definition of the velocity lead, Def. (A-2), to the important relation (A-9) which connects directly the speed and position of the particle. Let us then introduce some convenient abbreviations and terminology in order to express this relation in more compact form.

We begin by introducing the abbreviation

$$\boxed{K = \frac{1}{2}mv^2} \qquad \text{(A-10)}$$

and correspondingly this definition:

Def. | **Kinetic**: The kinetic energy K of a particle is $K = (1/2)mv^2$ where m is the mass of the particle and v is its speed. | (A-11)

Similarly, we introduce the abbreviation

$$\boxed{\delta W = F dr_F} \qquad \text{(A-12)}$$

and correspondingly this definition:

Def. **Small work**: The work δW done on a particle by *any* force $\vec{F}$ acting on the particle while it moves through a small enough displacement $d\vec{r}$ is $\delta W = F dr_F$ where F is the magnitude of $\vec{F}$ and dr_F is the numerical component of the particle's displacement along $\vec{F}$. * (A-13)

* Here we have used the symbol δ (small Greek letter delta) to indicate explicitly that the quantity δW is small enough (so that the ratio $\delta W/dr_F = F$ remains constant for any smaller value of dr_F). Note that δW is ordinarily *not* a difference since it is obtained by multiplying the difference dr_F by the quantity F.

By using the preceding definitions, we can write the relation (A-9) as

$$dK = \delta W(\text{by total force}) \qquad \text{(A-14)}$$

where $\delta W = F dr_F$ is the work done by the *total* force $\vec{F}$ acting on the particle. The result Eq. (A-14) can be stated in words:

During any small displacement of a particle, the change in the kinetic energy of the particle is equal to the work done by the *total* force on the particle. (A-15)

PROPERTIES OF KINETIC ENERGY AND WORK

According to its definition $K = (1/2)mv^2$, the kinetic energy of a particle depends only on its mass m and speed v. Since m and v are numbers which are either positive or zero, the kinetic energy is thus an ordinary number which is either positive or zero.

The work $\delta W = F dr_F$ done by any force $\vec{F}$ is also an ordinary number (and *not* a vector) since the magnitude F of the force and the numerical component dr_F of the displacement are numbers. The work may be positive, zero, or negative since dr_F can have any sign. If $\vec{F}$ is not zero, its magnitude F must be positive. Hence the sign of the work $\delta W = F dr_F$ is the same as that of the numerical component dr_F of the particle's displacement $d\vec{r}$ along the force $\vec{F}$ and depends thus on the

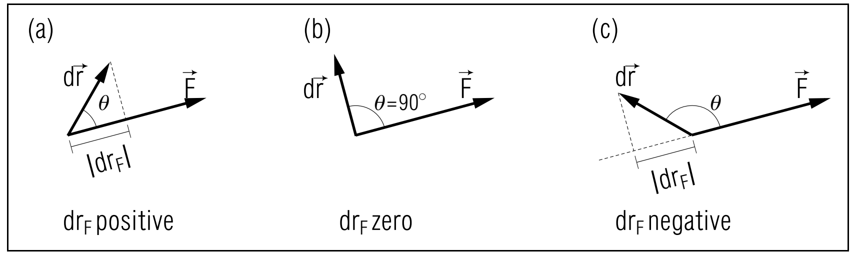


Fig. A-2: Dependence of the work $\delta W = F dr_F$ on the angle θ between the displacement $d\vec{r}$ and the force $\vec{F}$.

angle θ between $d\vec{r}$ and $\vec{F}$. If the particle moves along, or partly along, the force $\vec{F}$ (as shown in Fig. A-2a), the work is positive. If the particle moves in a direction perpendicular to the force $\vec{F}$ (as shown in Fig. A-2b), the work is zero. If the particle moves opposite, or partly opposite, to the force $\vec{F}$ (as shown in Fig. A-2c), the work is negative.

Irrespective of the magnitude of the force, the work $\delta W = F dr_F$ must be zero whenever the displacement $d\vec{r}$ of the particle is zero. For example, if a man is standing at a bus stop while holding a heavy suitcase in his hand, the work done on the suitcase by the force due to the hand is zero since the displacement of the suitcase is zero. (On the other hand, work is done by the forces acting on the moving atoms and electrons within the man's muscles since such "chemical" work must be provided to maintain the tension in the muscles. Hence the man gets tired even if he does no work on the suitcase.)

According to the relation $dK = \delta W$ of Eq. (A-14), the work δW done by the *total* force $\vec{F}$ on a particle is equal to the change dK in the kinetic energy of the particle. Hence the sign of the work determines whether the change dK in the kinetic energy is positive, negative, or zero, i.e., whether the kinetic energy K increases, decreases, or remains the same. For example, Fig. A-3 illustrates the motion of a ball moving under the sole influence of the gravitational force $\vec{F}$ due to the earth. During a small displacement $d\vec{r}$ from the point A, the force $\vec{F}$ does negative work on the ball since the ball moves partly *opposite* to the force. Hence the kinetic energy K of the ball (and thus also its speed v) *decreases* during this displacement. But during the small displacement $d\vec{r}$ from the point B, the force $\vec{F}$ does positive work on the ball since the ball moves partly *along* the force. Hence the kinetic energy K of the ball (and thus also its speed v) *increases* during this displacement.

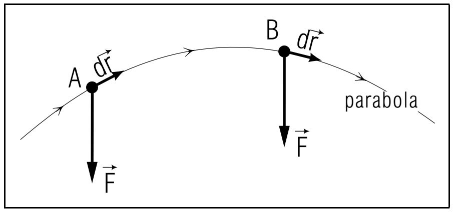


Fig. A-3: Work done during various small displacements along the path of a ball moving near the surface of the earth.

The unit of kinetic energy follows from its definition $K = (1/2)mv^2$. Thus

$$\boxed{\text{unit of } K = (\text{kg})\left(\frac{\text{meter}}{\text{sec}}\right)^2 = \frac{\text{kg meter}^2}{\text{sec}^2} = \text{joule}} \quad \text{(A-16)}$$

where the unit "joule" is merely a convenient abbreviation for kg meter^2/sec^2. [The unit "joule" is named in honor of the British physicist James P. Joule (1818-1889) who carried out fundamental studies of various forms of energy.] Because of the relation $dK = \delta W$ of Eq. (A-14), the unit of work must be the same as the unit of kinetic energy. Indeed, the definition $\delta W = F dr_F$ of work implies that the unit of work is

$$\text{unit of work} = \text{newton meter} = \left(\text{kg}\frac{\text{meter}}{\text{sec}^2}\right)(\text{meter}) = \text{joule} \quad \text{(A-17)}$$

since the combination of units appearing here is the same as that appearing in Eq. (A-16).

Now: Go to tutorial section A.

Knowing About Small Work

A-1 After it is thrown, a 0.2 kg ball moves along an arched trajectory, hits a tree branch, and then falls vertically to the ground. During its motion, the ball moves under the sole influence of gravity through the four small displacements, each of magnitude 0.3 meter, shown in Fig. A-4. For each of these displacements, what is the small work δW done on the ball by the total force acting on it? *(Answer: 106) (Suggestion: [s-3])*

Understanding the Definition of Kinetic Energy (Cap. 1a)

A-2 *Example:* What is the kinetic energy K of a 1000 kg car (a) with a velocity of 20 m/s north, (b) sitting at rest at a stop sign, and

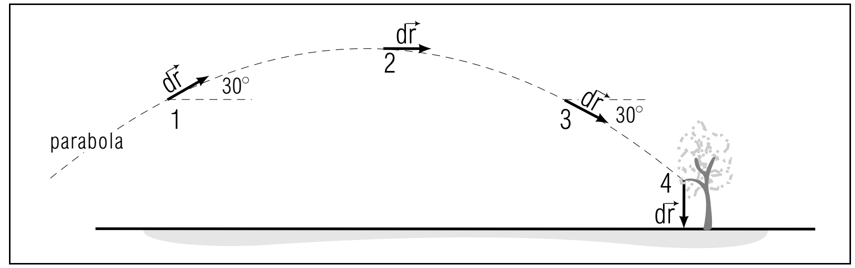


Fig. A-4.

(c) with a velocity of 20 m/s east? *(Answer: 103)*

A-3 *Properties:* List the following properties of the quantities kinetic energy and work: kind of quantity (number or vector), possible signs of numerical quantities, single SI unit, unit expressed in terms of kg, meter, and second. Which properties *differ* for the two quantities? *(Answer: 101)*

A-4 *Relating kinetic energy and speed:* The ball described in problem A-1 has a speed $v_0 = 2$ m/s at the beginning of the displacement labeled (1) in Fig. A-4. (a) What is the ball's initial kinetic energy K_0 at the beginning of this displacement? (b) The change $dK = K_c - K_0$ in the ball's kinetic energy during this displacement is given by $dK = \delta W$, where $\delta W = -0.3$ joule is the work done on the ball by the total force (as found in problem A-1). At the end of this displacement, what is the ball's final kinetic energy K_c? What is its final speed v_c? *(Answer: 105)*

A-5 *Dependence of kinetic energy on mass and speed:* A little boy sliding slowly down a playground slide has a kinetic energy of 10 joule. What is the kinetic energy of (a) the boy's big sister, who has twice the boy's mass, when she slides down with the same slow speed, and (b) the boy when he slides down with twice his original slow speed? *(Answer: 102)*

SECT.
B GENERAL MOTION

In the preceding section we considered only small displacements of a particle. We shall now extend this discussion to deal with *any* displacement of a particle.

Let us then consider a particle which moves along some path while acted on by a *total* force $\vec{F}$ which may depend on the position as well as on the velocity of the particle. Then the motion of the particle may be regarded as a series of successive small displacements (as illustrated in Fig. B-1) and the relation $dK = \delta W$ of Eq. (A-14) may be applied to each one of them. Thus we can write for the first of these displacements

$$(dK)_1 = (\delta W)_1 \qquad \text{(B-1)}$$

where $(\delta W)_1 = F_1(dr_F)_1$ is the work done on the particle by the total force during this first displacement. Similarly we can write for the second displacement

$$(dK)_2 = (\delta W)_2 \qquad \text{(B-2)}$$

where $(\delta W)_2 = F_2(dr_F)_2$ is the work done on the particle by the total force during this second displacement. By adding corresponding sides of these equations for all the successive displacements, we then obtain

$$(dK)_1 + (dK)_2 + \ldots = (\delta W)_1 + (\delta W)_2 + \ldots \qquad \text{(B-3)}$$

Here each sum is obtained by adding all successive changes of kinetic energy, or all successive small works, starting with the particle in its initial state a (specified by the particle's initial position P_a and its initial

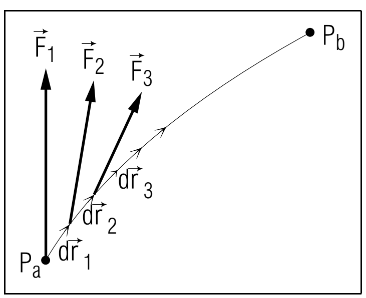


Fig. B-1: Work done on a particle along a path from a point P_a to a point P_b.

velocity $\vec{v}_a$) and ending with the particle in its final state b (specified by the particle's final position P_b and its final velocity $\vec{v}_b$).

On the left side of Eq. (B-3), the sum of all the successive changes in the kinetic energy is just equal to the *total* change $K_b - K_a$ in the kinetic energy. Hence Eq. (B-3) can be written as

$$\boxed{K_b - K_a = W_{ab} \text{ (by total force)}} \qquad \text{(B-4)}$$

if we introduce the abbreviation

$$W_{ab} = (\delta W)_1 + (\delta W)_2 + \ldots \qquad \text{(B-5)}$$

to denote the sum of all the small works $\delta W = F dr_F$ done by the *total* force $\vec{F}$ as the particle moves along its path from state a to state b. This sum can be written compactly as

$$W_{ab} = \int_a^b \delta W = \int_a^b F dr_F \qquad \text{(B-6)}$$

where the symbol $\int$ (which is a special form of the letter S) denotes the sum (or "integral") obtained by adding the indicated small enough quantities, starting in state a and ending in state b. *

* The added quantities are small enough if subdivision of the path into smaller displacements would leave the value of the sum unaffected within the desired precision.

The preceding sum can be defined for any individual force and is called the "work done along a path" in accordance with this definition:

Def. **Work**: The work W_{ab} done by *any* force $\vec{F}$ on a particle, along some specified path from state a to state b, is the sum of all the successive small works done by this force on the particle moving along this path. (B-7)

The relation $K_b - K_a = W_{ab}$ obtained in Eq. (B-4) is the central result of this unit. This relation asserts that the change in the kinetic energy of a particle is always equal to the work done by the *total* force on the particle along its path. Thus the change in the kinetic energy of a particle is positive, negative, or zero (i.e., the kinetic energy increases, decreases, or remains the same) depending on whether the work done on the particle by the *total* force is positive, negative, or zero. For example, when an object falls vertically downward, the total force on the particle is always along the direction of motion of the particle so that the work done on the particle by this total force is positive. Hence the kinetic energy of the particle increases.

Since the kinetic energy of a particle depends on its speed, and the work done on the particle can be found from a knowledge of its path, the relation $K_b - K_a = W_{ab}$ allows us to relate directly the speed of a particle to information about its position. In order to do this, we must, however, be able to find the work done on the particle. Accordingly, we shall use much of the remaining part of this unit to illustrate how the work can be found in various cases.

WORK DONE IN SOME SIMPLE CASES

If a force on a particle is always *along*, or partly along, the motion of the particle along its path, the work done on the particle by this force in any small displacement is positive. Hence the work done by this force along the entire path of the particle is also *positive*. If a force on a particle is always *opposite*, or partly opposite, to the motion of the particle along its path, the work done on the particle by this force in any small displacement is negative. Hence the work done by this force along the entire path of the particle is also *negative*. If a force on a particle is always *perpendicular* to the path of the particle, the work done on the particle by this force in any small displacement is zero. Hence the work done by this force along the entire path of the particle is then zero.

Example B-1: Work done by various forces on a skier

Figure B-2 illustrates a skier who is being pulled some distance up a slope by the rope of a ski tow. Since the tension force $\vec{F}_t$ exerted on the skier by the rope is along the uphill direction of motion of the skier, the work done on the skier by this tension force is positive. Since the friction force $\vec{F}_f$ exerted on the skier by the snow-covered surface of the hill is downhill *opposite* to the direction of motion of the skier, the work done on the skier by this frictional force is negative. Since the normal force $\vec{F}_n$ exerted on the skier by the surface of the hill is always perpendicular to the path of the skier, the work done on the skier by this normal force is zero.

Finally, suppose that a particle moves along some path from a point P_a to some point P_b and from there to some point P_c. (See Fig. B-3.) Then the definition of work, Def. (B-7), implies that the work W_{ac} done by any force on the particle moving along its entire path from P_a to P_c is simply the sum of the works W_{ab} and W_{bc} done by this force as the particle moves along the successive portions of its path. In other words,

$$\boxed{W_{ac} = W_{ab} + W_{bc}\,.} \tag{B-8}$$

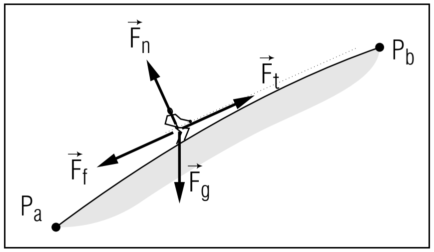


Fig. B-2: Forces acting on a skier being pulled up a hill.

Knowing About the Definitions of Work and State

B-1 A woman pushes a baby carriage through a park along the path shown in Fig. B-4, traveling 50 meter from the point A to the point B, 100 meter around a fountain to the point B again, and then 50 meter back to the point A. Let us divide the carriage's motion into a series of small displacements, each of magnitude 1 meter. During each small displacement $d\vec{r}$, the woman exerts on the carriage a force $\vec{F}_w$ having a constant magnitude of 10 N and a direction along the displacement. (a) What is the small work δW done on the carriage by $\vec{F}_w$ in each small displacement? (b) Using this result, find the work done on the carriage by $\vec{F}_w$ along the parts of the path from A to B, from B around the fountain to B again, and from B to A. (c) What is the work done on the carriage by $\vec{F}_w$ along the entire path through the park? *(Answer: 104) (Suggestion: [s-11])*

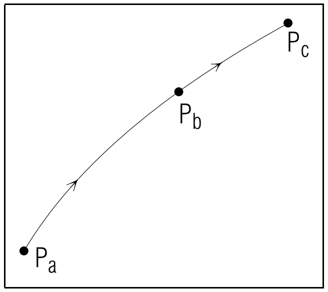


Fig. B-3: Work done on a particle along successive portions of a path.

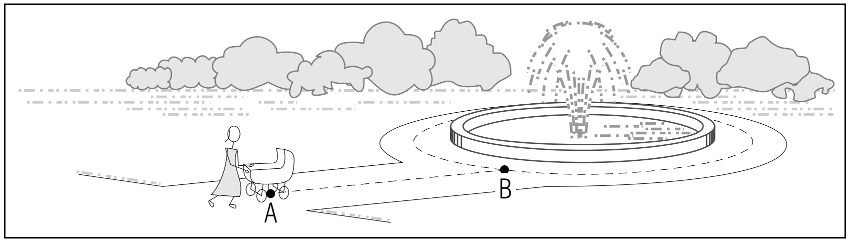


Fig. B-4.

B-2 The baby carriage described in problem B-1 has the same speed, mass, and position when it leaves the park as when it entered it. Is the state of the carriage the same at these two times? Why or why not? *(Answer: 112)*

Finding the Sign of Work Done by Special Forces (Cap. 2)

B-3 What is the sign $(+, 0, -)$ of the work done (a) by the sun's gravitational force $\vec{F}_g$ on a comet traveling directly away from the sun, (b) by the sun's gravitational force $\vec{F}_g$ on a planet traveling along its circular orbit around the sun, (c) by the frictional force $\vec{F}_f$ on a cereal box sliding along a grocery check-out counter, and (d) by the electric force $\vec{F}_e$ on an electron moving along the direction of $\vec{F}_e$ in the "electron gun" of an oscilloscope? *(Answer: 109) ([s-9], [p-1])*

Understanding the Relation $W_{ab} = K_b - K_a$ (Cap. b)

B-4 *Example:* A baseball has a kinetic energy $K_a = 90$ joule just after it is hit by a bat, and a kinetic energy $K_b = 45$ joule when it reaches the top of its trajectory. What is the work W_{ab} done by the total force on the baseball as it moves from the bat to the top of its trajectory? *(Answer: 107)*

B-5 *Meaning of W_{ab}:* For each of the following particles, either find the particle's final kinetic energy, or explain why this, cannot be done. (a) An elevator, initially at rest with zero kinetic energy, is hoisted upward by its cable. What is the kinetic energy of the elevator as it passes the next higher floor? During this ascent, the work done on the elevator by the cable force is 10^4 joule. (b) When a coin is flipped into the air, it moves up and down with negligible air resistance until it is caught at its initial position. During this motion, the work done on the coin by the gravitational force is zero. If the coin's initial kinetic energy is 0.05 joule, what is its final kinetic energy just before it is caught? *(Answer: 115)*

B-6 *Dependence of kinetic energy on work done by the total force:* In each of the examples of problem B-3, the force described is the total force on the particle. For each example, use your results for the sign of the work done by this total force to state whether the particle's final kinetic energy is larger than, equal to, or smaller than its original kinetic energy, and thus whether the particle's kinetic energy and speed increase, decrease, or remain constant during its motion. *(Answer: 111)*

Relating Work and Speed (Cap. 3)

B-7 A 2000 kg truck hits a highway safety barrier with a speed of 30 m/s and comes to rest. What is the work done on the truck by the total force during this collision? *(Answer: 108) (Suggestion: [s-4])*

B-8 Using a computer to calculate the sum of small works, an astronautics expert finds that the work done by the total force on a 100 kg space probe as it travels along a path from the earth's orbit to Mercury's orbit is 1×10^9 joule. If the probe's speed at the earth's orbit is 9×10^3 m/s, what is its speed when it reaches Mercury's orbit? (These speeds are measured relative to the inertial solar frame.) *(Answer: 114) (Suggestion: [s-2])*

B-9 The speed of a red blood cell in the aorta is about 50 cm/s, and the speed of the same cell in a capillary in the hand is about 0.1 cm/s. What is the sign of the work done by the total force on this cell as it travels from the aorta to the capillary? *(Answer: 110)*

(Practice: [p-2])

SECT.

C WORK DONE BY A CONSTANT FORCE

Suppose that a force $\vec{F}$ on a particle is constant (in both magnitude and direction) as the particle moves along its path. Then the work $F dr_F$ done on the particle in every small displacement $d\vec{r}$ of the particle depends only on the numerical component of this displacement along the fixed direction of the force $\vec{F}$ (a direction indicated by the unit vector $\hat{x}_F$ in Fig. C-1). Thus the work done on the particle is completely independent of how the particle moves perpendicularly to the fixed direction of the force.

What then is the work W done on the particle by the constant force $\vec{F}$ along the path from a point P_a to another point P_b? This work is the sum of the works done in all the successive small displacements of the particle along this path, i.e.,

$$W = F(dr_F)_1 + F(dr_F)_2 + \ldots = F[(dr_F)_1 + (dr_F)_2 + \ldots]$$

since the constant force has the same magnitude F in all displacements. Hence

$$\boxed{\text{if } \vec{F} \text{ is constant, } W = FD_F} \qquad \text{(C-1)}$$

where $D_F = (dr_F)_1 + (dr_F)_2 + \ldots$, the sum of the numerical components along $\vec{F}$ of all the successive displacements of the particle, is simply equal to the numerical component D_F along $\vec{F}$ of the *entire* displacement $\vec{D}$ of the particle (See Fig. C-1b). Note that the value of the component of a displacement will be positive if the component is in the same direction as $\vec{F}$, negative if in the opposite direction. Thus D_F can be either positive or negative (or zero).

Suppose now that the force $\vec{F}$ on a particle is constant throughout an entire region of space so that it has the same value along *any* path traversed by the particle in this region. (For example, $\vec{F}$ might be the gravitational force near the surface of the earth.) Since $\vec{F}$ has then everywhere the same value, the work FD_F done on the particle by this force has the same value along all paths for which the numerical component D_F of the displacement along the force has the same value. For example, the work done on a particle by the vertical gravitational force near the earth has the same value along *all* paths whose end points are separated by the same difference in vertical height. (Thus the work done by

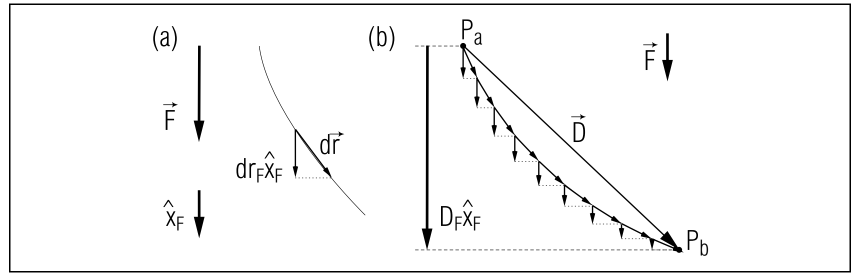


Fig. C-1: Work done by a constant force. (a) Work done in a small displacement. (b) Work done along an entire path.

the gravitational force is the same along all the different paths shown in Fig. C-2.)

Example C-1: Work along a path parallel to a constant force

The gravitational force on a brick of mass $m = 2\,$kg is $mg = (2\,\text{kg})(10\,\text{meter/sec}^2)$ downward $= 20\,$newton downward. If the brick falls vertically *downward* through a distance of 3 meter, the numerical component of the displacement along the gravitational force is $D_F = 3\,$meter. Hence the work done on the brick by the gravitational force is $FD_F = (20\,\text{newton})(3\,\text{meter}) = 60\,$joule. On the other hand, if the brick is thrown so that it moves vertically *upward* through a distance of 3 meter, the numerical component of the displacement of the brick along the gravitational force is $D_F = -3\,$meter since the upward displacement is opposite to the downward gravitational force. Hence the work done on the brick by the gravitational force is then $FD_F = (20\,\text{newton})(-3\,\text{meter}) = -60\,$joule.

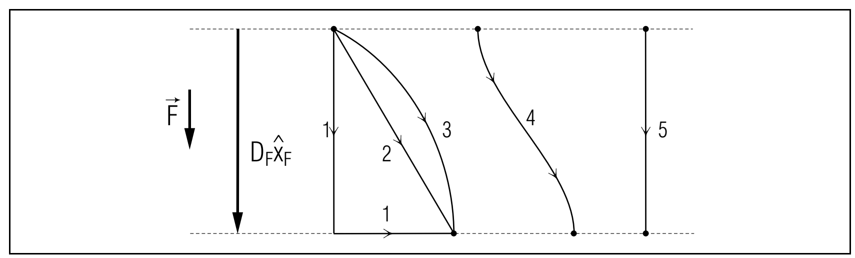


Fig. C-2: Different paths between the same two heights near the surface of the earth. The work done by the constant gravitational force is the same along each of these paths.

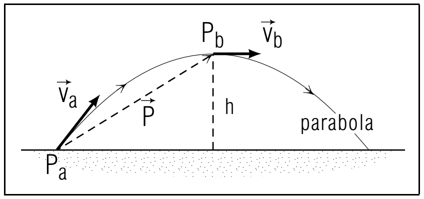


Fig. C-3: Motion of a ball near the earth.

Example C-2: Relation between speed and height of a ball

A ball is thrown from the ground with a speed of 15 meter/sec. If the ball moves under the sole influence of gravity, what is the speed of the ball when it is at the highest point of its trajectory 6 meter above the level ground?

Description: Fig. C-3 illustrates the situation where the ball of mass m starts from the point P_a with a speed $v_a = 15$ meter/sec. We should like to find the speed v_b when the ball is at its highest point P_b at a height $h = 6$ meter above the initial point P_a.

Planning: To relate the speed directly to the height, we can use the relation between kinetic energy and work

$$K_b - K_a = W_{ab} \tag{C-2}$$

To calculate the work, we note that the total force $\vec{F}$ on the ball is the constant gravitational force of magnitude $F = mg$. The numerical component of the displacement of the ball along the vertically downward direction of the gravitational force is $D_F = -h$. (Here D_F is negative because the component vector of the ball's displacement from P_a to P_b is in the upward direction *opposite* to the gravitational force.) Thus

$$W_{ab} = FD_F = (mg)(-h) = -mgh \tag{C-3}$$

Hence Eq. (C-2) becomes

$$\frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 = -mgh \tag{C-4}$$

or

$$v_b^2 - v_a^2 = -2gh \tag{C-5}$$

This relation allows us to find v_b from the known information about v_a and h.

Implementation: From Eq. (C-5) we find

$$v_b^2 = v_a^2 - 2gh \tag{C-6}$$

Substituting the known values, we get

$$v_b^2 = (15\,\mathrm{m/s})^2 - 2(10\,\mathrm{m/s^2})(6\,m) = (225 - 120)(\,\mathrm{m/s})^2 = 105(m/s)^2$$

Hence

$$v_b = 10.2\,\mathrm{m/s}$$

i.e., the speed of the ball at its highest point is 10.2 meter/sec.

Checking: As anticipated, the speed of the ball at its highest point is smaller than its initial speed at ground level. Furthermore, the result Eq. (C-6) does not involve the mass m of the ball, as expected in the case of motion under the sole influence of gravity.

Understanding the Relation $W = FD_f$ (Cap. 1c)

C-1 *Example:* A 1000 kg elevator moves through a displacement of 15 meter upward, and then a displacement of 15 meter downward, returning to its original position. (a) For each of these displacements, and for the displacement corresponding to the elevator's entire trip up and down, find the numerical component D_F of the displacement along the gravitational force $\vec{F}_g$ on the elevator, and the work W done on the elevator by $\vec{F}_g$. (b) Is the work done by $\vec{F}_g$ during the entire trip equal to the sum of the works done in the upward and downward parts of the trip? *(Answer: 118) (Suggestion: [s-5])*

C-2 *Interpretation:* (a) What is the work done by the gravitational force on the 1200 kg car shown in Fig. C-4a as the car travels a distance of 20 meter up the indicated slope? (b) Fig. C-4b shows the path of an electron traveling between two charged deflecting plates in an oscilloscope tube. As the electron travels from the point A to the point B, the total force acting on it is the constant electric force $\vec{F}_e = 1 \times 10^{-14}$ N upward due to the charged plates. What is the work done by the total force on the electron as it travels from A to B? (c) A 60 kg hiker walks from a ranger station over a mountain pass to a lake. If the elevation of the lake is 1000 meter higher than that of the ranger station, what is the work done by the gravitational force on the hiker during the journey? *(Answer: 116) (Suggestion: [s-16])*

C-3 *Comparison of work done by different forces:* To push a broom a distance of 5.0 meter in a straight line across a level floor, a man exerts on the broom a force $\vec{F}_m$ having a constant magnitude of 20 N

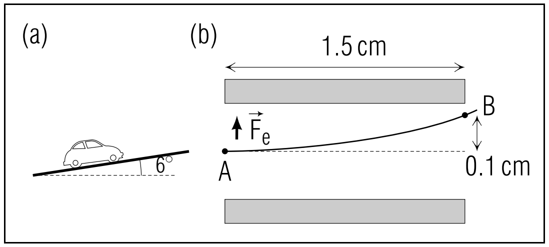


Fig. C-4.

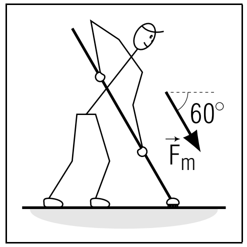


Fig. C-5.

and a direction along the broom handle (Fig. C-5). During this motion, three other constant forces act on the broom: the gravitational force $\vec{F}_g$ of magnitude 20 N, and the frictional force $\vec{F}_f$ of magnitude 10 N and normal force $\vec{F}_n$ of magnitude 37 N due to the floor. What is the work done on the broom by each of these forces during the broom's motion? *(Answer: 113) (Suggestion: [s-8])*

C-4 *Applicability:* Suppose the man described in problem C-3 now pushes the broom in the same way once around a circle on the level floor. All the forces on the broom have the same constant magnitudes as before. For which of these forces can you *not* use the relation $W = FD_F$ to find the work done on the broom, and why? *(Answer: 120)*

C-5 *Dependence of work on the direction of the displacement:* Figure C-6 shows six paths a bird might take from its original position on a roof. (a) For each path, describe the direction of the bird's displacement relative to the gravitational force $\vec{F}_G$ on the bird. (Use the terms "along," "partly along," "perpendicular," "partly opposite to," and "opposite to.") Then give the sign of the work done by $\vec{F}_g$ on the bird as it moves along each path. (b) For which pairs of paths is the work done by $\vec{F}_g$ the same? *(Answer: 117) (Suggestion: [s-15])*

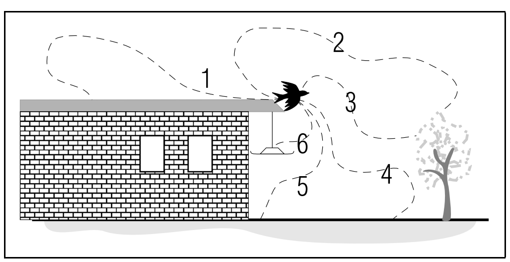


Fig. C-6.

Relating Work and Speed (Cap. 3)

C-6 A truck driver of mass M jumps off the back of his truck with an initial speed v_a. He moves with negligible air resistance, and lands on the level pavement, a vertical distance h below, at a horizontal distance L from the back of the truck. (a) Find the work W_{ab} done on the driver by the total force during this jump, and the driver's final speed v_b just before hitting the pavement. Using the values $M = 80$ kg and $h = 0.8$ meter, find the driver's final speed if he (b) simply drops vertically with an initial speed of zero, and (c) runs off the truck bed with an initial speed of 3 m/s, so that he lands a horizontal distance of 1 meter from the back of the truck. *(Answer: 124) (Suggestion: [s-13])*

C-7 The electron described in part (b) of problem C-2 has a speed of 1×10^7 m/s when it passes the point B. What was its speed when it passed the point A? (Use your previous result for the work done on the electron, and the value 1×10^{-30} kg for the mass of the electron.) *(Answer: 121)*

(Practice: [p-3])

SECT.

D SUPERPOSITION PRINCIPLE FOR WORK

The work done on a particle by a force $\vec{F}$ is equal to FD_F for any displacement $\vec{D}$ of the particle if the force on the particle is constant along its path. In particular, the relation $W = FD_F$ holds for any displacement $\vec{D} = d\vec{r}$ which is small enough (so that $\vec{F}$ remains constant within this displacement).

In practice, the work W can be calculated in various equivalent ways. For example, the numerical component D_F of the displacement $\vec{D}$ along the force $\vec{F}$ can be found from the magnitude D of the displacement and the angle θ between the displacement and the force. Indeed, from Fig. D-1a we see that $D_F = D\cos\theta$. Hence the work W is equal to

$$W = FD_F = FD\cos\theta \tag{D-1}$$

But from Fig. D-1b we also see that $F\cos\theta = F_D$, the numerical component of the force $\vec{F}$ along the displacement $\vec{D}$. Hence the work in Eq. (D-1) is also equal to

$$W = D(F\cos\theta) = DF_D \tag{D-2}$$

Hence the work can be found either from Eq. (D-1) by multiplying the magnitude of the force by the numerical component of the particle's displacement along this force, or from Eq. (D-2) by multiplying the magnitude of the particle's displacement by the numerical component of the force along this displacement.

Suppose that a particle is acted on by several forces, e.g., by a force $\vec{F}_1$ due to its interaction with one object and by a force $\vec{F}_2$ due to its interaction with a second object. Then we know from the superposition principle that the total force $\vec{F}$ on the particle due to its interaction with all the other objects is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 \tag{D-3}$$

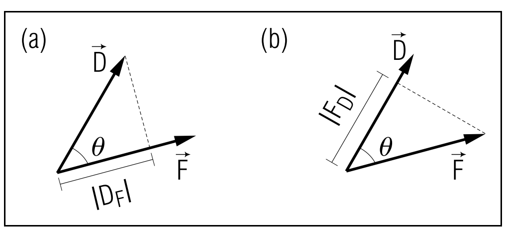


Fig. D-1: Displacement $\vec{D}$ of a particle acted on by a force $\vec{F}$.

How then is the work done on the particle by this total force $\vec{F}$ related to the works done by the individual forces $\vec{F}_1$ and $\vec{F}_2$?

According to Eq. (D-2), the work done by the total force $\vec{F}$ in any small displacement $\vec{D}$ of the particle is equal to DF_D. But the numerical component F_D of the total force along D is just equal to the sum of the numerical components of the individual forces along $\vec{D}$. *

* We recall from statement (D-2) of Unit 407 that the numerical component of the sum of vectors is just equal to the sum of their numerical components.

In other words $F_D = F_{1D} + F_{2D}$. Hence $DF_D = DF_{1D} + DF_{2D}$ so that the work done by the total force in any small displacement $\vec{D}$ is the sum of the works done by the individual forces. Since this relationship is true for every small displacement, it must also be true for the works done by these forces along any entire path. Thus the superposition principle Eq. (D-3) for forces implies correspondingly this conclusion:

> Superposition principle for work: The work done by several forces acting jointly is equal to the sum of the works done by the individual forces separately. (D-4)

Thus the work W done by the total force is related to the works W_1 and W_2 done by the individual forces so that

$$W = W_1 + W_2 \tag{D-5}$$

The conclusion in Rule (D-4) is extremely useful and can greatly simplify the calculation of work. Thus the work done by several forces acting jointly can be found without ever calculating the total force, i.e., without ever needing to calculate a cumbersome *vector* sum of individual forces. Instead, we need merely calculate the sum of the works done by the individual forces. Since these works are ordinary numbers, we need thus merely calculate a simple *numerical* sum without ever having to add any vectors.

Example D-1: Work done on a skier descending a hill

A skier of mass m descends along an icy frictionless hill. What is the total work done by all forces on the skier after he has descended through a vertical height h?

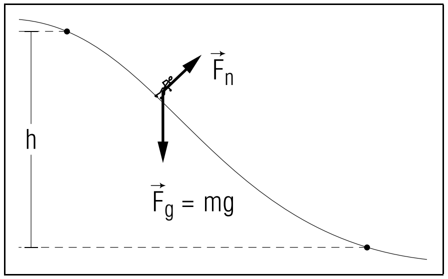


Fig. D-2: Skier descending along the frictionless surface of a hill.

As illustrated in Fig. D-2, the skier is acted on by the downward gravitational force $\vec{F}_g$ due to the earth and by the normal force $\vec{F}_n$ due to the surface. The total work done by these forces acting jointly (i.e., by the total force) is then, by Rule (D-4), simply the sum of the works done by these individual forces separately. But since the normal force $\vec{F}_n$ is always perpendicular to the path of the skier, the work done by this force is zero. Hence the total work consists merely of the work done by the constant gravitational force $m\vec{g}$, i.e., it is equal to mgh (since the numerical component of the skier's displacement along the downward gravitational force is just h).

Understanding the Superposition Principle for Work (Cap. 1d)

D-1 *Example:* Using your previous results for the work done by each individual force on the broom described in problem C-3, find the work done by all forces (i.e., the total work done). *Review:* Does the broom move with increasing, constant, or decreasing speed? *(Answer: 123)*

D-2 *Interpretation:* A simple pendulum (Fig. D-3) consists of a "bob" of mass m attached by a string of length L to a support S. Suppose the bob is initially released at the point A where the string is horizontal. It then swings downward along a circular arc to the point B where the string is vertical. During this motion, the gravitational force of magnitude mg and the string tension force of magnitude F_t act on the bob. What is the total work W_{ab} done on the bob by all forces as the bob moves from A to B? *(Answer: 126) (Suggestion: [s-6])*

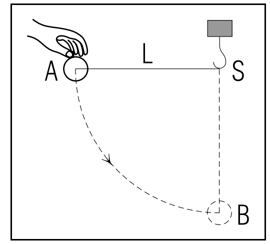


Fig. D-3.

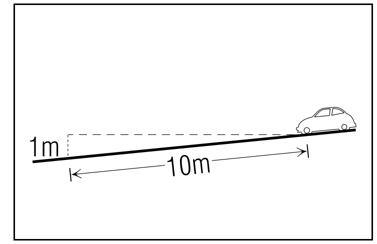


Fig. D-4.

Relating Work and Speed (Cap. 1,3)

D-3 (a) If the pendulum bob described in problem D-2 was initially at rest at the point A, what is its speed v_b when it reaches the point B? (b) Suppose the bob is detached from the string and is simply dropped from rest at the support S so that it falls vertically to the point B. What is the total work W_{sb} done on the bob during this motion? What now is the bob's speed v_b' at the point B? How does v_b' compare with v_b? *(Answer: 119)*

D-4 A 1000 kg car is descending a "10 percent grade," which means that the car descends 1 meter vertically for every 10 meter it travels along the road (Fig. D-4). The driver of the car wants to be able to bring the car to a stop within 50 meter from the point he applies the brakes. To find the largest speed the car should have when the driver applies the brakes, assume that the car does travel 50 meter down the road before coming to rest, and that the frictional force exerted by the road on the locked wheels of the car has a constant magnitude of 5.0×10^3 N. What is the total work done on the car during the braking process? What is the car's initial speed when the driver applies the brakes? *(Answer: 122) (Suggestion: [s-14])*

More practice for this Capability: [p-4], [p-5]

SECT.

E POWER

It is often important to specify how *rapidly* work is done. For example, to accelerate a car from rest to a speed of 45 mile/hour, it is necessary to do on the car a certain amount of work equal to the increase of the car's kinetic energy. But to do this work (and thus to achieve this final speed) in a time of 10 seconds requires a larger motor than to do this same work in a time of 20 seconds. Similarly, the strain on a person's heart is very different when the work the person does to haul his body up a flight of stairs is done rapidly rather than slowly.

Suppose then that a small amount of work δW is done during a small enough time interval dt. Then we can describe the *rate* of doing work by the ratio $\delta W/dt$ which is called "power" and which we shall denote by the script letter $\mathcal{P}$. Thus we have introduced this definition:*

Def. **Power**: $\mathcal{P} = \dfrac{\delta W}{dt}$ (E-1)

* The corresponding quantities δW and dt are supposed to be small enough so that the ratio $\delta W/dt$ remains unchanged for any smaller value of the chosen time interval dt.

The unit of power can be found from its definition, Def. (E-1). Thus

$$\text{unit of power} = \frac{\text{joule}}{\text{sec}} = \text{watt} \qquad \text{(E-2)}$$

where the combination of units joule/sec is given the special name "watt" [in honor of James Watt (1736-1819), the inventor of the steam engine]. For example, a 100 watt light bulb is designed so that, when it is turned on, 100 joule of electric work per second is done to move electrons through the filament inside the bulb (thus heating the filament sufficiently to produce light).

It is important to distinguish carefully between work and power. For example, suppose that a crane lifts an object of mass m through a vertical distance h by exerting on this object an upward force equal in magnitude to the weight mg of the object. Then the work done on the object by the crane is always equal to mgh. But the *rate* of doing this work, i.e., the

power delivered to the object by the crane, is larger if the crane lifts the object through this distance h in a shorter time.

Understanding the Definition of Power (Cap. 1e)

E-1 *Example:* To measure the power delivered to a dog sled by a team of nine huskies, a physiologist has the huskies pull the sled along level snow at a constant speed of 1.5 m/s. She finds that the horizontal force $\vec{F}_t$ exerted on the sled by the team (i.e., by the team's harness) has a constant magnitude of 800 N. (a) What is the work δW_t done on the sled by $\vec{F}_t$ in a time interval $dt = 2.0$ second, during which the sled moves 3.0 meter? What is the power P_t delivered to the sled by the dog team? (b) Since the sled moves with constant speed, the frictional force exerted on the sled by the snow is equal in magnitude but opposite in direction to $\vec{F}_t$. What is the work δW_s done on the sled by the frictional force due to the snow during the interval $dt = 2.0$ second? What is the power P_s delivered to the sled by the snow? *(Answer: 129)*

E-2 *Relating power to work:* The unit *horsepower* (hp) is approximately equal to 750 watt. In towing a 1500 kg car, a strong horse might be able to deliver 1.0 hp to the car for a small time interval $dt = 4.0$ second. During this interval, what is the work done on the car by the force exerted by the horse? *Review:* If the car is in neutral gear on a level road, this work is about equal to the total work done on the car. If the car is initially at rest, what is its speed after 4.0 second? *(Answer: 127)*

E-3 *Dependence of power on time interval:* During the part of the human heart cycle in which blood is expelled from the heart into the aorta, the force exerted by the walls of the heart does about 0.9 joule of work on the blood. (a) For a resting person, this part of the heart cycle has a duration of about 0.3 second. Assuming that this time interval is small enough, what is the power delivered to the blood by the heart during this part of the heart cycle? (b) When a person does mild exercise, the heart rate nearly doubles, so that the duration of each part of the heart cycle is about half that for rest. By assuming that the work done on the blood by the heart is the same during exercise as it is during rest, estimate the power delivered to the blood by the heart of an exercising person during the part of the heart cycle described previously. *(Answer: 125)*

SECT.

F SUMMARY

DEFINITIONS

kinetic energy; Def. (A-11)

work; Def. (A-13), Eq. (B-6)

joule; Eq. (A-16)

power; Def. (E-1)

watt; Rule (E-2)

IMPORTANT RESULTS

Definitions of kinetic energy and work: Eq. (A-10), Eq. (B-6)

$$K = \frac{1}{2}mv^2,\ W_{ab} = \int_a^b \delta W = \int_a^b F dr_F$$

Relation between kinetic energy and work: Eq. (B-4)

$K_b - K_a = W_{ab}$ where W_{ab} is the work done by *all* forces.

Work done by a constant force: Rule (C-1)

$$W = FD_F$$

Superposition principle for work: Eq. (D-5)

Work done by several forces jointly is equal to the sum of the works done by the individual forces.

Definition of power: Def. (E-1)

$$\mathcal{P} = \frac{\delta W}{dt}$$

NEW CAPABILITIES

You should have acquired the ability to:

(1) Understand these relations for a single particle:
- (a) the definition of kinetic energy (Sec. A),
- (b) the relation $W_{ab} = K_b - K_a$ (Sec. B),
- (c) the relation $W = FD_F$ (Sec. C),
- (d) the superposition principle for work (Sec. D)
- (e) the definition of power (Sec. E).

(2) Find the sign of the work done on a particle by a force everywhere parallel or perpendicular to the particle's path. (Sec. B, [p-1])

(3) Relate the speed of a particle and the work done on it by all forces. (Sec. B, [p-2] to [p-5])

Applying Work, Kinetic Energy, and Power (Cap. 1, 3)

F-1 To propel itself, the common squid *Loligo vulgaris* expels from its mantle about 50 grams of water in a single jet pulse lasting 0.1 second. During the pulse, the expelled water accelerates from rest to a speed of 4 m/s. (a) During this pulse, what is the work done on the expelled water by all forces? (b) By assuming that this work is entirely done by the force exerted on the expelled water by the walls of the squid's mantle, estimate the power delivered to the expelled water by the squid during the pulse. (c) The action of the squid's mantle in expelling water is similar to the action of the heart in expelling blood (see problem E-3). Is the power delivered to the expelled fluid by these organs about the same, or is one value much larger (i.e., more than 10 times larger) than the other? *(Answer: 134)*

F-2 The physiological stress of running is directly related to the power delivered by the legs. Suppose an 80 kg man jogs 100 meter in 20 second with a constant speed of 5 m/s. Let us estimate the power delivered to his torso by his legs if he jogs this distance in a straight path (a) along a level field, (b) up a very gentle 1 percent grade, so that he ascends a vertical distance of 1.0 meter, and (c) up an ordinary 10 percent grade, so that he ascends a vertical distance of 10 meter. The man's torso (including head and arms) can be considered as a particle of mass 50 kg moving with the man's constant speed. The torso is acted on by a force due to the leg muscles, a force due to gravity, and a constant force (due to air resistance) having a magnitude of 5 N and a direction opposite to the man's direction of motion. For each situation, use the total work W done on the torso by all forces, the work W_G done on it by the gravitational force, and the work W_A done on it by the force due to air resistance to find the work W_L done on the torso by the force due to the leg muscles, and thus the power P_L delivered to the torso by the legs. Why should physical fitness programs specify slope as well as distance and time in their jogging exercises? Use your results to explain. *(Answer: 131) (Suggestion: [s-7])*

SECT.
G PROBLEMS

Relating Speed, Displacement, and Total Force

G-1 A 0.2 kg baseball is hit with an initial speed of 20 m/s from home plate to a point in the grandstand 15 meter higher than home plate and 20 meter horizontally distant from it. Assuming that air resistance on the ball is negligible, estimate the ball's speed when it barely misses an unwary spectator in the grandstand. *(Answer: 133) ([s-1], [p-6])*

G-2 Figure G-1 shows a device used to measure the kinetic energy and speed of electrons moving in a vacuum along the direction $\hat{x}$. The electrons pass through a charged wire screen S and travel toward a charged plate P, 1 cm from S, where their arrival is detected by electrical measurements. During this motion, the total force on an electron is an electrical force $\vec{F}_e$ due to the charged screen and plate. The charges of the screen and plate are adjusted until every electron comes to rest just before it reaches the plate, so that no more electrons are detected. If the electric force $\vec{F}_e = (-5 \times 10^{-13}\,\mathrm{N})\hat{x}$ in this situation, what is the kinetic energy and the speed of an electron when it passes through the screen S? Use the value 1×10^{-30} kg for the mass of an electron. *(Answer: 130) (Practice: [p-7])*

G-3 *Stopping distance for highway vehicles:* Suppose a vehicle comes to a panic stop on a straight level highway. Let us relate the vehicle's mass m, its speed v when the brakes are applied, and the "stopping distance" D it travels before coming to rest. (a) The constant frictional force exerted on the vehicle's locked wheels has a magnitude $F_f = \mu F_n = \mu mg$, where $F_n = mg$ is the magnitude of the upward normal force on the vehicle due to the highway, and μ is the "coefficient of friction" between the tires and the highway. Using this result, find an expression for D in terms of m, v, μ, and g. (b) The value of μ depends only on the condition of the vehicle's tires and of the highway surface, Does the stopping distance D depend on the mass of the vehicle? (c) The value of μ is about 0.50 for a car with good tires traveling on dry pavement, What is the stopping distance D for this car if its initial speed is 20 m/s (45 mph)? (d) What is D if its initial speed is twice this value? *(Answer: 128)*

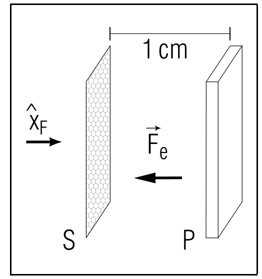


Fig. G-1.

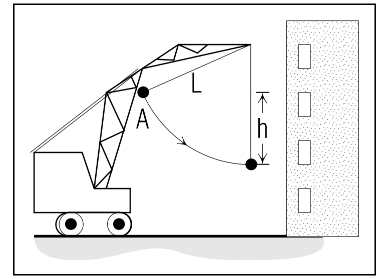


Fig. G-2.

G-4 *Tension in a wrecking-ball cable:* A "wrecking ball" of mass m is attached to a crane by a cable of length L. In use, the ball swings like a pendulum to strike and collapse the wall of a building being demolished (Fig. G-2). (a) Suppose the ball is initially at rest at the point A, and then swings downward through the point B at the bottom of its arc, If A is a height h above B, what is the ball's speed v when it passes B? (b) As the ball passes B, it is moving momentarily along a circular path with constant speed. At this time, what is the magnitude F_t of the tension force exerted on the ball by the cable? Express your answer in terms of m, g, h, and L. (c) Suppose that $h = (1/2)L$, as shown in Fig. G-2, What is the value of F_t, expressed in terms of the weight $w = mg$ of the ball? *(Answer: 132) (Suggestion: [s-12])*

TUTORIAL FOR A

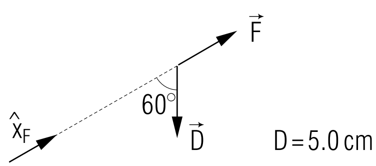
FINDING AND DESCRIBING THE NUMERICAL COMPONENT OF A DISPLACEMENT

a-1 *PURPOSE:* In this tutorial section we shall review briefly how to find or describe the numerical component of a particle's displacement along the direction of a force. We do so because we need these abilities to find the work done on the particle by the force. (You may also want to refer to text section B of Unit 407, where we first discussed numerical components.)

a-2 *FINDING THE COMPONENT OF A DISPLACEMENT ALONG A FORCE:* Suppose a particle moves through a displacement $\vec{D}$ while acted on by some force $\vec{F}$ that is constant during the displacement. (For example, this displacement $\vec{D}$ might be the small displacement $d\vec{r}$ discussed in text section A.) Let us begin by recalling the method for finding the component D_F of $\vec{D}$ along $\vec{F}$. (We shall henceforth use the word "component" as a shorthand for "numerical component.")

(1) On a diagram, construct the component vectors of $\vec{D}$ parallel and perpendicular to the direction of $\vec{F}$.

►



(2) Using the triangle formed by $\vec{D}$ and its component vectors, express the component vector parallel to $\vec{F}$ as a multiple of the unit vector $\hat{x}_F$ indicating the direction of $\vec{F}$.

► ____________________

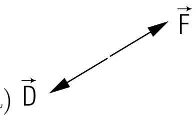
(3) The component D_F of $\vec{D}$ along $\vec{F}$ is the quantity, including sign and unit, that multiplies $\hat{x}_F$ in the previous expression. Find D_F.

► $D_F =$ ____________________

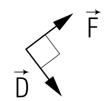
(Answer: 4) (Suggestion: [s-10])

For comparison, let us find the component D_F of several other displacements having the same magnitude $D = 5.0\,\text{cm}$ as the previous one, but different directions relative to $\vec{F}$.

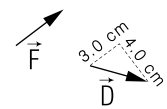
For each of the following displacements, find its component D_F along the force $\vec{F}$.

► (a) 

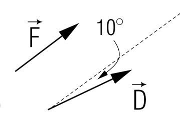
$D = 5.0\,\text{cm}$, $D_F =$ ________________

(b) 

$D = 5.0\,\text{cm}$, $D_F =$ ________________

(c) 

$D = 5.0\,\text{cm}$, $D_F =$ ________________

(d) 

$D = 5.0\,\text{cm}$, $D_F =$ ________________

(Answer: 1)

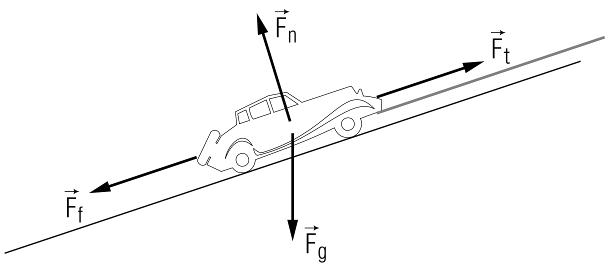
a-3 *DESCRIBING THE COMPONENT OF A DISPLACEMENT ALONG A FORCE:* Let us summarize the characteristics of the component D_F of a displacement $\vec{D}$ along a force $\vec{F}$, and then use this summary to describe the sign and magnitude of several components in an example.

For each of the directions of the displacement $\vec{D}$ indicated in the following table, give the sign of the component D_F of $\vec{D}$ along $\vec{F}$, and state whether the magnitude $|D_F|$ of this component is larger than, equal to, or smaller than the magnitude D of the displacement.

►

Direction of $\vec{D}$	Sign of D_F	Comparison of $|D_F|$ to D
Along $\vec{F}$		
Partly along $\vec{F}$		
Perpendicular to $\vec{F}$		
Partly opposite to $\vec{F}$		
Opposite to $\vec{F}$		

Suppose a car is being towed up a hill. As it moves through a small displacement $\vec{D} = 1$ meter uphill, it is acted on by the gravitational force $\vec{F}_g$, the normal and frictional forces $\vec{F}_n$ and $\vec{F}_f$ due to the road, and the towing force $\vec{F}_t$ shown in this drawing:



For each of these forces, describe the component of the car's displacement along the force by giving its sign and stating whether its magnitude is larger than, equal to, or smaller than 1 meter.

► Component along $\vec{F}_g$: __________ , __________

Component along $\vec{F}_n$: __________ , __________

Component along $\vec{F}_f$: __________ , __________

Component along $\vec{F}_t$: __________ , __________

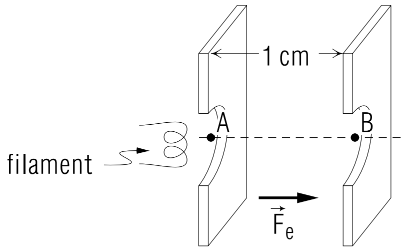
(Answer: 7) Now: Go to text problem A-1.

PRACTICE PROBLEMS

p-1 *DESCRIBING WORK DONE BY SPECIAL FORCES (CAP. 2):* An otter slides down a slippery mud bank into the water. During this motion, what is the sign of the work done on the otter by the frictional and normal forces $\vec{F}_F$ and $\vec{F}_n$ due to the surface of the bank? The otter then dives vertically downward to the bottom of a pool. During this motion, what is the sign of the work done on the otter by the gravitational force $\vec{F}_g$? *(Answer: 11) (Suggestion: See suggestion frame [s-9] or review text problem B-3.)*

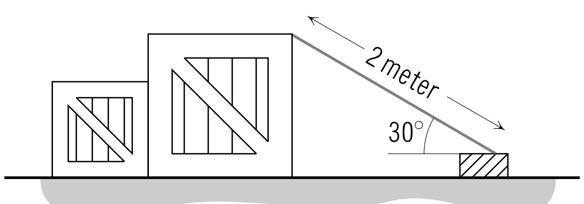
p-2 *RELATING WORK AND SPEED (CAP. 3):* The so-called "magnetic force" $\vec{F}_m$ on a moving charged particle is always perpendicular to the particle's velocity, and thus $\vec{F}_m$ is everywhere perpendicular to the particle's path. In the "magnetic lens" of an electron microscope, this force is the total force on an electron, and it causes the electron to travel along a curved path. As the electron travels through the lens, is the work done on it by the total force positive, zero, or negative? When the electron leaves the lens, is its speed larger than, equal to, or smaller than its speed when it enters the lens? *(Answer: 6) (Suggestion: review text problem B-9.)*

p-3 *RELATING WORK AND SPEED (CAP. 3):* The following drawing shows the path of an electron in the "electron gun" of an oscilloscope tube, where electrons emitted from a hot filament are accelerated so that they emerge from the gun with a high speed v. After leaving the filament, an electron, of mass 1×10^{-30} kg, enters the region between two charged plates through a hole at the point A, travels in a straight line for a distance of 1 cm across this region, and emerges from the gun through a hole at the point B. During this motion, the total force on the electron is an electric force $\vec{F}_e$ having a magnitude of 8×10^{-16} N and the direction indicated in the drawing. (a) What is the work done on the electron by the total force as the electron travels from A to B? (b) If the speed of the electron at the point A is 1×10^4 m/s, what is its speed when it emerges from the gun at the point B? *(Answer: 9) (Suggestion: review text problems C-6 and C-7.)*



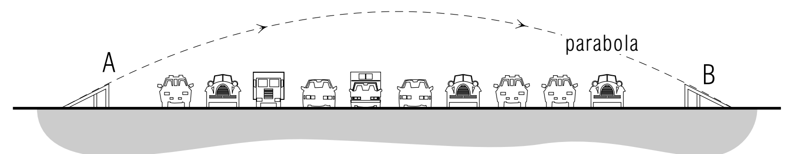
p-4 *RELATING WORK AND SPEED (CAPS. 1 AND 3):* A 200 kg rocket used in atmospheric research is launched vertically upward from rest on the earth's surface. Until the rocket reaches a height of 1 kilometer above the earth's surface, the rocket engine produces a constant force on the rocket of 1.8×10^4 N upward. To make a rough estimate of the rocket's speed at this height, let us assume that air resistance on the rocket is negligible. (a) What is the total work done on the rocket as it ascends vertically to this height? (b) What is the rocket's speed when it reaches this height? *(Answer: 5) (Suggestion: Review text problems D-3 and D-4.) (Further practice: [p-5])*

p-5 *RELATING WORK AND SPEED (CAPS. 1 AND 3):* A girl of mass 20 kg slides down the home-made slide shown in the following drawing. During this motion, the slide exerts on the girl frictional and normal forces $\vec{F}_f$ and $\vec{F}_n$ having constant magnitudes $F_f = 100$ N and $F_n = 170$ N. What is the total work done on the girl as she moves from the top to the bottom of the slide? If the girl's speed at the bottom of the slide is 1.0 m/s, what was her speed at the top of the slide? *(Answer: 8) (Suggestion: Review text problems D-3 and D-4.)*



More Difficult Practice Problems (Text Section G)

p-6 *RELATING SPEED, DISPLACEMENT, AND TOTAL FORCE:* The following drawing shows the path of a stunt car of mass 1.5×10^3 kg as it makes a jump over ten parked cars. The stunt car leaves the ramp at the point A, 3.0 meter above the horizontal track, and lands on the second ramp at the point B, 1.0 meter above the track. The car's speed at the point A is 19 m/s. Assuming that the car moves with negligible air resistance, estimate its speed when it lands at the point B. *(Answer: 2) (Suggestion: Review text problem G-1.)*



p-7 *RELATING SPEED, DISPLACEMENT, AND TOTAL FORCE:* An alpha particle, of mass 7.0×10^{-27} kg, travels through a tube 2.0 meter long which forms part of a particle accelerator. During this motion, the only force on the alpha particle is a constant electric force having a magnitude of 1.4×10^{-15} N and a direction along the direction of the alpha particle's motion. If the alpha particle's speed when it enters the tube is 1.0×10^5 m/s, what is its speed when it emerges from the tube? *(Answer: 10) (Suggestion: review text problem G-2.)*

SUGGESTIONS

s-1 *Text problem G-1:* Systematically apply the relation $W_{ab} = K_b - K_a$, using text example C-2 as a guide.

s-2 *(Text problem B-8):* Apply the relation $W_{ab} = K_b - K_a$. First find the probe's initial kinetic energy K_a at the earth's orbit, and then find its final kinetic energy K_b and speed at Mercury's orbit by using the work W_{ab} done by the total force on the probe.

s-3 *(Text problem A-1):* To find the small work $\delta W = F dr_F$ done on the ball by the total force, first find the *component* dr_F of each displacement along this downward force, and then multiply by the *magnitude* F of this force. If you need help finding the components, review tutorial frame [a-2].

s-4 *(Text problem B-7):* Use the initial and final speeds of the truck to find its initial and final kinetic energies K_a and K_b. Then use the relation $W_{ab} = K_b - K_a$ to find the work W_{ab} done on the truck by the total force. (Be careful with signs.)

s-5 *(Text problem C-1):* Since the elevator returns to its original position, its *displacement* (i.e., the vector from its original position to its final position) is zero, even though it traveled a *distance* of 30 meter.

s-6 *(Text problem D-2):* To find the work done on the bob by the gravitational force $\vec{F}_g$, sketch the bob's displacement $\vec{D}$ and find its component D_F along $\vec{F}_g$ by noting that the bob descends a vertical distance L between the points A and B. To find the work done by the tension force $\vec{F}_t$, note that $\vec{F}_t$ is everywhere perpendicular to the bob's circular path.

s-7 *(Text problem F-2):* According to the superposition principle for work, the work W done on the torso by all forces is the sum $W = W_L + W_G + W_A$ done by each individual force on the torso. But W must be zero, since the man's torso moves with constant speed and hence constant kinetic energy. By calculating the works W_G and W_A, you can thus find W_L.

s-8 *(Text problem C-3):* For each force, make a quick sketch showing the broom's displacement $\vec{D}$ and the direction of the force. Then use your sketch to find the component D_F and the work $W = FD_F$ for each force

separately. (For help in finding the components, review tutorial section A.)

s-9 *(Text problem B-3):* In each case, compare the direction of the particle's motion along its path (i.e., the direction of successive small displacements of the particle) with the direction of the force. If the directions are the same, the work done by the force is positive; if the directions are perpendicular, the work done by the force is zero; if the directions are opposite, the work done by the force is negative.

s-10 *(Tutorial frame [a-2]):* To find the component vectors of $\vec{D}$ parallel and perpendicular to $\vec{F}$, first draw a line from the beginning of $\vec{D}$ parallel to $\vec{F}$. Then draw a line from the end of $\vec{D}$ perpendicular to $\vec{F}$. These lines will intersect at a point P. The component vector of $\vec{D}$ parallel to $\vec{F}$ is the vector drawn from the beginning of $\vec{D}$ to the point P. The component vector of $\vec{D}$ perpendicular to $\vec{F}$ is the vector drawn from the point P to the end of $\vec{D}$. (If you need more help, review text section D of Unit 407.)

s-11 *(Text problem B-1):* In each small displacement $d\vec{r}$, the small work done on the carriage by the force $\vec{F}_w$ is $\delta W = F_w dr_F = (10\,\text{N})(+\,1\,\text{meter}) = 10\,\text{joule}$, since each displacement is along the direction of $\vec{F}_w$. In traveling the 50 meter distance from A to B, the carriage makes 50 of these small displacements, so the sum of the small works done along this path (i.e., the work done along this path) is $50(\delta W) = 5.0 \times 10^2$ joule.

s-12 *(Text problem G-4):* Part (b): Since the ball is moving momentarily with constant speed v along a circular path of radius L, its acceleration is $a = v^2/L$ upward. Use your value for v to express this acceleration in terms of m, g, h, and L. Then apply the equation of motion to relate this acceleration to the downward gravitational force and the upward tension force $\vec{F}_t$, using an upward unit vector to indicate directions, and solve for F_t.

s-13 *(Text problem C-6):* Apply the relation $W_{ab} = K_b - K_a$ systematically in the manner outlined in text example C-2. First make a quick sketch showing the truck driver's path and the distances h and L. Then use your sketch to express the work W_{ab} in terms of the symbols provided by finding the component D_F of the driver's displacement along the gravitational force. Complete the relation $W_{ab} = K_b - K_a$ by expressing the driver's initial and final kinetic energies in terms of the symbols provided. *Simplify* this equation, and then solve for v_b.

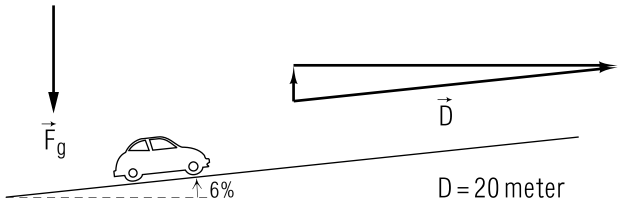
s-14 *(Text problem D-4):* Make a sketch showing the car's displacement $\vec{D}$ and the direction of the three constant forces acting on the car. Use your sketch to find the component of $\vec{D}$ along the *two* forces that are not everywhere perpendicular to the car's path. (Note that if the car moves 50 meter down the road, it descends 5 meter vertically.) Find the total work done on the car by adding the work done by each force (being careful with signs), and then apply $W_{ab} = K_b - K_a$ to find the car's initial kinetic energy and hence its initial speed.

s-15 *(Text problem C-5):* Part (a): Sketch the bird's displacement $\vec{D}$ (the vector from its original position to its final position) for each path. Then use your sketch to find the sign of the component D_F of this displacement along the gravitational force $\vec{F}_g$. (You may want to refer to your summary in tutorial frame [a-3].) Then answer the questions, recalling that the work done by $\vec{F}_g$ has the same sign as the component D_F.

Part (b): Compare the components for the displacements corresponding to the bird's paths: if the component D_F along $\vec{F}_g$ is the same for two paths, the work done by $\vec{F}_g$ is the same. (Several paths having the same component along $\vec{F}_g$ are illustrated in Fig. C-2.)

s-16 *(Text problem C-2):* In each case, make a sketch showing the displacement $\vec{D}$ of the particle and the force $\vec{F}$ in question. Remember that the displacement is the vector from the particle's initial position to its final position, irrespective of the path the particle takes between these two points. Then use the information provided, and the methods reviewed in tutorial section A, to find the component D_F of the particle's displacement along the force.

For example, the following drawing shows the displacement $\vec{D}$ of the car described in part (a), the gravitational force $\vec{F}_g$, and the component vectors of $\vec{D}$ parallel and perpendicular to $\vec{F}_g$. By finding one of the angles in the triangle formed by $\vec{D}$ and its component vectors, find the component D_F of $\vec{D}$ along $\vec{F}_g$.



► $D_F =$ ________________

(Answer: 3) Now: Return to text problem C-2.

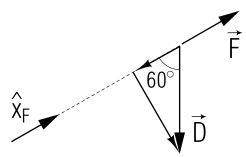
ANSWERS TO PROBLEMS

1. a. $D_F = -5.0\,\text{cm}$
   b. $D_F = 0$
   c. $D_F = 3.0\,\text{cm}$
   d. $D_F = 4.9\,\text{cm}$

2. $20\,\text{m/s}$

3. $D_F = -(\sin 6°)(20\,\text{meter}) = -(\cos 84°)(20\,\text{meter}) = -2.1\,\text{meter}$

4. (1)



   (2) $(-2.5\,\text{cm})\,\hat{x}_F$ (3) $D_F = -2.5\,\text{cm}$

5. a. 1.6×10^7 joule (NOT 1.8×10^7 joule
   b. 4×10^2 m/s

6. zero, equal to

7.

Direction of $\vec{D}$	Sign of D_F	Comp. of $\lvert D_F \rvert$ to D
Along $\vec{F}$	+	equal to
Partly along $\vec{F}$	+	smaller than
Perpendicular to $\vec{F}$	0	smaller than
Partly opposite to $\vec{F}$	−	smaller than
Opposite to $\vec{F}$	−	equal to

   $\vec{F}_g$: −, smaller; $\vec{F}_n$: 0, smaller; $\vec{F}_f$: −, equal; $\vec{F}_t$: +, equal.

8. zero, 1.0 m/s

9. a. 8×10^{-18} joule
   b. 4×10^6 m/s

10. 9×10^5 m/s

11. $\vec{F}_f$: $-\vec{F}_n : 0$ $\vec{F}_g : +$

101.

	Kinetic energy	Work
Kind of quantity	number	number
Possible signs	+, 0	+, 0, −
SI unit	joule	joule
Unit (kg, m, s)	kg m^2/s^2	kg m^2/s^2

   The possible signs are different for the two.

102. a. 20 joule
     b. 40 joule

103. a. 2.0×10^5 joule
     b. zero
     c. 2.0×10^5 joule

104. a. 10 joule
     b. A to B: 5.0×10^2 joule. B to B: 1.0×10^3 joule. B to A: 5.0×10^2 joule.
     c. 2.0×10^3 joule

105. a. $K_0 = 0.4$ joule
     b. $K_c = K_0 + \delta W = 0.1$ joule. $v_c = 1$ m/s

106. (1) −0.3 joule, (2) zero, (3) + 0.3 joule, (4) + 0.6 joule

107. $W_{ab} = -45$ joule (note sign)

108. -9.0×10^5 joule (note sign)

109. a. −
     b. 0
     c. −
     d. +

110. negative

111. Comet: smaller, decrease. Planet: equal, constant. Box: smaller, decrease. Electron: larger, increase.

112. No. Although the positions are the same, the velocities are different (opposite directions).

113. $\vec{F}_m$: +50 joule. $\vec{F}_g$: zero. $\vec{F}_f$: −50 joule. $\vec{F}_n$: zero.

114. 1×10^4 m/s

115. a. Cannot be found without knowing the work done on the elevator by the *total* force, which is due to both the cable and the earth.
 b. 0.05 joule

116. a. -2.5×10^4 joule
 b. 1×10^{-17} joule
 c. -6.0×10^5 joule

117. a. 1: perpendicular, zero. 2 and 3: partly opposite, negative. 4: partly along, positive. 5 and 6: along, positive.
 b. 2 and 3, 4 and 5

118. a. Upward: $D_F = -15$ meter, $W = -1.5 \times 10^5$ joule. Downward: $D_F = +15$ meter, $W = +1.5 \times 10^5$ joule. Entire Trip: $D_F = 0$, $W = 0$.
 b. Yes

119. a. $v_b = \sqrt{2gL}$
 b. $W_{sb} = mgL$, $v'_b = \sqrt{2gL}$, $v'_b = v_b$

120. $\vec{F}_f$ and $\vec{F}_m$, because they are not constant in direction

121. 9×10^6 m/s

122. -2×10^5 joule, 20 m/s (if you differ, you are wrong)

123. zero, constant speed

124. a. $W_{ab} = Mgh$, $v_b = \sqrt{v_a^2 + 2gh}$
 b. 4 m/s
 c. 5 m/s

125. a. 3 watt
 b. 6 watt

126. $W_{ab} = mgL$

127. 3.0×10^3 joule, 2.0 m/s

128. a. $D = v^2/2\mu g$
 b. No.
 c. $D = 40$ meter

 d. $D = 160$ meter!

129. a. $\delta W_t = 2.4 \times 10^3$ joule, $P_t = 1.2 \times 10^3$ watt
 b. $\delta W_s = -2.4 \times 10^3$ joule, $P_s = -1.2 \times 10^3$ watt

130. 5×10^{-15} joule, 1×10^8 m/s

131. a. $W_L = 500$ joule, $P_L = 25$ watt
 b. $W_L = 1.0 \times 10^3$ joule, $P_L = 50$ watt
 c. $W_L = 5.5 \times 10^3$ joule, $P_L = 280$ watt
 Because even a small slope greatly increases the power required and thus the physiological stress. (In fact, only athletes in top condition could hope to manage (c).)

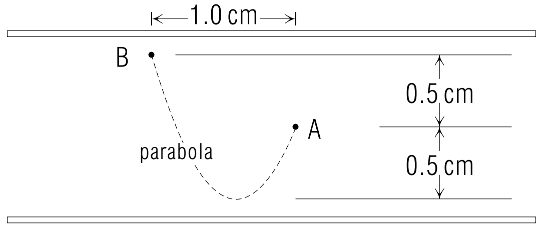
132. a. $v = \sqrt{2gh}$
 b. $F_t = mg + ma = mg + 2mgh/L$
 c. $F_t = 2w$, *twice* the ball's weight!

133. 10 m/s

134. a. 0.4 joule
 b. 4 watt
 c. about the same

MODEL EXAM

1. **Work done on a charged oil drop.** As a student performs the Millikan oil drop experiment, a charged oil drop moves from the point A to the point B along the path shown in the following drawing. The drop is acted on by a constant electric force $\vec{F}_e = 4 \times 10^{-15}$ N upward due to the charged plates.



What is the work done on the drop by the force $\vec{F}_e$ as the drop moves along this path?

2. **Motion of a car ascending a hill.** A woman is driving her 1000 kg car up a "10 percent grade," which means that the car ascends 1.0 meter vertically for every 10 meter it travels along the road. Seeing a police car ahead, the woman gently applies the brakes, so that the road surface exerts a constant frictional force of magnitude 1.0×10^3 N on the car as it travels a distance of 50 meter up the hill.

a. As the car travels this distance, what is the work done on it by all forces?

As the car travels up the hill from the point the woman applies the brakes to a point opposite the police car, the work done on the car by all forces is -1.5×10^5 joule.

b. If the car has a speed of 20 m/s when the woman applies the brakes, what is its speed when it passes the police car?

3. **Decay of a tritium nucleus.** The nucleus of a tritium atom (a radioactive isotope of hydrogen) decays to a helium nucleus by emitting a high-speed electron. During part of the decay process, the emitted electron travels directly away from the positively charged helium nucleus under the sole influence of the Coulomb electric force due to this nucleus.

a. During this motion, is the work done on the electron by all forces positive (+), zero (0), or negative (−)?

b. During this motion, does the electron's kinetic energy increase, remain the same, or decrease?

Brief Answers:

1. 2×10^{-17} joule
2. a. -1.00×10^5 joule
   b. 10 m/s
3. a. negative (−)
   b. If answer (a) is +, increase
   If answer (a) is 0, remain same
   If answer (a) is −, decrease