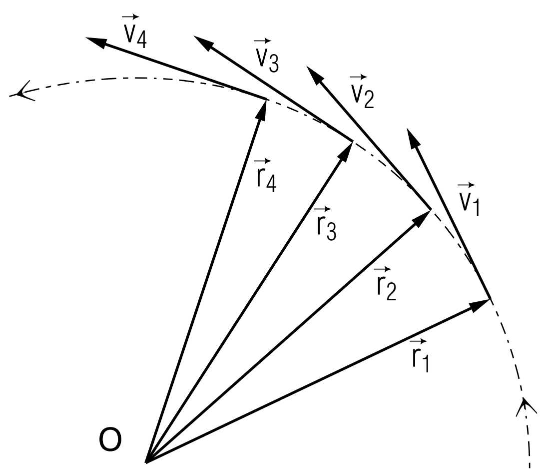
MISN-0-406

DESCRIPTION OF MOTION



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DESCRIPTION OF MOTION

by

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CONTENTS

Title: **Description Of Motion**

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Version: 4/30/2002 Evaluation: Stage 0

Length: 1 hr; 60 pages

Input Skills:

1. Vocabulary: position, reference frame, coordinate system, displacement, position vector (MISN-0-405).
2. Define velocity for motion in a straight line (MISN-0-404).

Output Skills (Knowledge):

K1. Define velocity and acceleration in both words and symbols.

K2. Write the equations of motion for a particle moving in a straight line with constant velocity or with constant acceleration.

K3. Describe the gravitational acceleration of a particle: its direction, its approximate magnitude and how it varies with altitude.

Output Skills (Problem Solving):

S1. Calculate velocity, speed, and acceleration using their definitions.

S2. Calculate the position or velocity of a particle moving in a straight line with constant or zero acceleration.

S3. Determine the position vector of a particle relative to a coordinate system, and state whether the particle is moving relative to this system.

S4. Determine a particle's velocity and acceleration from information about its speed and path.

S5. Use the relations for constant acceleration along a straight line; (a) to calculate initial or final values of position or velocity; (b) to solve problems involving the vertical motion of particles subject only to gravitational interaction with the earth.

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MISN-0-406

DESCRIPTION OF MOTION

Abstract:
In the present unit we shall exploit our familiarity with vectors to discuss useful ways of describing motion. Our knowledge of an effective *description* of motion will then prepare us for the more ambitious task of formulating a theory for the *prediction* of motion.

SECT.

A MOTION OF A PARTICLE

We shall focus primary attention on describing the motion of a simple kind of object called a "particle."

Def. | **Particle**: An object whose position can be adequately described by the position of a single point. | (A-1)

Whether an object can be considered as a particle depends on the desired precision of description. Thus we can regard any object as a particle if we are not interested in describing the relative positions of its parts, e.g., if the size of the object is very small compared to all relevant distances. (For instance, we could consider a ship on the far ocean as a particle, although we could not consider it as a particle when it is maneuvering near a pier.)

Any complicated object (e.g., a bicycle or a man) can always be regarded as a collection of many particles (each of which can be described by the position of a single point). In principle, an understanding of the motion of particles is thus sufficient to achieve an understanding of the motion of *all* objects, no matter how complex.

The position of a particle, like that of a point, must be described by comparison with some reference frame. We shall henceforth assume that the specification of a reference frame includes also some conveniently chosen coordinate system fixed with respect to this frame. As discussed in text section B of Unit 405, we may then describe the position of a particle by specifying its position vector r relative to the coordinate system associated with the reference frame. (See Fig. A-1.)

If the position vector $\vec{r}$ of the particle changes with time, the particle is said to be "moving" relative to the specified reference frame. If the position vector $\vec{r}$ does *not* change with time, the particle is said to be "at rest" relative to the reference frame. (Rest is thus merely a special case of motion corresponding to *no* change in $\vec{r}$.) It is important to note that motion is a *relative* concept involving a comparison with some reference frame.

Hence any description of motion *must* always include a specification of the reference frame used to describe the motion. (For example, consider a man sitting on a plane which is moving relative to the ground. Then

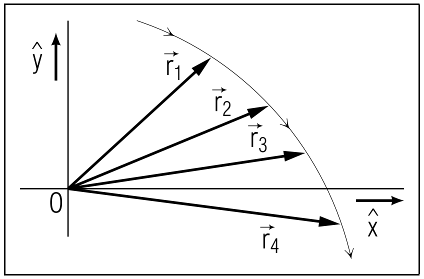


Fig. A-1: Position vector $\vec{r}$ of a particle at several successive times.

the man is moving relative to the ground, but is *not* moving relative to the plane since his position relative to the plane does not change.)

The motion of a particle relative to some reference frame can be described by specifying the position vector $\vec{r}$ of the particle at all times of interest. This can be done by a table listing corresponding numerical values of the time t and the position vector $\vec{r}$, or by a formula which relates $\vec{r}$ to the time t. Experimentally, the motion of a visible particle in the laboratory can be conveniently studied by taking a "stroboscopic" photograph, i.e., by illuminating the particle (moving in darkness) at regular intervals with brief flashes of light while focusing a fixed camera (with open shutter) on the particle. The developed film in the camera will then show a series of pictures indicating the successive positions of the particle at times separated by the time interval between the successive flashes of light. Fig. E-1 shows such a stroboscopic photograph of a freely falling object.

Describing Position and Motion (Cap. 2)

A-1 An elevator in an office building travels with a constant velocity of 8.9 m/s upward. A man in the elevator holds a cup of coffee 1.5 meter above the elevator floor. What is the position vector of this cup (a) relative to an origin O fixed below it on the elevator, and (b) relative to an origin O' fixed on the basement floor of the building 150 meter below the cup? (c) Is the cup moving relative to the elevator, relative to the building, relative to both, or relative to neither? *(Answer: 102) ([s-13], [p-1])*

* As problem A-1 illustrates, quantities describing motion depend on the reference frame used to measure these quantities. For simplicity we shall always use the earth's surface as a reference frame, unless another frame is explicitly indicated.

SECT. B VELOCITY

As a particle moves, its position vector $\vec{r}$ changes with time. To describe the *rate* of change of position with time, consider the particle at some time t_0 when it is located at some point P_0 described by the position vector $\vec{r}_0$, as indicated in Fig. B-1. For purposes of comparison, consider a slightly different (or changed) time t_c when the particle is located at some nearby point P_c described by the position vector $\vec{r}_c$. If the time difference $t_c - t_0$ is small enough, we shall denote it by $dt = t_c - t_0$ and shall denote the corresponding displacement of the particle during this time by $d\vec{r} = \vec{r}_c - \vec{r}_0$. (See Fig. B-1.) The rate of change of the position of the particle with time is then specified by the ratio $d\vec{r}/dt$. This ratio is a vector (since it is the result of dividing the vector $d\vec{r}$ by the number dt). It is denoted by the symbol $\vec{v}$ and is called the "velocity of the particle at the time t_0," according to this definition:*

Def. **Velocity**: $\vec{v} = \dfrac{d\vec{r}}{dt}$ (B-1)

* This definition supersedes our earlier Def. (B-8) of the word "velocity" in Unit 404.

The preceding definition specifies the condition that the time difference dt must be chosen small enough. In other words, the value of dt must be so small that, if one chose any smaller value of dt and calculated the ratio $d\vec{r}/dt$ with the correspondingly smaller value of $d\vec{r}$, the value obtained for dr/dt remains the same (within the desired precision). The reason that the *direction* of $d\vec{r}$ remains the same is that the path of the particle is nearly straight during a small enough time dt. Thus the direction of $d\vec{r}$, and thus of $\vec{v}$, is at any time t_0 *along* the path (i.e., tangent to it).

Note that the description of the velocity $\vec{v}$ must include the specification of a reference frame since the description of the position vector $\vec{r}$ requires the specification of such a reference frame. (For example, the velocity of a man sitting in a flying plane is zero relative to the plane, but is *not* zero relative to the ground.)

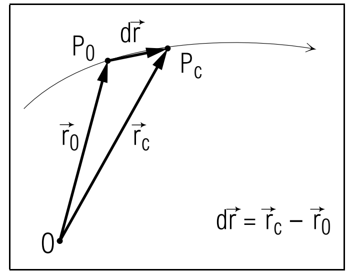


Fig. B-1: Position vectors of a moving particle at two slight1y different times (The magnitude of the displacement dr has been exaggerated to make it easily visible.)

Let us briefly examine some simple properties of the velocity defined by Def. (B-1). As already mentioned, the velocity $\vec{v}$ is a *vector* (and differs thus from the word "velocity" loosely used in everyday life). The *direction* of $\vec{v}$ is the same as that of the displacement $d\vec{r}$ of the particle during a short time interval dt between the original time t_0 and a slightly later time t_c. The *magnitude* of $\vec{v}$ is equal to the magnitude of the displacement $d\vec{r}$ divided by the magnitude of the corresponding time difference dt. Hence the magnitude of $\vec{v}$ is expressed in terms of a unit of length divided by a unit of time. The SI unit of the velocity $\vec{v}$ is thus meter/second.

The magnitude v or $|\vec{v}|$ of the velocity $\vec{v}$ of a particle is commonly called its "speed."

Def. **Speed**: Magnitude of the velocity (B-2)

(Thus the speed v is the ratio $|d\vec{r}|/|dt|$ of the small distance $|d\vec{r}|$ divided by the magnitude $|dt|$ of the small time required to move this distance.) Since the speed denotes a magnitude, it is a positive (or zero) *number*, unlike the velocity $\vec{v}$ which is a vector. When a particle moves, its position vector $\vec{r}$ ordinarily changes both in magnitude *and* direction. (See Fig. B-2.) The velocity $\vec{v}$, which describes the rate of this change, has then at any instant a direction along the path of the particle, i.e., a direction ordinarily different from $\vec{r}$. (It is often convenient to represent the velocity $\vec{v}$ of a particle by an arrow drawn from the particle.)

Example B-1: Directions of the velocity

Consider a car traveling along a *straight* road. (As indicated in Fig. B-3, the position of this car may be specified by a position vector $\vec{r}$ measured from some origin O on this road.) Then any small displacement $d\vec{r}$ of the car, and hence also its velocity $\vec{v}$, is always parallel to the road (and thus also parallel to $\vec{r}$).

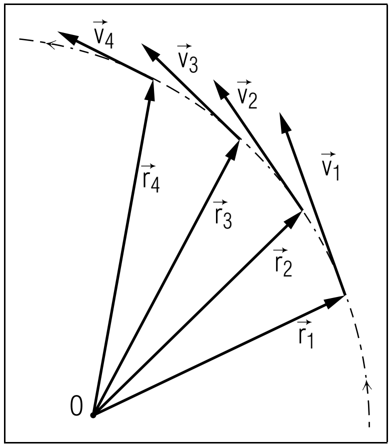


Fig. B-2: Velocity of a particle at several successive instants of time.

Consider now a car traveling along a *curved* road. As usual, the velocity of the car is then at any instant directed along the road. As the car moves along the curved road, the direction of its velocity thus changes. Hence the velocity of the car is *not* constant, even if its speed (i.e., the magnitude of its velocity) remains constant.

Now: Go to tutorial section B.

Understanding the Definitions of Velocity and Speed (Cap. 1a)

B-1 *Statement and interpretation:* (a) State an equation defining velocity. (b) After a DC-10 aircraft touches down on a runway, it travels from the point P_0 to the point P_c in the small enough time interval of 0.50 sec (Fig. B-4). What are the velocity and speed of the aircraft at the time it passes P_0? *(Answer: 106)*

B-2 *Properties:* (a) Answer each of the following questions first for velocity and then for speed. Is this quantity a vector or a number?

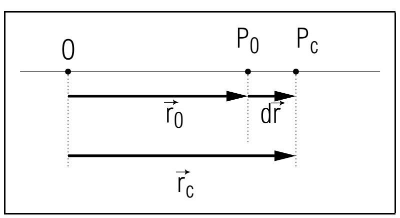


Fig. B-3: Motion of a particle along a straight path.

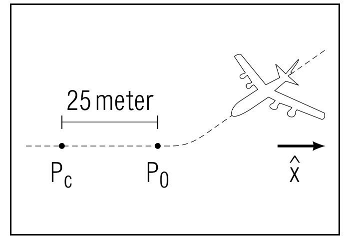


Fig. B-4.

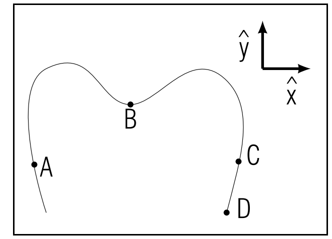


Fig. B-5.

What is its SI unit? What algebraic symbol commonly represents this quantity? (b) For which of the preceding questions are the answers for velocity and speed different? *(Answer: 109)*

Using Path and Speed to Describe Velocity (Cap. 3)

B-3 A bicycle moves from A to C along the path shown in Fig. B-5. (a) If the bicycle's speed has the constant value 7 m/s, is its velocity constant? (b) What is the velocity of the bicycle at each of the points A, B, and C? (c) Suppose the bicyclist stops at a point D, and remains there at rest during a time interval dt. What is the displacement $d\vec{r}$ of the bicycle during this time? What is the bicycle's velocity as it stands at rest at D? *(Answer: 107)*

B-4 The drawing in Fig. B-6 shows a roller coaster track. The passenger car is first pulled with constant speed along the straight part of the track from A to B, and then along the curved part from B to C. After being released, it then travels with increasing speed along the straight path from C to D. (a) For which of the following pairs of points are the velocities of the car equal: (A and B), (B and C), (C and D)? (b) For which pairs of points are the speeds of the car equal? *(Answer: 101)*

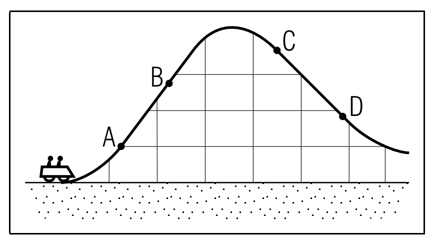


Fig. B-6.

SECT.

C ACCELERATION

As a particle moves, its velocity ordinarily changes with time. To describe the *rate* of change of velocity with time, consider the particle at some time t_0 when it is located at some point P_0 and has some velocity $\vec{v}_0$ (as indicated in Fig. C-1a.) For purposes of comparison, consider a slightly different time t_c when the particle is located at some other point P_c and has some slightly different velocity $\vec{v}_c$. If the time difference $t_c - t_0$ is small enough, we denote it by $dt = t_c - t_0$ and denote the corresponding change of velocity during this time by $d\vec{v} = \vec{v}_c - \vec{v}_0$. (The vector difference $d\vec{v}$ between these two velocities is shown in Fig. C-1b.) The rate of change of the velocity of the particle with time is then specified by the ratio $d\vec{v}/dt$. This ratio is a vector (since it is the result of dividing the vector $d\vec{v}$ by the number dt). It is denoted by the symbol $\vec{a}$ and is called the "acceleration of the particle at the time t_0," according to this definition:

Def. $$\left| \textbf{Acceleration}\text{: } \vec{a} = \frac{d\vec{v}}{dt} \right| \tag{C-1}$$

Note that the time interval dt in this definition must be chosen small enough so that any smaller value of dt would leave the ratio $d\vec{v}/dt$ unchanged. Note also that the description of the acceleration $\vec{a}$ must include a specification of a reference frame since the description of the velocity $\vec{v}$ requires such a specification.

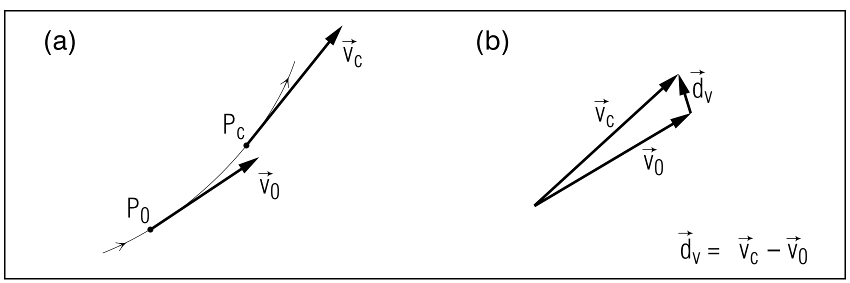


Fig. C-1: Velocities of a particle at two slightly different times. (a) Velocity vectors indicated along the path. (b) Difference between the velocities. (The magnitude of $d\vec{v}$ has been exaggerated.)

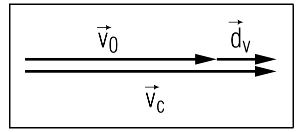


Fig. C-2: Velocities of a particle moving along a straight path.

Let us briefly examine some simple properties of the acceleration defined by Def. (C-1). As already mentioned, the acceleration $\vec{a}$ is a *vector* (and differs thus from the word "acceleration" loosely used in everyday life.) The *direction* of $\vec{a}$ is the same as that of the velocity change $d\vec{v} = \vec{v}_c - \vec{v}_0$ of the particle during a short time interval dt between the original time to and a slightly later time t_c. The *magnitude* of $\vec{a}$ is the magnitude of the velocity change $d\vec{v}$ divided by the magnitude of the corresponding time interval dt. Hence the magnitude of $\vec{a}$ is expressed in terms of a unit of velocity divided by a unit of time. The SI unit of the acceleration $\vec{a}$ is thus $(\text{meter/second})/\text{second} = \text{meter/second}^2$.

When a particle moves, its velocity $\vec{v}$ changes ordinarily *both* in magnitude and in direction, as illustrated in Fig. C-1. Furthermore, the velocity change $d\vec{v}$ of the particle, and hence also its acceleration $\vec{a}$, has a direction which is ordinarily *different* from the direction of the velocity $\vec{v}$ of the particle. Hence the acceleration $\vec{a}$ is ordinarily *not* along the path of the particle. (It is often convenient to indicate the acceleration of a particle by an arrow drawn from the particle.)

Example C-1: Directions of the acceleration

Consider a car traveling along a *straight* road. As indicated in Fig. C-2, the velocity $\vec{v}$ of the car is then always parallel to the road. Hence any velocity change $d\vec{v}$ of the car, and thus also its acceleration $\vec{a} = d\vec{v}/dt$, must also be parallel to the road (and hence also parallel to the velocity).

Consider now a car traveling along a *curved* road, as illustrated in Fig. C-1a. Because the direction of the velocity changes, the acceleration $\vec{a}$ of the car is then *not* along the road. Indeed, Fig. C-1b indicates that the direction of $d\vec{v}$, and hence of $\vec{a}$, is different from $\vec{v}_0$ (such that it points toward the inside of the curved path traversed by the car).

Understanding the Definition of Acceleration (Cap. 1a)

C-1 *Statement and example:* (a) State an equation defining acceleration. (b) A baseball moves upward along a curved path from P_0 to P_c in the small enough time interval $dt = 0.5\,\text{sec}$ (Fig. C-3). Use the indicated values for the velocities $\vec{v}_0$ and $\vec{v}_c$ at these points to construct the velocity change $d\vec{v}$ during the time dt. (c) What is the ball's acceleration $\vec{a}$ at the time it passes P_0 ? (d) Make a sketch of the ball's path showing the point P_0. Beginning at P_0 draw arrows showing the direction of the ball's velocity and acceleration at this point. (e) For this *curved* path, describe the directions of the ball's velocity and its acceleration by stating whether each is parallel to the path, towards the inside of the path, or towards the outside of the path. *(Answer: 104) (Suggestion: [s-1])*

$\boxed{\text{C-2}}$ *Interpretation:* A baseball rolls along a straight path with a velocity $\vec{v}_0 = 1.5\,\text{m/s}\;\hat{x}$ at time $t = 5.2\,\text{sec}$ (where $\hat{x}$ is a unit vector directed towards the right). After a small enough time interval, at a time $t_c = 5.4\,\text{sec}$, the ball has a velocity $\vec{v}_c = 0.9\,\text{m/s}\;\hat{x}$. (a) What is the acceleration $\vec{a}$ of the baseball at time t_0? (b) Make a sketch of the ball's path, showing its location at time t_0. Then from this location, draw arrows indicating the directions of the ball's velocity and acceleration at this time. (c) For this straight path, describe the directions of the ball's velocity and its acceleration by stating whether each is parallel to the path. *(Answer: 108) (Suggestion: [s-9])*

$\boxed{\text{C-3}}$ *Properties and comparison with velocity:* (a) Answer each of the following questions both for acceleration and for velocity. Is this quantity a vector or a number? What is its SI unit? What algebraic symbol commonly represents this quantity? Is the direction of this quantity always parallel to the path, or is it parallel to a straight path but toward the inside of a curved path? (b) For which of the preceding questions are the answers for velocity and acceleration different? (c) Which of the following are reasonable magnitudes for the velocity of a car on a highway? (d) Which are reasonable magnitudes for its acceleration? (2.5 m/s, 25 m/s, 250 m/s, 2.5 m/s^2, 25 m/s^2) *(Answer: 110) (Suggestion: [s-11])*

Now: Go to tutorial section C.

Using Path and Speed to Describe Acceleration (Cap. 3)

$\boxed{\text{C-4}}$ Consider again the motion of the roller coaster car shown in Fig. B-6. During which of the following motions of the car is its acceleration equal to zero? (a) Motion with constant speed along the straight path from A to B; (b) Motion with constant speed along the curved path from B to C; (c) Motion with increasing speed along the straight path from C to D. *(Answer: 103)*

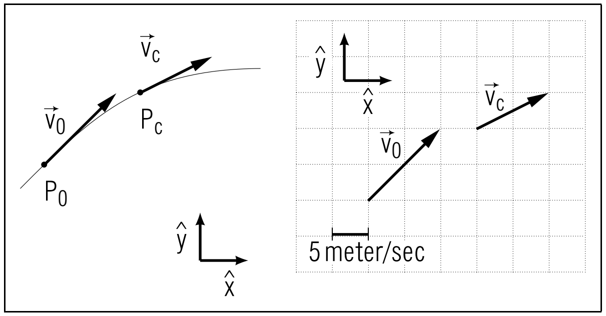


Fig. C-3.

$\boxed{\text{C-5}}$ A ball, attached by a rubber band to a paddle (Fig. C-4), can move along a vertical path. Consider six combinations of position and speed: position above the paddle or below the paddle, and for each of those regions its speed increasing, decreasing, or zero. Make a chart with six lines, one for each of the six combinations just described. Label each line with its combination of those region and speed descriptors. Now fill in new information on each line: the directions of the velocity and acceleration vectors. Use the terms "up," "down," or "zero," where "zero" means the length of the vector is zero so it has no direction. *(Answer: 105)*

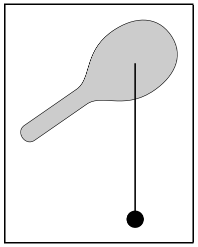


Fig. C-4.

SECT.

D MOTION ALONG A STRAIGHT LINE

There are many familiar situations where a particle moves along a straight line. The motion can then be described most conveniently relative to a one-dimensional coordinate system chosen so that its origin O lies on the line and so that its direction (specified by a unit vector $\hat{x}$) is parallel to this line. (See Fig. D-1.) The position vector $\vec{r}$ must then always be parallel to this line. Hence the *difference* between the position vectors of the particle at two different times, and thus the velocity $\vec{v}$ of the particle, must also be parallel to the line. Then the *difference* between the velocities of the particle at two different times, and thus also the acceleration $\vec{a}$ of the particle, must also be parallel to the line. In short, *all* vectors which describe the motion (such as $\vec{r}$, $\vec{v}$, and $\vec{a}$) are parallel to the line, and thus parallel to $\hat{x}$.

In accordance with the comments at the end of text section C of Unit 405, all these vectors can then be simply expressed as multiples of the unit vector $\hat{x}$. Thus the position vector $\vec{r}$ is conventionally written as

$$\vec{r} = x\hat{x} \tag{D-1}$$

where the *number* x is called the "*numerical component* of $\vec{r}$ along $\hat{x}$" or the "position coordinate" of the particle along $\hat{x}$. [For example, in Fig. D-1 the position of the particle at P_1 can be described by $\vec{r} = (3\,\text{meter})\hat{x}$ or by $x = 3\,\text{meter}$; its position at the point P_2 can be described by $\vec{r} = -(2\,\text{meter})\hat{x}$ or by $x = -2\,\text{meter}$.] Similarly, the other vectors parallel to $\hat{x}$ can be written as

$$\vec{v} = v_x\hat{x} \text{ and } \vec{a} = a_x\hat{x} \tag{D-2}$$

where the *number* v_x is called the "numerical component of $\vec{v}$ along $\hat{x}$" and where the *number* a is called the "numerical component of $\vec{a}$ along $\hat{x}$."

The definition $\vec{v} = d\vec{r}/dt$ of the velocity then implies correspondingly that $v_x\hat{x} = (dx/dt)\hat{x}$ so that *

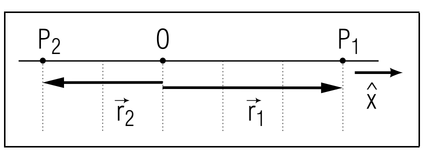


Fig. D-1: Coordinate system for describing motion along a straight line.

$$v_x = \frac{dx}{dt}\,. \tag{D-3}$$

* This result follows since $d\vec{r} = \vec{r}_c - \vec{r}_0 = x_c\hat{x} - x_0\hat{x} = (x_c - x_0)\hat{x} = dx\hat{x}$

Similarly, the definition $\vec{a} = d\vec{v}/dt$ of the acceleration implies that

$$a_x = \frac{dv_x}{dt} \tag{D-4}$$

The preceding comments indicate how our general vector definitions of position and velocity are related to the numerical relations used earlier in Unit 404. [For example, Relation (B-7) of Unit 404 was merely a special definition for the *numerical component* Eq. (D-3) of the velocity.]

Since the velocity $\vec{v} = d\vec{r}/dt$ and acceleration $\vec{a} = d\vec{v}/dt$ are vector rates, we can apply the same arguments as those of text section D of Unit 404 to use information about these rates to find information about the corresponding functions. Thus we can use information about the velocity of a particle to predict its position, or can use information about its acceleration to find its velocity. Let us illustrate how to make such predictions in the simple cases where either the velocity or the acceleration is constant.

MOTION WITH CONSTANT VELOCITY

Consider the simple case where the velocity $\vec{v}$ of a particle is constant. Suppose that, at some initial time t_A the position vector of the particle is $\vec{r}_A$. What then is the position vector $\vec{r}_B$ of the particle at any other time t_B?

The definition $\vec{v} = d\vec{r}/dt$ of the velocity implies that $d\vec{r} = \vec{v}dt$. Then the total displacement $\Delta\vec{r}$ is merely the sum of the successive small displacements $d\vec{r}$ (as indicated in Fig. D-2). But if the velocity $\vec{v}$ is constant, *all* corresponding small changes are connected by the same relation $d\vec{r} = \vec{v}dt$ with the *same* value of $\vec{v}$. Hence the *total* changes are similarly connected by the relation: *

$$\boxed{\Delta\vec{r} = \vec{v}\Delta t\,.} \tag{D-5}$$

* This result is analogous to that of Relation (D-2) of Unit 404 since $\Delta\vec{r} = d_1\vec{r} + d_2\vec{r} + \ldots = \vec{v}d_1t + \vec{v}d_2t + \ldots = \vec{v}(d_1t + d_2t + \ldots) = \vec{v}\Delta t$.

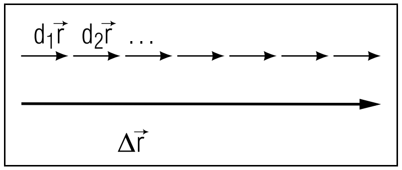


Fig. D-2: Successive displacements of a particle moving along a straight line with constant velocity.

Thus the *total* change $\Delta\vec{r} = \vec{r}_B - \vec{r}_A$ of the position vector (i.e., the total displacement) of the particle is simply equal to the constant velocity $\vec{v}$ multiplied by the *total* change $\Delta t = t_B - t_A$ of time. The position vector $\vec{r}_B$ of the particle at any time $t_B = t_A + \Delta t$ is then equal to

$$\vec{r}_B = \vec{r}_A + \Delta\vec{r} = \vec{r}_A + \vec{v}\Delta t \qquad \text{(D-6)}$$

Example D-1: Distance traveled during driver reaction time

The physiological reaction time required by a driver between the instant he perceives an emergency and the instant he can apply his brakes is known to be about 0.75 second. Suppose that a car is driving down a straight highway with a constant velocity of $(30\,\text{meter/second})\hat{x}$ [or 67 mile/hour in the $\hat{x}$ direction. What then is the displacement of the car during the reaction time of the driver?

The displacement $\Delta\vec{r}$ of the car during the time interval $\Delta t = 0.75\,\text{sec}$ is simply

$$\Delta\vec{r} = [(30\,\text{meter/sec})\hat{x}](0.75\,\text{sec}) = (22.5\,\text{meter})\hat{x}$$

i.e., a distance of 22.5 meter (or about 74 foot) along the $\hat{x}$ direction.

MOTION WITH CONSTANT ACCELERATION

Consider the case where the *acceleration* $\vec{a}$ of a particle is constant. Suppose that, at some initial time t_A the velocity of the particle is $\vec{v}_A$ and its position vector is $\vec{r}_A$. What then is the velocity $\vec{v}_B$ and position vector $\vec{r}_B$ of the particle at any other time t_B?

The definition $\vec{a} = d\vec{v}/dt$ of the acceleration implies that $d\vec{v} = \vec{a}dt$. But because $\vec{a}$ is constant, the relation $d\vec{v} = \vec{a}dt$ for all small changes implies again a similar relation for the *total* changes so that:

$$\boxed{\Delta\vec{v} = \vec{a}\Delta t} \qquad \text{(D-7)}$$

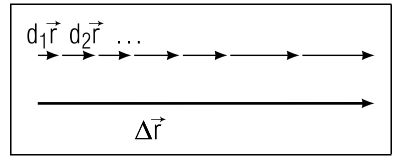


Fig. D-3: Successive displacements of a particle moving along a straight line with uniformly changing velocity.

where $\Delta\vec{v} = \vec{v}_B - \vec{v}_A$ and $\Delta t = t_B - t_A$. Hence

$$\vec{v}_B = \vec{v}_A + \Delta\vec{v} = \vec{v}_A + \vec{a}\Delta t \qquad \text{(D-8)}$$

We can now use our information about the velocity $\vec{v}$ to find the position vector $\vec{r}$ of the particle at any time. The definition $\vec{v} = d\vec{r}/dt$ of the velocity implies that $d\vec{r} = \vec{v}dt$. But since the acceleration $d\vec{v}/dt$ is constant, the velocity changes at a constant rate, i.e., the velocity changes uniformly with time. Hence the successive uniformly changing displacements $d\vec{r}$ occurring in equal small intervals of time can be added as indicated in Fig. D-3. The argument of text section D of Unit 404 then leads to the conclusion that the total change of position is simply equal to the *middle* value of the velocity multiplied by the total change of time. But if the particle starts at the time t_A with a velocity $\vec{v}_A$ and reaches after a total time Δt the velocity $\vec{v}_A + \vec{a}\Delta t$ found in Eq. (D-8), the middle value of its velocity during this time is $(\vec{v}_A + 1/2\vec{a}\Delta t)$.

Thus the total change $\Delta\vec{r} = \vec{r}_B - \vec{r}_A$ of the position vector of the particle is simply

$$\Delta\vec{r} = (\vec{v}_A + \frac{1}{2}\vec{a}\Delta t)\Delta t$$

or

$$\boxed{\Delta\vec{r} = \vec{v}_A\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2} \qquad \text{(D-9)}$$

The position vector $\vec{r}_B$ at the time $t_B = t_A + \Delta t$ is then simply equal to $\vec{r}_B = \vec{r}_A + \Delta\vec{r}$. *

* In the special case where $\vec{v} = \vec{v}_A$ is constant, $\vec{a} = 0$ and the result Eq. (D-9) reduces properly to Eq. (D-5).

Note that, if the particle starts from rest so that $\vec{v}_A = 0$, its displacement $\Delta\vec{r}$ increases proportionately to $(\Delta t)^2$. For example, if its travel time Δt is 3 times as large, its displacement $\Delta\vec{r}$ is $3^2 = 9$ times as large.)

Example D-2: Distance traveled after applying brakes

Published data indicate that a driver, applying his brakes as much as feasible, can produce a constant acceleration of magnitude $6.0\,\text{meter/sec}^2$ in a direction opposite to the velocity $\vec{v}$ of his car. [Thus $\vec{a} = -(6.0\,\text{meter/sec}^2)\hat{x}$ if $\vec{v}$ is along $\hat{x}$.] Suppose that a driver, traveling with an initial velocity $\vec{v}_A = (30\,\text{meter/sec})\hat{x}$ (or 67 mile/hour along $\hat{x}$), applies his brakes at some time t_A. After what time Δt does the car come to rest?

When the car comes to rest, $\vec{v}_B = 0$. Hence the change of velocity of the car during the time Δt is $\Delta\vec{v} = 0 - \vec{v}_A = -(30\,\text{meter/sec})\hat{x}$. Using the relation $\Delta\vec{v} = \vec{a}\Delta t$ of Eq. (D-7), we then find

$$-(30\,\text{meter/sec})\hat{x} = [-(6.0\,\text{meter/sec}^2)\hat{x}]\Delta t$$

Hence

$$\Delta t = 5.0\,\text{sec} \tag{D-10}$$

To find the displacement $\Delta\vec{r}$ of the car during this time, we need only use Eq. (D-10). Thus

$$\begin{aligned}\Delta\vec{r} &= [(30\,\text{meter/sec})\hat{x}](5.0\,\text{sec}) - \frac{1}{2}[(6.0\,\text{meter/sec}^2)\hat{x}](5.0\,\text{sec})^2\\ &= (150\,\text{meter})\hat{x} - (75\,\text{meter})\hat{x}\end{aligned}$$

or

$$\Delta\vec{r} = (75\,\text{meter})\hat{x} \tag{D-11}$$

The car travels thus a distance of 75 meter (or 246 ft) along $\hat{x}$ before it can be brought to rest. This frighteningly large distance is in addition to the distance of 22.5 meter traveled by the car during the reaction time before the driver can apply his brakes!

Now: Go to tutorial section D.

Understanding Relations for Motion Along a Straight Line (Cap. 1b)

D-1 *Statement and example:* Before answering each of the following questions, state the equation you will use, choosing one of

Eqs. (D-5), (D-7), or (D-9). Just after jumping from a plane, a sky-diver falls for 2.0 sec with a constant acceleration of $10\,\text{m/s}^2$ downward, beginning his fall with an initial velocity $\vec{v}_A = 0$ (a) What is the change $\Delta\vec{v}$ in the sky-diver's velocity during these 2.0 seconds? (b) Through what displacement $\Delta\vec{r}$ does he move during these 2.0 seconds? (c) After a time, air resistance causes the sky-diver to fall with a constant "terminal velocity" of 53 m/s downward. Through what displacement $\Delta\vec{r}\,'$ does he move during 2.0 sec of motion with this terminal velocity? *(Answer: 112)*

D-2 *Relating quantities:* A rapid-transit train moving with an acceleration $\vec{a} = -1.2\,\text{m/s}^2\hat{y}$ has an initial velocity $\vec{v}_A = 36\,\text{m/s}\hat{y}$ (where $\hat{y}$ is a unit vector pointing north). (a) What time interval is required for the train to come to rest, i.e., for its velocity to change by an amount $\Delta\vec{v} = \vec{v}_B - \vec{v}_A = 0 - 36\,\text{m/s}\hat{y} = -36\,\text{m/s}\hat{y}$? (b) Through what displacement $\Delta\vec{r}$ does the train travel during this time? *(Answer: 114) (Suggestion: p-3)*

Relating Values of Position and Velocity (Cap. 4a)

D-3 A car enters a highway with an initial velocity $5\,\text{m/s}\hat{x}$ (where $\hat{x}$ is a unit vector parallel to the road). Then for 10 sec, the car moves with a constant acceleration of $2.0\,\text{m/s}^2\ \hat{x}$. (a) What is the car's final velocity at the end of this time interval? (b) Through what displacement $\Delta\vec{r}$ does the car move during the time required to reach this velocity? *(Answer: 117) ([s-5], [p-4])*

SECT.

E GRAVITY NEAR THE EARTH

A particle can interact with many other things which influence its motion. Thus it interacts with all the things with which it is in contact, e.g., with the ground on which it is lying, with the hand by which it is held, or with the surrounding air. But even when a particle near the earth is not in contact with the earth or anything else, it nevertheless interacts with the earth since it is observed to fall toward it. This remarkable phenomenon is called “gravity.” Accordingly, the “gravitational interaction” of a particle with the earth is that interaction not due to contact with the earth.

To study the motion of a particle subject only to the gravitational interaction, we shall consider the situation where the particle moves in a vacuum so that its interaction with the surrounding air has been eliminated. However, our discussion will also apply approximately in cases where the interaction of the moving particle with the surrounding air is negligibly small. *

> * The interaction with the surrounding air depends on the nature of the particle; e.g., it is fairly small if the particle is a steel ball, but quite appreciable if it is a feather.

Let us then examine more closely the gravitational interaction of a particle with the neighboring earth. If such a particle is released from rest, it is observed to fall along the vertically “downward” direction, i.e., the direction specified by a “plumb line” (a string supporting a heavy object). The downward velocity $\vec{v}$ of the particle is observed to increase *uniformly* with time. (See Fig. E-1.) Hence the acceleration $\vec{a}$ of the particle has a downward direction and a magnitude which is *constant*. (To be precise, the acceleration is constant as long as the particle moves through a region of linear size much less than the radius of the earth.)

A particle influenced solely by the gravitational interaction with the earth may also move in more complicated ways.

For example, the particle might have been thrown vertically upward, or it might move in some curved path like a baseball. Nevertheless, the acceleration a of the particle is observed to be the *same* as if it were simply falling vertically downward. These experiments suggest this general

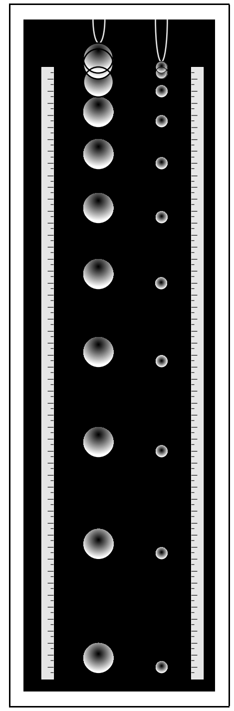


Fig. E-1: Stroboscopic photograph showing two different balls falling after being released from rest at the same time. Successive positions of the balls are portrayed at intervals of 1/30 second. The numbers on the meter stick indicate lengths expressed in terms of centimeter. [Photograph taken by Physical Science Study Committee, published in J. Orear, Fundamental Physics (2nd ed.), (John Wiley & Sons, New York, 1967), p.29.]

conclusion:

> A particle subject only to the gravitational interaction with the earth moves always with a *constant downward* acceleration (within any region of size much less than the radius of the earth). (E-1)

This acceleration is called the “gravitational acceleration of the particle due to the earth” and is commonly denoted by $\vec{g}$.

How does the gravitational acceleration $\vec{g}$ of a particle depend on its properties, e.g., on the size or shape of the particle or on the substance of which it is made? Observations show that in a vacuum all particles, irrespective of their substance or size, fall with the same acceleration (as illustrated in Fig. E-1). This remarkable conclusion has been confirmed by extremely precise experiments * and can be summarized:

* A recent experimental comparison of the magnitudes g and g' of the gravitational accelerations of a piece of platinum and a piece of aluminum shows that, if g and g' differ at all, the difference $(g - g')/g$ must be less than 10^{-12}.

The gravitational acceleration of a particle is independent of all its properties. (E-2)

Thus we can simply talk about the "gravitational acceleration $\vec{g}$ due to the earth" *without* needing to specify the nature of the particle which has this acceleration. There are many far-reaching implications of Rule (E-2) and it is also the basis of Einstein's general theory of relativity. The magnitude g of the gravitational acceleration due to the earth is slightly different at different positions near the earth's surface, being somewhat smaller at points which are farther from the center of the earth. Thus g is slightly smaller at the top of mountains than at lower altitudes. At sea level, the measured value of g is found to decrease from $9.8822\,\text{meter/sec}^2$ at the poles to $9.7805\,\text{meter/sec}^2$ at the equator (partly because the earth bulges at the equator instead of being completely spherical). At intermediate latitudes the value of g is approximately

$$g = 9.80\,\text{meter/sec}^2 \qquad \text{(E-3)}$$

When one is satisfied with less accuracy, one may conveniently use the value $g = 10\,\text{meter/sec}^2$. Unless stated otherwise, we shall use this value throughout the rest of the book.

Suppose that a particle moves vertically while being subject only to the gravitational interaction with the earth. Then we know that this particle moves with a *constant* downward acceleration having the known value $\vec{a} = \vec{g}$. Hence the arguments at the end of the preceding section allow us immediately to predict the velocity and position of such a particle at any time. [The direction $\hat{x}$ is then most conveniently chosen vertically (i.e., parallel to $\vec{g}$), either downward or upward.]

Knowing About Acceleration due to Gravity

E-1 (a) What is the acceleration of each of the following objects while it is moving subject only to gravitational interaction with the earth? A thrown baseball traveling upward along a curved path. A dropped dish falling along a vertical path. A dime tossed in the air so that it travels upward along a vertical path. (b) What is the acceleration

a of a golf ball which lies at rest in the weeds? Is this ball's motion subject only to gravitational interaction with the earth? *(Answer: 111)*

Now: Go to tutorial section E.

Solving Problems Involving Vertical Motion (Cap. 4b)

$\boxed{\text{E-2}}$ A hiker, wanting to measure the height of a cliff he has climbed, releases a stone from rest and measures the time interval Δt required for it to fall vertically to the ground below. (a) Express the height of the cliff (the magnitude $|\Delta r|$ of the displacement through which the stone moves) in terms of known quantities, including the magnitude g of the gravitational acceleration and the time interval Δt. (b) If the time interval is 2.0 sec, what is the height of the cliff? (c) Suppose the time interval for a stone to drop from a second cliff is twice as large as the interval for the first. Is the second cliff twice as high or four times as high as the first one? *(Answer: 119)*

$\boxed{\text{E-3}}$ At the top of his jump (at a height of 5.5 meter) the velocity of a pole vaulter is nearly zero. He then falls vertically downward for 1.0 sec before striking the ground. If we consider the pole vaulter as a particle moving subject only to the gravitational interaction, what is his final velocity as he strikes the ground? (Express vectors by using a *downward* unit vector $\hat{y}$.) *(Answer: 115)*

More practice for this Capability: [p-5], [p-6]

SECT.

$\boxed{\text{F}}$ SUMMARY

DEFINITIONS

particle; Def. (A-1)

velocity; Def. (B-1)

speed; Def. (B-2)

acceleration; Def. (C-1)

IMPORTANT RESULTS

Definitions of velocity and acceleration: Def. (B-1), Def. (C-1)

$\vec{v} = d\vec{r}/dt$, $\vec{a} = d\vec{v}/dt$

Motion along a straight line: Eq. (D-5), Eq. (D-7), Eq. (D-9)

If $\vec{v}$ = constant, $\Delta\vec{r} = \vec{v}\Delta t$

If $\vec{a}$ = constant, $\Delta\vec{v} = \vec{a}\Delta t$, $\Delta\vec{r} = \vec{v}_A\Delta t + (1/2)\vec{a}(\Delta t)^2$

Gravity due to the earth: Rule (E-1), Rule (E-2)

Near the earth, every particle has a constant downward acceleration independent of all properties of the particle. ($\vec{g} \approx 10\,\text{meter/sec}^2$.)

NEW CAPABILITIES

You should have acquired the ability to :

(1) Understand these relations: (a) The definitions of velocity, speed, and acceleration (Sects. B and C); (b) The relations describing motion with constant [or zero] acceleration along a straight line, $\Delta\vec{v} = \vec{a}(\Delta t)$ and $\Delta\vec{r} = \vec{v}_A(\Delta t) + (1/2)\vec{a}(\Delta t)^2$ [or $\Delta\vec{r} = \vec{v}(\Delta t)$] (Sec. D, [p-3]).

(2) Find the position vector of a particle relative to a coordinate system, and decide whether the particle is moving relative to this system (Sec. A, [p-1]).

(3) Describe a particle's velocity and acceleration from information about its speed and path (Sects. B and C, [p-2]).

(4) Use the relations describing motion with constant acceleration along a straight line (a) to find initial or final values of position or velocity (Sec. D, [p-4]); (b) to solve systematically problems involving the vertical motion of particles subject only to gravitational interaction with the earth (Sec. E, [p-5], [p-6]).

Now: Go to tutorial section F

SECT.
G PROBLEMS

G-1 *Effect of a safety barrier on accelerations in a crash:* The magnitude of a car's acceleration in a crash critically affects the probability of injury to the occupants. Suppose a car has an initial velocity $\vec{v}_A$ and that during the crash it moves with a constant acceleration $\vec{a}$, coming to rest after a time Δt. (a) Find an algebraic expression for the magnitude a of the car's acceleration in terms of the initial speed v_A and the distance $|\Delta\vec{r}|$ traveled by the car during the time Δt. (b) Suppose two cars each have the same initial speed of $30\,\mathrm{m/s}$, but car A strikes a solid highway divider and travels through a distance $0.3\,\mathrm{m}$ before coming to rest. Car B strikes a safety barrier composed of flexible drums (Fig. G-1) so that it travels through a distance of $3\,\mathrm{m}$ before coming to rest. What is the magnitude of each car's acceleration? Even if the occupants are restrained by seat-belts, injury is very likely to occur if the magnitude of a car's acceleration exceeds $4 \times 10^2\,\mathrm{m/s^2}$. *(Answer: 125) (Suggestion: [s-8])*

Note: Tutorial section G includes further biological applications of the description of motion.

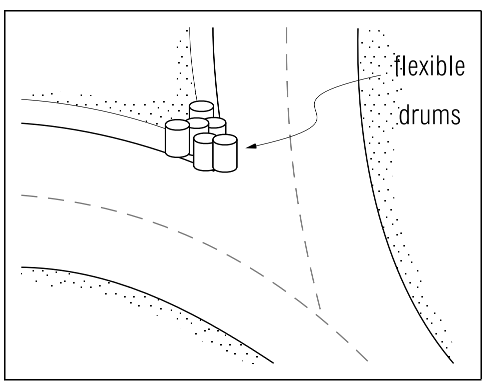


Fig. G-1.

TUTORIAL FOR B

UNDERSTANDING THE DEFINITIONS OF VELOCITY AND SPEED

b-1 *PLAN:* The capability "Understand the definitions of velocity and speed," means: be able to demonstrate this understanding by answering about these relations the questions on understanding listed in Appendix B and discussed in Unit 404. Answering the following questions (based on Appendix B) should help you develop this capability.

b-2 *STATEMENT:* Write an equation or brief sentence defining each of these quantities:

► velocity ________________________________ .

speed ________________________________ .

(Answer: 3)

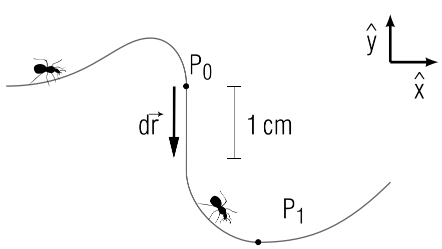
b-3 *EXAMPLE:* An ant crawling along the path indicated in the following drawing reaches the point P_0 at time t_0, and then moves through the displacement $d\vec{r}$ during the small enough time interval $dt = 0.5\,\text{sec}$.

What is the velocity $\vec{v}$ of the ant at t_0?

$\vec{v} =$ ______________________ .

Indicate the *directions* of the ant's velocity as it passes the points P_0 and P_1 by drawing from each point an arrow about one inch long.

►



(Answer: 5) (Suggestion: [s-4])

b-4 *PROPERTIES AND COMPARISON OF PROPERTIES:* Complete the following chart summarizing the properties of velocity and speed. Then compare these quantities by placing a check to the left of each property for which velocity and speed *differ*.

√	Property	Velocity	Speed
	Kind of quantity (vector or number)		
	SI unit		
	Common algebraic symbol		

After solving a problem, a quantitative result should be checked by asking if it is "reasonable" in magnitude. Therefore understanding a relation includes being able to state typical magnitudes for quantities in the relation.

Very roughly estimate a typical walking speed for a person, and a typical highway speed for a car, using the familiar unit mile/hour. Then use the relations $1.0\,\text{mile} = 1.6 \times 10^3$ meter and $1.0\,\text{hour} = 3.6 \times 10^3$ sec to state these speeds using the unit meter/sec (or m/s).

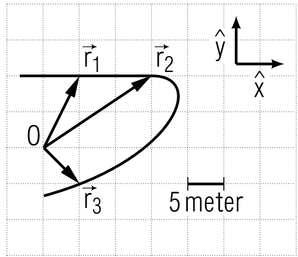
► walking speed: __________ mile/hour = __________ meter/sec.

highway speed: __________ mile/hour = __________ meter/sec.

(Answer: 7)

b-5 *INTERPRETATION:* Velocity is related to $d\vec{r}$ and dt. The following problem asks you to interpret these symbols by using information about positions and times.

A car on a flat winding road moves along the path shown in the following drawing, so that at the time $t_1 = 10\,\text{sec}$ its position vector is $\vec{r}_1$. After a small enough time interval, at $t_2 = 15\,\text{sec}$, the car's position vector is $\vec{r}_2$. Still later, at the time $t_3 = 35\,\text{sec}$, the car's position vector is $\vec{r}_3$.



What is the velocity $\vec{v}$ of the car at the time t_1?

$\vec{v} =$ ________________________ .

(Answer: 2)

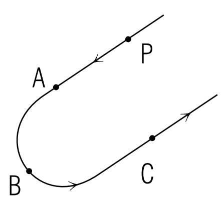
b-6 *MEANING OF $d\vec{r}$ AND dt:* In working the preceding example, a student using the relation $\vec{v} = d\vec{r}/dt$ *mistakenly* interprets dt as the difference $dt = t_3 - t_1 = 20\,\text{sec}$, and $d\vec{r}$ as the corresponding change in position $d\vec{r} = \vec{r}_3 - \vec{r}_1$.

Briefly explain why this interpretation of $d\vec{r}$ and dt is not correct.

►

(Answer: 4)

b-7 *COMPARING VELOCITY AND SPEED:* A model airplane travels with constant speed in the indicated direction along the following path. We compare the plane's motion at the point A, B, and C with its motion at P.



(1) At which points is the plane's *velocity* the same as it is at P?

A, B, C

(2) At which points is the plane's *speed* the same as it is at P?

A, B, C

(3) Indicate all of the following statements which correctly describe the velocity of a particle moving with constant speed.

(a) As the particle moves along a straight path segment, its velocity remains constant.

(b) At any point on the path, a particle's velocity has the same direction as a small displacement drawn along the path from that point.

(c) As a particle moves along a curved path, its velocity may change (even though its speed is constant).

► (a) (b) (c)

(Answer: 6) Now: If wrong, go to [s-3].

b-8 *RELATING QUANTITIES:* The following example illustrates how the definition of velocity can be used to find the values of quantities other than velocity.

A car approaching a stoplight has a velocity $15\,\text{m/s}\ \hat{x}$ at the instant the stoplight turns red. (The unit vector $\hat{x}$ has direction south.) The inattentive driver fails to notice a red light during the time interval 0.80 sec.

Through what displacement does the car travel during this small enough time interval?

► ____________________________

(Answer: 8)

b-9 *DEPENDENCE OF $\vec{v}$ ON $d\vec{r}$:* A boy rides his tricycle with varying speed along a curved path, so that he moves first through a displacement $d\vec{r}_1$ and later through a displacement $d\vec{r}_2$, both displacements requiring the *same* small enough time interval dt.

If the magnitude of $d\vec{r}_2$ is three times the magnitude of $d\vec{r}_1$, compare the corresponding speeds v_2 and v_1 by expressing v_2 as a number times v_1.

► $v_2 =$ ______________________ v_1

(Answer: 10)

b-10 *SUMMARY:* Velocity is both a rate and a vector. Because velocity is a rate, the time interval dt in the definition $\vec{v} = d\vec{r}/dt$ must always be small enough. Therefore $d\vec{r}$, and hence $\vec{v}$, are directed from the position of the particle along its path. Because velocity is a vector, its value must include a direction. If two velocities have different directions, they cannot be equal.

Speed, the magnitude of velocity, is a number, and does *not* include a direction.

Now: Go to the text problems for Section B. You may omit text problem B-2 on properties.

TUTORIAL FOR D

UNDERSTANDING RELATIONS FOR MOTION ALONG A STRAIGHT PATH

d-1 *STATEMENT:* This tutorial section should help you to develop an understanding of three important relations describing the motion of a particle moving with constant (or zero) acceleration along a straight line. [These relations are summarized by equations (D-5), (D-7) and (D-9) in the text.] In addition, this section should help you acquire the ability to apply these relations to find initial or final values for position or velocity.

(1) For each of the following descriptions of a relation, state an equation summarizing this relation.

► Describes the displacement $\Delta\vec{r}$ of a particle moving with constant velocity: ______________________

Describes the displacement $\Delta\vec{r}$ of a particle moving with constant acceleration: ______________________

Describes the change $\Delta\vec{v}$ in the velocity of a particle moving with constant acceleration: ______________________

(2) Express the changes $\Delta\vec{r}$ and $\Delta\vec{v}$ in terms of the initial and final values of position and velocity $\vec{r}_A$, $\vec{r}_B$, $\vec{v}_A$, and $\vec{v}_B$

► $\Delta\vec{r} =$ ______________________

$\Delta\vec{v} =$ ______________________

(Answer: 13)

d-2 *EXAMPLE:* Because people find large accelerations uncomfortable, the magnitude of an elevator's acceleration should not exceed about $2.0\,\mathrm{m/s^2}$. A high speed elevator travels with a constant upward acceleration $\vec{a}$ of this magnitude during a time interval $\Delta t = 3.0\,\mathrm{sec}$. Its initial velocity, at the beginning of this interval, is $\vec{v}_A = 2.0\,\mathrm{m/s}$ upward.

What is the change $\Delta\vec{v}$ in the elevator's velocity during these three seconds?

► $\Delta\vec{v} =$ ____________

Through what displacement $\Delta\vec{r}$ does the elevator move during these three seconds?

► $\Delta\vec{r} =$ ____________

Now suppose that the elevator travels with a constant velocity of 6.0 m/s downward during a time interval of 10 sec.

What is the change $\Delta\vec{v}$ in the elevator's velocity during these ten seconds?

► $\Delta\vec{v} =$ ____________

Through what displacement $\Delta\vec{r}$ does the elevator move during this time:

► $\Delta\vec{r} =$ ____________

(Answer: 15)

d-3 *PROPERTIES:* Summarize the properties of the quantities in these relations.

►

	Common symbol	Kind of quantity	SI unit
displacement:			
acceleration:			
velocity change:			
time interval:			

UNIT CONSISTENCY

Illustrating the unit consistency of a relation provides practice in using the units of the relation. To do this, just find the unit of each term in the relation, and verify that these units are consistent (i.e., the same).

Illustrate the unit consistency of the relation:

$\Delta\vec{r} = \vec{v}_A(\Delta t) + 1/2\vec{a}(\Delta t)^2$

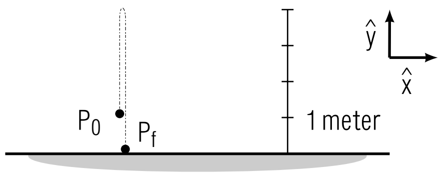
► $\Delta\vec{r}$: ____________

$\vec{v}_A(\Delta t)$: (________) (________) = ____________

$1/2\vec{a}(\Delta t)^2$: (________) (________)^2 = ____________

(Answer: 18)

d-4 *COMPARISON OF DISPLACEMENT AND DISTANCE TRAVELED:* A ball thrown upward and travels along a vertical path before landing on the ground as shown in this drawing:



What is the displacement $\Delta\vec{r}$ of the ball from its initial position P_0 to its final position P_f?

► $\Delta\vec{r} =$ ____________

What is the distance $|\Delta\vec{r}|$ between the initial and final positions?

► $|\Delta\vec{r}| =$ ____________

What is the distance traveled by the ball along its path between P_0 and P_f?

► ____________

(Answer: 20)

d-5 *COMPARISON OF SIMILAR RELATIONS:* The two relations $\vec{v} = d\vec{r}/dt$ and $\Delta\vec{r} = \vec{v}\Delta t$ [or $\vec{v} = \Delta\vec{r}/\Delta t$] look very similar. The following statements summarize the conditions under which each is applicable.

(1) $\vec{v} = d\vec{r}/dt$ describes motion during any *small enough* time interval dt. The velocity $\vec{v}$ may be constant or changing for times near this interval.

(2) $\Delta\vec{r} = \vec{v}\Delta t$ describes motion during *any* time interval Δt (however large), so long as $\vec{v}$ is constant during Δt.

Compare the relations $\vec{a} = d\vec{v}/dt$ and $\Delta\vec{v} = \vec{a}\Delta t$ by writing one of these equations after each of the following descriptions of applicability.

► (1) Applies to any time interval (however large) during which a particle's acceleration is constant: ______________

(2) Applies to any small enough time interval, even if the particle's acceleration is not constant for times near this interval: ______________

(Answer: 17)

d-6 *ORGANIZATION OF RELATIONS:* If you can work problems like the preceding ones in this section, you understand the relations describing motion with constant acceleration along a straight line, and the quantities appearing in these relations. The following problems ask you to apply these relations together with others applicable in the same situation.

FINDING INITIAL AND FINAL VALUES

The driver of a car, approaching a town with an initial velocity $\vec{v}_A = (24\,\mathrm{m/s})\hat{x}$ applies his brakes, so that the car moves with a constant acceleration $\vec{a} = (-1.4\,\mathrm{m/s^2})\hat{x}$ for 10 sec. (The unit vector $\hat{x}$ points east.)

What is the final velocity $\vec{v}_B$ of the car at the end of these 10 seconds?

► $\vec{v}_B =$ ______________

Through what displacement $\Delta\vec{r}$ does the car travel during this time?

► $\Delta\vec{r} =$ ______________

(Answer: 23) (Suggestion: [s-5].)

d-7 *USING AVERAGE VELOCITY TO FIND DISTANCE TRAVELED:* According to text section D of Unit 404, the distance traveled by a particle J with uniformly changing velocity during a time Δt is just the average of the particle's initial and final velocities multiplied by Δt.

If a particle moves with constant acceleration, is its velocity uniformly changing?

► yes, no

If it is correct to do so, apply this result from Unit 404 to the car described in tutorial frame [d-6], so as to find the distance traveled by the car from its initial and final velocities.

► distance traveled: ______________

Does this result agree with the value of $\Delta\vec{r}$ you found earlier?

► yes, no

(Answer: 26) Now: Go to text problem D-2. (You may omit problem D-1, Statement and example.)

TUTORIAL FOR E

A STRATEGY FOR SOLVING PROBLEMS IN VERTICAL MOTION

e-1 *A PROBLEM SOLVING STRATEGY:* The capabilities you should acquire through work with Unit 406 include being able to systematically solve problems involving the vertical motion of particles subject only to gravitational interaction with the earth. This means you should have a plan or strategy for solving such problems, so that your work is clear and easy to check (both for you and for another person). We suggest the following general strategy, which is illustrated by a sample problem in the next frame.

DESCRIPTION

Describe the essential features of the problem by:

(1) *Making a sketch* showing the moving particle and the path it follows. Include a unit vector for specifying vector values.

(2) *Summarizing known and desired information*, choosing a symbol for each quantity, and listing known values.

PLANNING

Choose the relation you will use to solve the problem. Because we consider here motion along a straight path with the constant acceleration $\vec{a} = \vec{g} = 10\,\mathrm{m/s^2}$ downward, both of the following relations are applicable. Choose the one which relates the known and desired information.

$$\Delta\vec{v} = \vec{a}(\Delta t),\ \Delta\vec{r} = \vec{v}_A(\Delta t) + \frac{1}{2}\vec{a}(\Delta t)^2$$

IMPLEMENTATION

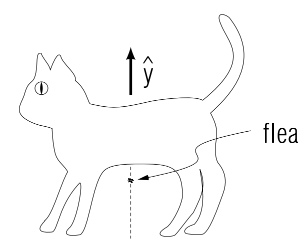
Write the equation you have chosen using your symbols for known and desired quantities. Then solve for the desired information. (To avoid recopying long numerical values, first find an algebraic expression for the desired quantity in terms of symbols. Then substitute numerical values, including units.)

CHECKING

Check each step of arithmetic and algebra. Then make sure that the unit you obtained is correct, that the direction (if any) makes sense, and that the magnitude is not unreasonably large or small.

e-2 *ILLUSTRATION OF THE PROBLEM-SOLVING STRATEGY:* A flea hoping to land on a cat jumps upward with an initial velocity of 2.0 m/s. If only gravity affects the flea's motion after leaving the ground (i.e., if the flea's interaction with the air is negligible), after what time interval does the flea reach its maximum height (where its velocity is zero)?

DESCRIPTION



Known:

initial velocity $\vec{v}_A = (2.0\,\mathrm{m/s})\hat{y}$

after a time Δt, $\vec{v}_B = 0$

Desired: the time interval Δt

PLANNING

The relation $\Delta\vec{v} = \vec{a}(\Delta t)$ relates known and desired quantities, if we use the definition $\Delta\vec{v} = \vec{v}_B - \vec{v}_A$, and the acceleration $\vec{a} = -10\,\mathrm{m/s^2}\hat{y}$ due to gravity.

IMPLEMENTATION

$\Delta\vec{v} = \vec{a}(\Delta t)$

Expressing this relation in terms of known and desired quantities we obtain:

$$\vec{v}_B - \vec{v}_A = \vec{a}(\Delta t).$$

We use $\vec{v}_B = 0$, take the magnitudes of both sides of the equation, and solve for the desired quantity Δt:

$$-\vec{v}_A = \vec{a}(\Delta t),$$

$$|\vec{v}_A| = |\vec{a}|(\Delta t),$$

$$\Delta t = \vec{v}_A/a$$

Substituting values for known quantities we obtain:

$$\Delta t = (2.0\,\mathrm{m/s})/(10\,\mathrm{m/s^2}) = 0.20\,\mathrm{sec}$$

CHECKING

Each step is correct. The unit obtained is a unit of time. A small hopping insect probably stays in the air less than one second, so the magnitude is reasonable.

Now: Return to text section E and systematically solve problems E-2 and E-3.

TUTORIAL FOR F

ORGANIZATION OF RELATIONS

f-1 *PLAN:* Unit 406 has introduced a large number of definitions and relations. To demonstrate the capabilities of this unit, you must be able to apply them without confusion. Therefore, read carefully the summary in text section F. If any of the definitions or relations are unfamiliar, or, if you cannot recall under what conditions each can be applied, take the time to review the indicated text section. When you have completed your review, test your knowledge by answering the following questions.

f-2 *DEFINITIONS:* The summary in text section F lists the following important terms defined in this unit: *particle, velocity, speed, acceleration.* Complete the following statements by writing one term from this list in each blank.

► (1) The direction of a particle's ____________ is along the path in the direction of the particle's motion.

(2) An object whose position is described by the position of a single point is called a ____________ .

(3) ____________ is the magnitude of velocity.

(4) ____________ is the rate of change of velocity with respect to time.

(Answer: 33)

f-3 *RELATIONS AND THEIR CONDITIONS:* These important relations were discussed in Unit 406:

$$\vec{v} = d\vec{r}/dt$$

$$\vec{a} = \vec{g} = 10\,\mathrm{m/s^2} \text{ downward}$$

$$\vec{a} = d\vec{v}/dt$$

$$\Delta\vec{v} = \vec{a}(\Delta t)$$

$$\Delta\vec{r} = \vec{v}_A(\Delta t) + 1/2\vec{a}(\Delta t)^2$$

Each of these relations can correctly be applied only under certain conditions. Summarize these conditions by using one or more of the preceding relations to answer each of these questions:

(1) Which relation(s) apply *only* to particles moving with constant or uniformly changing speed along a straight path?

► ________________________________ .

(2) Which relation(s) apply *only* to particles moving near the earth's surface under the sole influence of gravity?

► ________________________________ .

(3) Which relation(s) apply whenever dt is small enough?

► ________________________________ .

(4) Which relation(s) apply only to particles moving with constant acceleration along a straight path?

► ________________________________ .

(Answer: 30) (Suggestion: Check your ability to apply these relations by working text problem F-1.)

TUTORIAL FOR G

BIOLOGICAL APPLICATIONS OF THE DESCRIPTION OF MOTION

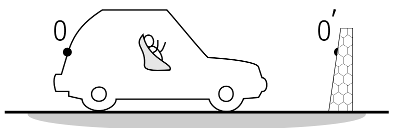
g-1 *ACCELERATION OF THE HEAD IN A FALL:* Injury of the head and brain occurs when the acceleration of the head has a large magnitude. In the following problem we estimate this magnitude for falls in which the head strikes the ground.

(a) We approximate the motion of a person's head by assuming that as the head and body fall together, the head moves along a vertical path with the gravitational acceleration $\vec{g}$. Express the speed v of the head just before it strikes the ground in terms of the height h from which it fell and known quantities. (b) During the time interval Δt in which the head is brought to rest by its contact with the ground, the head has an approximately constant upward acceleration $\vec{a}$ and moves through a downward displacement $\Delta\vec{r}$. Express the magnitude $|\vec{a}|$ of the head's acceleration in terms of the speed v, and the distance $|\Delta\vec{r}|$. (c) Combine the preceding results to express the magnitude a of the head's acceleration in terms of the height h from which it fell, the distance $|\Delta\vec{r}|$ through which it moves as it strikes the ground, and known quantities.

Use your expression for a to answer these questions: (d) Why do small children, although they often fall and strike their heads, rarely injure themselves? (e) A mother tells her son it is too dangerous for him to climb a tree over a stone patio, but he may more safely climb a tree of equal height over a grassy lawn. Is she right? Why? (f) A skater falls on ice so that $h = 1.5$ meter and $|\Delta\vec{r}| = 0.1$ cm. What is the magnitude of a for the skater's head as it strikes the ice? *(Answer: 32)*

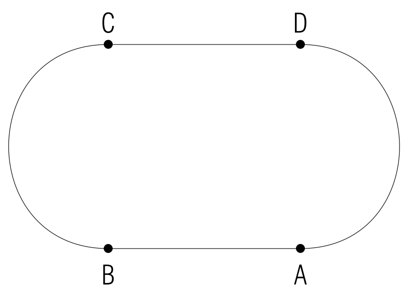
PRACTICE PROBLEMS

p-1 *DESCRIBING POSITION AND MOTION (CAP. 2):* The following drawing shows an infant in a car seat fastened to the inside of a car. Draw arrows indicating the position vectors of the infant relative to the origin O fixed at the rear of the car, and relative to the origin O' fixed on a railroad crossing gate ahead of the car.



As the car travels towards the gate, is the infant moving relative to the car? Is it moving relative to the gate? *(Answer: 1) Now: Return to text problem A-1 and make sure your work is correct.*

p-2 *RECOGNIZING ZERO AND NON-ZERO ACCELERATION (CAP. 3):* A jogger runs around the track shown in this drawing: running with increasing speed along the straight section from A to B, and then with constant speed from B to C and along the straight section from C to D.



As he travels along each of the following path segments, when is the jogger's acceleration zero? A to B, B to C, C to D. *(Answer: 11) Now: Return to tutorial frame [c-2] and make sure your work is correct.)*

p-3 *UNDERSTANDING RELATIONS FOR STRAIGHT LINE MOTION (CAP. 1B):* The brakes fail for a car initially parked at rest, and it rolls with a constant acceleration $\vec{a} = 1.0\,\text{m/s}^2\hat{x}$ (where $\hat{x}$ is a unit vector

directed down a hill). What time interval is required for the car to travel through a displacement $\Delta\vec{r} = 4.5\,\text{meter})\hat{x}$ from the car's initial position to the position of a pedestrian who wishes to leap out of the way before the car arrives)? *(Answer: 21) (Suggestion: If you need help, review text problem D-2.))*

p-4 *RELATING VALUES OF POSITION AND VELOCITY (CAP. 4A):* A driver applies his brakes so that his car moves with a constant acceleration $\vec{a} = (2.0\,\text{m/s}^2)$ north, and comes to rest (with a final velocity of 0) after a time interval of 6.0 sec. What was the initial velocity of the car? *(Answer: 19) (Suggestion: If you need help, review text problem D-3.))*

p-5 *SOLVING PROBLEMS INVOLVING VERTICAL MOTION (CAP. 4B):* Visitors to the Empire State building are warned *not* to drop anything from the observation decks. The following calculations illustrate why. Suppose a stone is released at rest from the observation deck a distance $|\Delta\vec{r}| = 245\,\text{meter}$ above the ground. We estimate the stone's motion by assuming that its interaction with the air is negligible, and so it moves subject only to the gravitational interaction. (Express vector quantities by using the downward unit vector $\hat{y}$.) (a) What is the time interval required for the stone to reach the ground? (b) Use this time interval to find the final velocity of the stone just before it strikes the ground. (Express your answer using the unit mile/hour where $1\,\text{m/s} = 2.3\,\text{mile/hour}$.) *(Answer: 24) (Suggestion: Review tutorial section E or text problems E-2 and E-3.)*

p-6 *SOLVING PROBLEMS INVOLVING VERTICAL MOTION (CAP. 4B):* A boy wants to know with what speed he can throw a ball upward. To find out, he throws a ball with an (unknown) initial upward velocity $\vec{v}_A$ while a friend measures the time interval of 3.0 sec until the ball returns to its original position in the boy's hand. Thus the displacement of the ball during this time interval is $\Delta\vec{r} = 0$. If during this time the ball moves subject only to gravitational interaction with the earth, what is the initial velocity of the ball? (Specify vectors by using an upward unit vector $\hat{y}$.) *(Answer: 27) (Suggestion: Review tutorial section E or text problems E-2 and E-3.))*

SUGGESTIONS

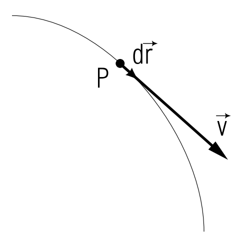
s-1 *(Text problem C-1):* The velocity change $d\vec{v}$ is the *vector* difference $\vec{v}_c - \vec{v}_0$. Therefore, to find $d\vec{v}$, draw the arrows representing $\vec{v}_0$ and $\vec{v}_c$ with their tails together, and construct their difference.

Recall from text section B that the velocity of a particle is always directed along its path, i.e., in the same direction as a very small displacement drawn from the position of the particle along the path.

s-2 *(Text problem F-1):* A particle's velocity is always along its path, i.e., it has the same direction as a very small displacement from the position of the particle along the path. A particle's acceleration is always directed from the particle's position towards the inside of any curved path. In particular, if the particle moves with constant speed along a circular path, its acceleration is directed towards the center of the circle.

s-3 *(Tutorial frame [b-7]):* Velocity is a vector with values including a magnitude and a direction. Therefore a particle's velocity is the same at two points on its path only if: (a) The magnitude of the velocity (i.e., the speed) is the same at these points. (b) The direction of the velocity (i.e., the particle's direction of motion) is the same at these two points.

s-4 *(Tutorial frame [b-3]):* The velocity $\vec{v}$ of a particle at a point P is directed along its path, i.e., in the same direction as a very small displacement $d\vec{r}$ drawn along the path from P.



s-5 *(Text problem D-3, and tutorial frame [d-6]):* Rewrite the relation $\Delta\vec{v} = \vec{a}(\Delta t)$, describing motion with constant acceleration along a straight line, by expressing the change $\Delta\vec{v}$ in terms of the initial and final velocities $\vec{v}_A$ and $\vec{v}_B$. You can simplify computation, and avoid problems with signs, by first finding an algebraic expression for the desired quantity $\vec{v}_B$, and then substituting known values into this expression.

s-7 *(Tutorial frame [c-2]):* The change $d\vec{v}$ is the change in velocity a *vector* quantity. Thus $d\vec{v}$ is zero only if *both* the magnitude of the car's velocity (its speed) *and* the direction of its velocity remain constant. When a car travels along a curved road section, the direction of its velocity does change. When it travels along a straight road section, this direction does not change.

Now: Go to [p-2].

s-8 *(Text problem G-1):* Eliminate Δt from the equation $\Delta\vec{r} = \vec{v}_A(\Delta t) + (1/2)\vec{a}(\Delta t)^2$ by using the relation between magnitudes $|\Delta\vec{v}| = a(\Delta t)$, where $|\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = |\vec{v}_A|$. Then solve for the magnitude a, remembering that the vectors $\vec{v}_A$ and $\vec{a}$ have opposite directions.

s-9 *(Text problem C-2):* To find $\vec{a} = d\vec{v}/dt$, you need values for $d\vec{v} = \vec{v}_c - \vec{v}_0$, and for $dt = t_c - t_0$. To subtract the velocities (which are vectors) either draw arrows and subtract them graphically, or subtract the expressions for these velocities in terms of $\hat{x}$ to obtain an expression for $d\vec{v}$ in terms of $\hat{x}$. With either method, be sure that you subtract $\vec{v}_0$ from $\vec{v}_c$, and be careful of the resulting sign.

s-11 *(Text problem C-3):* Estimate a reasonable speed (magnitude of velocity) first in terms of the familiar unit mile/hour. Then use the relations 1.0 mile $= 1.6 \times 10^3$ meter and 1 hour $= 3.6 \times 10^3$ sec to express your estimate in terms of meter/sec.

To estimate a reasonable magnitude for acceleration, use the common observation that the velocity of a car can change from 45 mile/hour (north) to 65 mile/hour (north) in a few seconds (let's estimate 5 sec).

What is the resulting estimate of the magnitude $|d\vec{v}|$ of the change in the car's velocity?

► $|d\vec{v}| =$ ____________ mile/hour $=$ ____________ meter/sec.

What is the resulting estimate of the magnitude $a = |d\vec{v}|/|dt|$ of the car's acceleration?

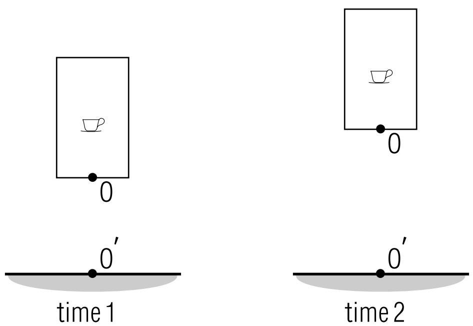
► $a =$ ____________ meter/sec^2

(Answer: 12) Now: Return to text problem C-3.

s-13 *(Text problem A-1):* The position vector of a particle (relative to a reference frame with origin O) is the displacement from O to the particle.

(1) Use the sketch at the left to draw arrows indicating the position vectors $\vec{R}$ and $\vec{R}\prime$ of the cup relative to O and relative to O'.

►



The sketch at the right shows the elevator, cup, and origins O and O' at a slightly later time.

(2) Comparing the two times shown in the sketches, has the value of the position vector $\vec{R}$ changed? Has the value of the position vector $\vec{R}\prime$ changed?

► $\vec{R}$: changed, unchanged

$\vec{R}\prime$: changed, unchanged

A particle moves (relative to a reference frame) if its position vector relative to that frame changes with time.

(3) Is the cup moving relative to the elevator? Is it moving relative to the building?

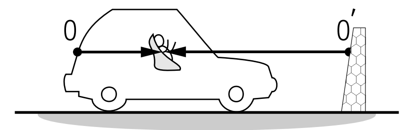
► Relative to the elevator: moving, not moving

Relative to the building: moving, not moving

(Answer: 9) Now: Go to [p-1].

ANSWERS TO PROBLEMS

1.



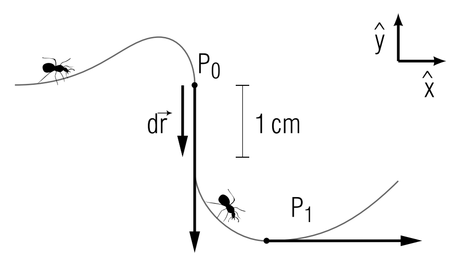
The infant is not moving relative to the car, but is moving relative to the gate.

2. $\vec{v} = 2\,\mathrm{m/s}\hat{x}$ (Answer must include direction.)

3. velocity : $\vec{v} = d\vec{r}/dt$; speed: $|v| = |d\vec{r}/dt| = |d\vec{r}|/|dt|$, or speed is the magnitude of velocity. (Equivalent statements in terms of symbols *you* can interpret are also correct.)

4. The time interval chosen as "dt" is not small enough, because the path of the particle is not even approximately straight during this interval.

5. $\vec{v} = (-2\,\mathrm{cm/sec})\hat{y}$ (Be sure you have specified a direction.)



6. (1) A (2) A, B, C (3) All three statements are correct.

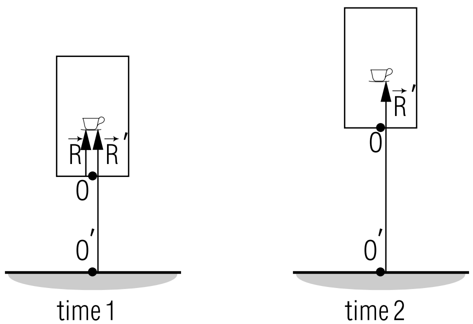
7.

Check:	*Property*:	*Velocity*:	*Speed*:
√	kind (vector, number)	vector	number
	SI unit	meter/sec	meter/sec
√	common algebraic symbol	$\vec{v}$	v

Walking speed: any value between 2 and 5 mile/hour, or between 1 and 2 m/s; Highway speed: any value between 40 and 70 mile/hour or between 18 and 30 m/s.

8. $d\vec{r} = \vec{v}(dt) = (12\,\text{meter})\hat{x}$, or 12 meter south.

9.(1)



(2) $\vec{R}$: unchanged, $\vec{R}\prime$: changed

(3) Relative to elevator: not moving; Relative to the building: moving

10. $\vec{v}_2 = 3\vec{v}_1$

11. Acceleration is zero only for the motion from C to D.

12. $|d\vec{v}| = 20\,\text{mile/hour} = 9$ (or 8.9) m/s, $a = 2\,\text{m/s}^2$

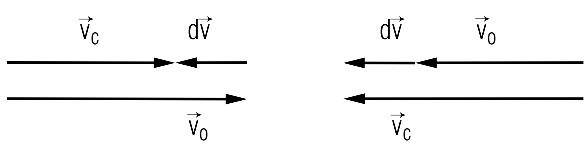
13.(1) $\Delta\vec{r} = \vec{v}(\Delta t)$, $\Delta\vec{r} = \vec{v}_A(\Delta t) + 1/2\vec{a}(\Delta t)^2$, $\Delta\vec{v} = \vec{a}(\Delta t)$

(2) $\Delta\vec{r} = \vec{r}_B - \vec{r}_A$, $\Delta\vec{v} = \vec{v}_B - \vec{v}_A$

14. (a), (c)

15. $\Delta\vec{v} = 6.0\,\text{m/s}$ upward, $\Delta\vec{r} = 15\,\text{meter}$ upward, $\Delta\vec{v} = 0$, (Velocity is constant.) $\Delta\vec{r} = 60\,\text{meter}$ downward.

16.



left

17. (1) $\Delta\vec{v} = \vec{a}(\Delta t)$ or $\Delta\vec{v}/\Delta t = \vec{a}$ (2) $\vec{a} = d\vec{v}/dt$

18.

Quantity:	*Common Symbol*:	*Kind of Quantity*:	*SI Unit*:
displacement:	$\Delta\vec{r}$	vector	meter
acceleration:	$\vec{a}$	vector	meter/sec^2
velocity change:	$\Delta\vec{v}$	vector	meter/sec
time interval:	Δt	number	sec

$\Delta\vec{r}$: meter; $\vec{v}_0(\Delta t)$: (meter/sec)(sec) = meter
$1/2\vec{a}(\Delta t)^2$: (meter/sec^2)(sec)^2 = meter

19. 12 m/s south (Check that direction is correct.)

20. $\Delta\vec{r} = (-1\,\text{meter})\hat{y}$ or 1 meter downward, $|\Delta\vec{r}| = 1\,\text{meter}$, 7 meter

21. 3.0 sec

22. larger, (d)

23. $\vec{v}_B = 10\,\text{m/s}\hat{x}$, $\Delta\vec{r} = (170\,\text{meter})\hat{x}$

24. a. 7.0 sec
    b. $70\,\text{m/s}\ \hat{y} = (1.6 \times 10^2\,\text{mile/hour})\hat{y}$

25. a. faster
    b. Centrifuge 1
    c. twice as large

26. yes, 170 meter. Yes, the magnitude of $\Delta\vec{r}$ is the distance traveled.

27. $\vec{v}_A = 15\,\text{m/s}\hat{y}$

28. $a = 1.5\,\text{m/s}^2$, $v = 0.47\,\text{m/s}$

29. $v = 2\pi r/T$, $v = 2\pi r\nu$

30.(1) $\Delta v = a(\Delta t)$, $\Delta\vec{r} = \vec{v}_A(\Delta t) + 1/2\vec{a}(\Delta t)^2$

(2) $\vec{a} = \vec{g} = 10\,\text{m/s}^2$ downward

(3) $\vec{v} = d\vec{r}/dt$, $\vec{a} = d\vec{v}/dt$

(4) $\Delta\vec{v} = \vec{a}(\Delta t)$ $\Delta\vec{r} = \vec{v}_A(\Delta t) + 1/2\vec{a}(\Delta t)^2$

31. $a = 10\,\text{m/s}^2$, as expected for an object moving due to gravitational interaction near the earth's surface.

32. a. $v = \sqrt{2gh}$

b. $a = v^2/(2|\Delta\vec{r}|)$

c. $a = gh/(|\Delta\vec{r}|)$

d. h is small for a short child.

e. Yes. $|\Delta\vec{r}|$ is larger for striking grass than for striking stone.

f. 1.5×10^4 m/s

33. (1) velocity (2) particle (3) speed, (4) acceleration

101. a. (A and B)

b. (A and B), (B and C)

102. a. 1.5 meter upward

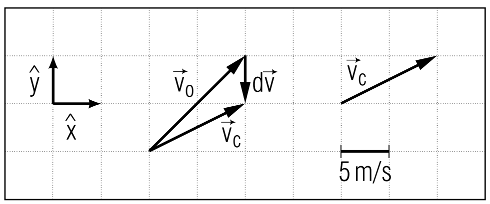
b. 150 meter upward.

c. The cup is not moving relative to the elevator, but is moving relative to the building.

103. (a)

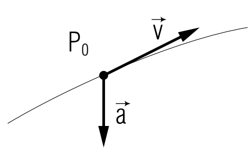
104. a. $\vec{a} = d\vec{v}/dt$ (or equivalent in terms of other vector symbols).

b.



c. $\vec{a} = -10\,\text{m/s}^2\hat{y}$ (Check unit and direction)

d.



e. Velocity parallel to the path, acceleration towards the inside of the path.

105.

Position:	*Speed*:	*Velocity*:	*Acceleration*:
above equil.	decreasing	up	down
above equil.	zero	zero	down
above equil.	increasing	down	down
below equil.	decreasing	down	up
below equil.	zero	zero	up
below equil.	increasing	up	up

106. a. $\vec{v} = d\vec{r}/dt$ (or equivalent in terms of other vector symbols)

b. Velocity: $-50\,\text{m/s}\hat{x}$ Speed: 50 m/s

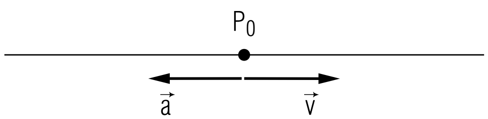
107. a. no, direction of $\vec{v}$ changes.

b. A: $7\,\text{m/s}\ \hat{y}$, B: $7\ \text{m/s}\ \hat{x}$, C: $-7\,\text{m/s}\hat{y}$

c. $d\vec{r} = 0$, Velocity is zero.

108. a. $\vec{a} = -3\,\text{m/s}^2\hat{x}$ or $3\,\text{m/s}^2$ toward the left

b.



c. Velocity and acceleration both parallel to the path.

109.

	Velocity:	*Speed*:	*Different Properties*:
Kind:	vector	number	yes
SI Unit:	meter/sec	meter/sec	no
Algebraic Symbol:	$\vec{v}$	v	yes

110.

	Acceleration:	*Velocity*	*Different Properties*:
Kind:	vector	vector	no
SI unit:	meter/sec^2	meter/sec	yes
Algebraic Symbol:	$\vec{a}$	$\vec{v}$	yes

Direction: Acceleration is parallel to a straight path and towards the inside of a curved path; Velocity is always parallel to the path.

c. 25 m/s

d. $2.5\,\mathrm{m/s^2}$

111. a. All have the acceleration $10\,\mathrm{m/s^2}$ downward

 b. $\vec{a} = 0$. No, it also interacts with the ground surface.

112. a. $\Delta\vec{v} = \vec{a}(\Delta t)$, $\Delta\vec{v} = 20\,\mathrm{m/s}$ downward

 b. $\Delta\vec{r} = \vec{v}_A(\Delta t) + 1/2\vec{a}(\Delta t)^2$, $\Delta\vec{r} = 20$ meter downward

 c. $\Delta\vec{r} = \vec{v}(\Delta t)$, $\Delta\vec{r}\prime = 1.1 \times 10^2$ meter downward

113. a. $a = v^2/r$

 b. At A: $\vec{a} = 1.0\,\mathrm{m/s^2}\hat{x}$, $\vec{v} = 1.2\,\mathrm{m/s}\hat{y}$; At B: $\vec{a} = 1.0\,\mathrm{m/s^2}\hat{y}$, $\vec{v} = -1.2\,\mathrm{m/s}\hat{x}$

 c. $1.0\,\mathrm{m/s}\ \hat{x}$, $-1.2\,\mathrm{m/s}\hat{y}$

 d. yes, no

114. a. 30 sec

 b. $\Delta\vec{r} = (540\,\text{meter})\ \hat{y}$

115. $\vec{v}_B = \vec{v}_A + \vec{a}(\Delta t) = 0 + (10\,\mathrm{m/s^2})(1.0\,\mathrm{sec})\hat{y} = 10\,\mathrm{m/s}\hat{y}$. Your solution should be systematic and clear, so that another person could understand it readily.

116. $a = 3 \times 10^{-2}\,\mathrm{m/s^2}$, $a/g = 3 \times 10^{-3}$

117. a. $25\,\mathrm{m/s}\ \hat{x}$

 b. $\Delta\vec{r} = (150\,\text{meter})\hat{x}$

118. a. Larger for "tight curve"

 b. $a_A = 9a_B$

119. a. $|\Delta\vec{r}| = (1/2)g(\Delta t)^2$

 b. 20 meter

 c. four times as high

120. $a = 3 \times 10^6\,\mathrm{m/s^2}$, $a/g = 3 \times 10^5$

121. a. at C: velocity and acceleration along $\hat{x}$; at E: velocity along $\hat{y}$, acceleration opposite to $\hat{x}$.

 b. at E: $(0.2\,\mathrm{m/s})\hat{y}$, $(-0.4\,\mathrm{m/s^2})\hat{x}$

 at G: $(-0.2\,\mathrm{m/s})\hat{y}$, $(0\,\mathrm{m/s^2})\hat{x}$ or 0

 c. No unless you regard a straight path as a circular path with an infinitely large radius r. Then $a = v^2/r = 0$ correctly describes the motion between F and A.)

122. $40\,\mathrm{m/s}$

123. a. decrease

 b. $v' = (1/1.50)v_0 = 0.667v_0$

124. 55 mile/hour

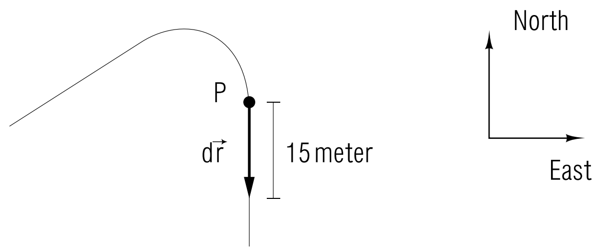
125. a. $a = (v_A)^2/(2|\Delta r|)$

 b. For car A, $a = 1.5 \times 10^3\,\mathrm{m/s^2}$. For car B, $a = 1.5 \times 10^2\,\mathrm{m/s^2}$.

MODEL EXAM

1. See Output Skills K1-K3 in this module's *ID Sheet*.

2.



A bird flies along the path shown in the diagram, as viewed from above.

The bird moves through the displacement $d\vec{r}$ from the point P during a small enough time interval of 2.0 seconds.

a. What is the bird's velocity at the time it passes point P?

b. What is the bird's speed at the time it passes point P?

3. An elevator moves downward with decreasing speed before coming to rest at a floor. What is the direction of the elevator's acceleration?

4. A particle moves with a constant acceleration of $2.5\,\text{m/s}^2\,\hat{x}$. The particle's initial velocity is $10.4\,\text{m/s}\,\hat{x}$.

a. Determine the particle's velocity after 3.0 seconds.

b. Calculate the distance traveled by the particle.

Brief Answers:

1. See this module's *text*.
2. a. 7.5 m/s, south

   b. 7.5 m/s
3. upward
4. a. $18\,\text{m/s}\,\hat{x}$

   b. 42 meters