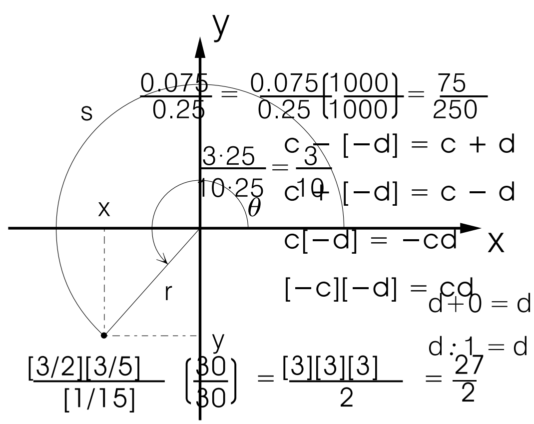
MISN-0-401

REVIEW OF BASIC MATHEMATICAL SKILLS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

REVIEW OF BASIC MATHEMATICAL SKILLS

by

F. Reif, G. Brackett and J. Larkin

CONTENTS

Title: **Review of Basic Mathematical Skills**

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Length: 1 hr; 80 pages

Output Skills (Knowledge):

K1. Vocabulary: real number, magnitude of a number, sign of a number, base, exponent, power, scientific notation, radian, degree, similar triangles, Pythagorean theorem, Cartesian coordinates, acute angle, sine, cosine, tangent, algebraic symbols, algebraic expressions, algebraic quantities.

Output Skills (Rule Application):

R1. Perform elementary arithmetic.

R2. Calculate the value of a number raised to a power.

R3. Express numbers in scientific notation and use them in calculations.

R4. Solve problems using the definitions of "radian" and "similar triangles."

R5. Use a calculator to calculate common trigonometric functions and their inverses, and, given two appropriate quantities among the angles and sides of a right-angle triangle, calculate other angles and/or sides.

R6. Solve algebraic expressions.

Post-Options:

1. "Scientific Goals and Methods" (MISN-0-402).
2. "Physical Description and Measurement" (MISN-0-403).

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

MISN-0-401

REVIEW OF BASIC MATHEMATICAL SKILLS

Abstract:
In the present unit we shall exploit our familiarity with vectors to discuss useful ways of describing motion. Our knowledge of an effective *description* of motion will then prepare us for the more ambitious task of formulating a theory for the *prediction* of motion.

SECT.
A INTRODUCTION AND ADVICE

THE REASON WHY

Some basic mathematical skills are used so often in introductory physics that they must be more than familiar to you: they must be automatic. If you have lost an easy and accurate facility with any of these basic skills, you should spend time to regain that facility. Otherwise, you will find your progress in learning new physics concepts continually cursed with old mathematical difficulties.

HOW TO STUDY THIS MODULE

Part II of this module contains a summary of important definitions, relations and techniques. Part III contains a self-assessment test: Your skills are "up-to-snuff" if you can easily complete this test within the indicated time without reference to any other material and without a single error. Part III also contains answers and solutions for each of the test problems. Part IV contains extra practice problems along with their answers. To use these materials to your best advantage, follow these steps:

1. Carefully go over the definitions, relations and techniques in Part II. If any statement seems unfamiliar, review a mathematics text until you understand it.

2. Work carefully through the Self-Assessment Test in Part III, allowing yourself only the indicated time for each section. Do not check your answers until you have completely finished the test.

3. Check your answers using the Answers following the test. All acceptable answers are given so if your answer does not agree it is incorrect.

4. If you had difficulty with a problem (even if you worked it correctly) or if you made an error on a problem, look up its solution in the Solutions section. After reading the solution and doing any other review you think necessary, try working the practice problems in Part IV. Continue your review or seek extra help until you can do such problems correctly and confidently.

USING THE TESTS EFFECTIVELY

It is worth your while to brush up your basic mathematics now. You may be tempted to look at the test and answers and say, "I remember how to do that." If you work the self test correctly you can be sure that you really do remember. Otherwise, you may deceive yourself and pay the exasperating price of a lot of "simple" errors later. Unfortunately, "simple" errors are no less wrong for being "simple."

SECT.

B DEFINITIONS, RELATIONS, TECHNIQUES

ELEMENTARY ARITHMETIC

1a. The Real Number Line. Real or algebraic numbers include all positive and negative numbers and zero. These numbers can be indicated on a line with 0 at a point called the origin. Then the positive numbers are on one side of the origin and the negative numbers on the other, as demonstrated in Figure B-1.

1b. Magnitude and Sign of Real Numbers. The "magnitude of a number r," represented by $|r|$, is its distance from the origin on the number line, (a positive number). A real number also has a sign telling on which side of the origin it is located.

$$|5| = 5; \qquad |-5| = 5\,. \tag{B-1}$$

1c. Definition of Larger and Smaller. If the number line is positioned so that negative numbers are on the left and positive numbers are on the right, then:
(a) c is larger than d ($c > d$) if c is to the right of d.
(b) c is smaller than d ($c < d$) if c is to the left of d.
(Notice that a positive number is always larger than a negative number. As in Figure B-2)

1d. Properties of Addition and Multiplication. Multiplication is written in several ways:

(c multiplied by d) $= c \times d$ or $c \cdot d$ or $(c)(d)$ or cd.

Addition and multiplication satisfy these properties:

Commutative property: $a + b = b + a$, $ab = ba$

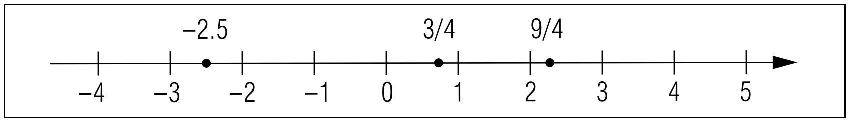


Fig. B-1: Graphical representation of a real number line.

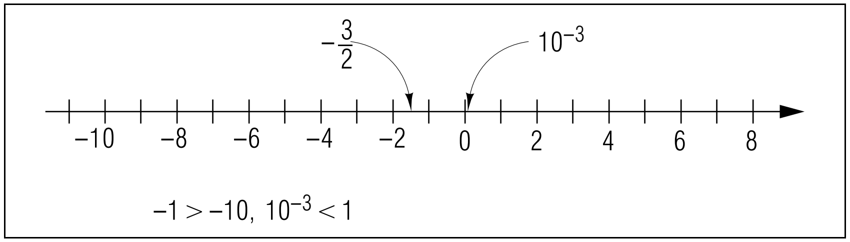


Fig. B-2: Example demonstrating how a positive number is larger than a negative number.

Associative property: $(a+b)+c = a+(b+c)$, $(ab)c = a(bc)$

Distributive property: $a(b+c) = ab + ac$.

1e. Operations with Positive and Negative Numbers. If c and d are any numbers:

$$c + (-d) = c - d$$

$$c - (-d) = c + d$$

$$c(-d) = -cd$$

$$(-c)(-d) = cd$$

$$d + 0 = d$$

$$d \cdot 1 = d$$

1f. Order of Operations. If several operations must be done to evaluate one expression, they are performed in the order: multiplication and division, then subtraction, then addition. However, parenthesis ( ), or brackets, [ ], { }, indicate that the operations inside the brackets should be performed first. If more than one pair of brackets appear, the innermost pair is evaluated first.In evaluating fractions, usually the numerator and denominator should each be computed separately before dividing the results.

1g. Addition and Subtraction of Fractions. Two fractions can be added or subtracted *only* if they have the same denominator. To change the denominator of any fraction, multiply it by a fraction equal to 1.

$$\frac{5}{6} + \frac{1}{4} = \frac{5}{6}\left(\frac{2}{2}\right) + \frac{1}{4}\left(\frac{3}{3}\right) = \frac{10}{12} + \frac{3}{12} = \frac{13}{12}$$

The common denominator, 12, must be a multiple of each original denominator, 6 and 4.

1h. Simplifying Fractions. Any fraction can be multiplied by 1 without changing its value ($b \cdot 1 = b$). This property can be used in many ways to simplify fractions.

(i) A fraction is in *simplest fraction form* if the numerator and denominator have no factors in common. The fraction 15/25 is not in simplest fraction form because 15 and 25 have the common factor 5. To change any fraction to simplest fraction form, find all the common factors, then use the property $(c/c) = 1$ and $b \cdot 1 = b$.

$$\frac{15}{25} = \frac{3 \cdot 5}{5 \cdot 5} = \left(\frac{3}{5}\right)\left(\frac{5}{5}\right) = \frac{3}{5} \cdot 1 = \frac{3}{5}$$

(ii) If either the numerator or denominator (or both) contain fractions, the expression can be simplified by multiplying by 1 in the form (c/c) where c is a multiple of each denominator of each fraction which appears.

$$\frac{(3/2)(3/5)}{(1/15)} = \frac{(3/2)(3/5)}{(1/15)}\left(\frac{30}{30}\right).$$

The expression $\left(\frac{30}{30}\right)$ is used because 30 is a multiple of $2, 5$, and 15.

$$\frac{(3/2)(3/5)}{(1/15)}\left(\frac{30}{30}\right) = \frac{(3)(3)(3)}{2} = \frac{27}{2}.$$

(iii) A fraction including decimals can be simplified by multiplying by (c/c) where c is a power of 10.

$$\frac{.075}{.25} = \frac{.075}{.25}\left(\frac{1000}{1000}\right) = \frac{75}{250} = \frac{3 \cdot 25}{10 \cdot 25} = \frac{3}{10}.$$

(iv) The quotient of two fractions can be written as a fraction, for example:

$$\frac{2}{3} \div \frac{2}{9} = \frac{2/3}{2/9}.$$

This expression can then be simplified by multiplication by $1 = (9/9)$, since 9 is a multiple of each denominator.

$$\frac{2/3}{2/9}\left(\frac{9}{9}\right) = \frac{2\cdot 3}{2} = 3\ .$$

1i. Definition of Proportional. Two numbers A and B are proportional, written $A \propto B$, if changing B by multiplying it by a certain factor implies A also changes by the same factor.

POWERS

2a. Definitions. The expression a^m is read "a exponent m" or "a to the mth power." The number a is called the base and m the exponent or power.

$$a^m = \underbrace{a \cdot a \cdot a \cdots a}_{m \text{ factors}}$$

$$a^{-m} = 1/a^m$$

2b. Multiplication and Division of Powers.

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

2c. Special Powers.

$$a^0 = 1$$

$$a^1 = a$$

2d. Powers of Powers.

$$(a^m)^p = a^{mp}$$

$$(a^m b^n)^p = a^{mp} b^{np}$$

2e. Fractional Powers.

$$a^{1/m} = x \text{ if } x^m = a$$

$$8^{1/3} = 2 \text{ because } 2^3 = 8$$

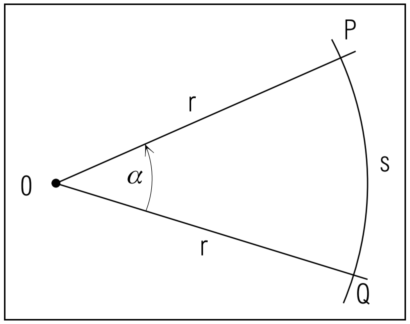


Fig. B-3: Arc $\overset{\frown}{PQ}$ with radius r.

SCIENTIFIC NOTATION

3a. Definition. A number in scientific notation consists of two factors. The left factor is a decimal number (positive or negative) with one non-zero digit to the left of the decimal point. The right factor is an integral power of ten. Examples: 3.2×10^2, 4.75×10^2, -2.0×10^1

3b. Order of Operations: IMPORTANT. In the absence of parentheses or brackets, exponentiation takes precedent over other operations, e.g. $-10^2 = -100$ (in this example, exponentiation is carried out before negation).

GEOMETRY

4a. Definition of Radian Measure[]. Consider two straight lines which intersect at a point O to form an angle α (Fig. B-3). If s is the length of an arc $\overset{\frown}{PQ}$ of any radius r, drawn about O, which intersects the lines at points P and Q, then the measure of angle α in radians is $\alpha = s/r$. This is commonly written $\alpha = s/r$ rad. However the lengths s and r should be in the same units, so α has no units. The abbreviation rad is a reminder that α is a radian measure, even though it has no units.

4b. Radian Measure of Common Angles. Several common angles are used so frequently, that it becomes useful to have them committed to memory.

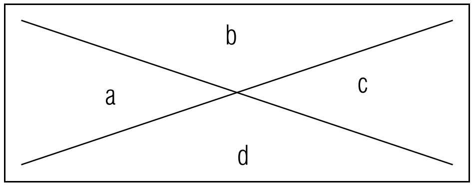
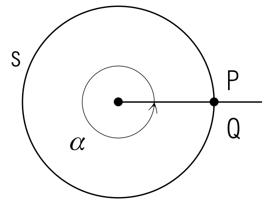
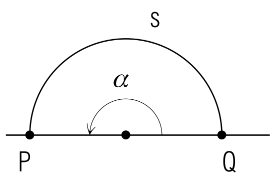


Fig. B-5.

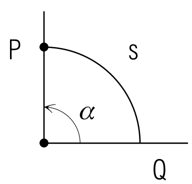
The radian measure of a full circular angle is 2π: $\alpha = 2\pi\,\text{rad} = 360°$.



The radian measure of the angle on one side of a straight line is π. The degree measure is 180°.



A right angle has radian measure $\pi/2$ and degree measure 90°.



4c. Intersecting Lines. When two lines intersect as in Figure B-5, opposite angles are equal, and the sum of adjacent angles is π rad or 180°. In the sketch: $a = c$, $b = d$, $a + b = b + c = c + d = d + a = \pi$ rad.

4d. Definition of Similar Triangles. Two triangles are similar if there exists a number k such that if the length of any side of one triangle is multiplied by k, the result is the length of the corresponding side of the

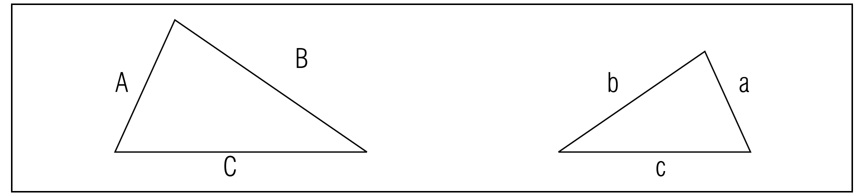


Fig. B-6: If $A = ka$, $B = kb$, and $C = kc$ for some fixed k, these triangles are similar.

other triangle. If $A = ka$, $B = kb$, and $C = kc$ for some fixed k, the triangles in Figure B-6 are similar.

4e. Angles in Similar Triangles. If two triangles are similar, then for every angle of one triangle, there is a corresponding angle in the other triangle which is the same. If one triangle has two angles which are equal to two angles in another triangle, then the two triangles are similar.

4f. Right Similar Triangles. If one acute angle in a right triangle is equal to one acute angle in another right triangle, then the two triangles are similar.

4g. Pythagorean Theorem. In a right triangle $\ell^2 + L^2 = h^2$ where ℓ and L are the lengths of the two legs and h is the length of the hypotenuse, as demonstrated in Figure B-7.

4h. Cartesian Coordinate Systems. A two-dimensional Cartesian coordinate system consists of two number lines called axes, drawn at right angles in a plane. The zero point of each number line is at the intersection, called the origin. Every point in the plane can then be labeled by two numbers called coordinates. The x coordinate of a point P is the number determined by the intersection with the x axis of a line through P perpendicular to the x axis. Similarly, the y coordinate is the number determined by the intersection with the y axis of a line through P and perpendicular to the y axis. In Figure B-8, the point P has x

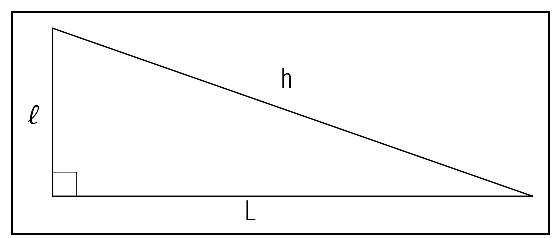


Fig. B-7: Right triangle with hypotenuse h and with legs ℓ and L.

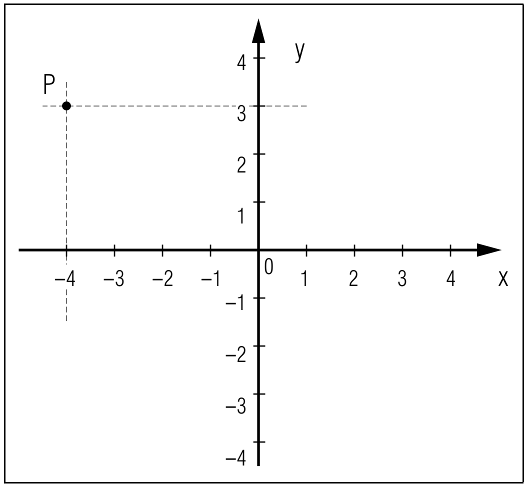


Fig. B-8: A standard representation of a Cartesian coordinate system.

coordinate $x = -4$ and y coordinate $y = 3$.

TRIGONOMETRY

5a. Sine, Cosine, and Tangent for Acute Angles. For an *acute* angle θ (See Fig. B-9), the sine, cosine, and tangent are the following ratios of the lengths of sides of *any* right triangle containing angle θ:

$$\sin\theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{x}{r}$$

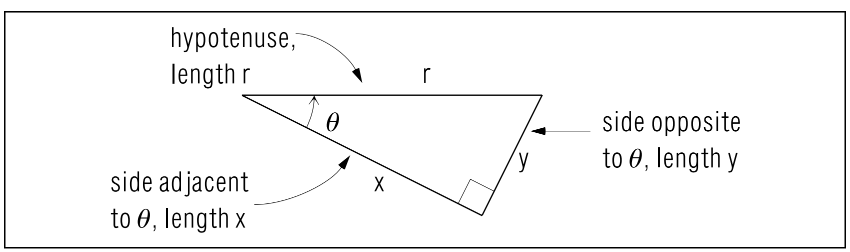


Fig. B-9: Sine, cosine, and tangent for an acute angle θ, can be defined in terms of the length of the side opposite θ, the side adjacent to θ, and the hypotenuse.

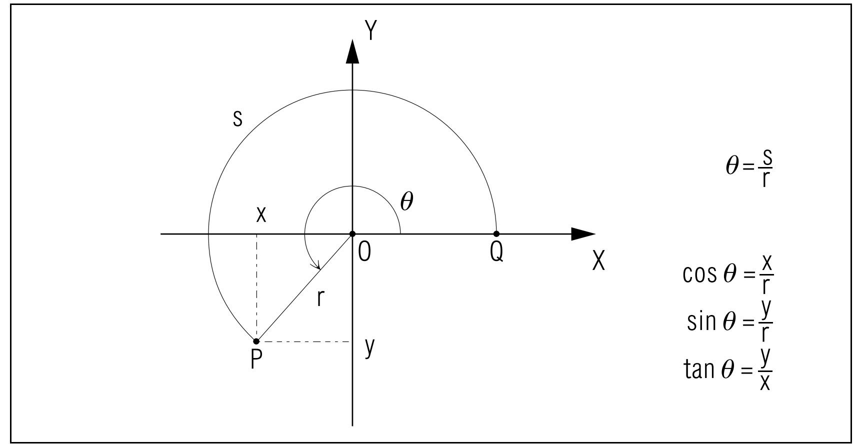


Fig. B-10: Definition of the ratios; sine, cosine, and tangent.

$$\tan\theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta} = \frac{y}{x}$$

Note that since x, y, and r are all understood to be positive lengths in the same units, $\sin\theta$, $\cos\theta$, and $\tan\theta$ are positive numbers (no units) for acute angles θ.

5b. Sine, Cosine, and Tangent for any Angle. Consider a circular arc $\overset{\frown}{QP}$, of radius r and length s, drawn about a point O. By convention, the arc is drawn counterclockwise from a point Q on the positive X-axis of a Cartesian reference frame to a point P with coordinates x and y (note that $r = +\sqrt{x^2 + y^2}$). If the angle θ is the angle measured counterclockwise between arc radii $\overset{\frown}{OQ}$ and $\overset{\frown}{OP}$, then θ, $\sin\theta$, $\cos\theta$, and $\tan\theta$ are defined as shown in Figure B-10. These ratios are understood to be pure numbers (no units). Note that x and y may be positive, negative, or zero, but r is *always* positive.

ALGEBRA

6a. Algebraic Symbols, Expressions, and Quantities. Algebraic *symbols* (letters of an alphabet) represent algebraic or real numbers, different symbols being used for distinct numbers. Algebraic expressions are written to prescribe a sequence of arithmetic operations applied to algebraic numbers, some of which are represented by symbols. Algebraic expressions, therefore, also represent algebraic numbers. The term algebraic quantity is used to mean an algebraic number, symbol, or expression.

6b. Equivalent Algebraic Expressions. To obtain an expression of equal numerical value, an algebraic expression can be manipulated by the following procedures:

(a) *Carry out indicated operations* according to the rules of arithmetic (especially 1d through 1h, 2a through 2e).

(b) *Substitute* one algebraic quantity (a number, symbol, or expression) for an equivalent quantity.

6c. Simplifying an Algebraic Expression. To obtain an equivalent algebraic expression in simpler (and often more useful) form, this sequence of steps may be applied to the original expression:

(a) *Combine* all fractions, using a common denominator, into a single fraction (see 1g).

(b) *Simplify* this fraction by removing expressions equal to zero or one, by combining and factoring terms, and by reducing the result to simplest form (see 1h).

(c) *Combine and simplify powers* of the same quantities (see 2a through 2e).

In carrying out these steps, it is advisable to follow the fail-safe rule for manipulation (6d).

6d. Fail-Safe Rule for Algebraic Manipulation. To provide the least possible chance for error in any sequence of algebraic manipulations:

(a) Perform only *one* operation at a time.

(b) *Write* each step.

(c) When in doubt, *check* a sequence of steps by substituting numbers into the first and last expressions of the sequence and then comparing the results. If the results are equal, the sequence is *probably* correct.

6e. Writing Algebraic Equations from Words. An algebraic equation states that one algebraic quantity (a number, symbol, or expression) has the same numerical value as another algebraic quantity. To write an algebraic equation (or equations) corresponding to a situation described by statements in words, these steps are advisable:

(a) Choose a *different* (but easily recognized and remembered) algebraic symbol for *every distinct* numerical quantity.

(b) Write as many equations as possible directly from the verbal statements. (Note that "is" or any similar verb often means "equals," so one sentence is often equivalent to one equation.) A diagram may help.

(c) Finally, search the situation described by the verbal statements for any new relationships that are not directly stated. Then write equations equivalent to these relationships. Diagrams are especially useful.

6f. Equivalent Algebraic Equations. To obtain from a given equation another equation which is equivalent (an equation satisfied by the same numerical values as the original one), these operations are allowed:

(a) The *same operation* (e.g., multiplication by a quantity, addition of a quantity, taking a square root) can be applied to *both sides* of the equation.

(b) Any *expression* in the equation may be replaced by an *equivalent expression.*

6g. Solving a Linear Algebraic Equation. To solve an algebraic equation for a given symbol x, the original equation is manipulated systematically to obtain an equivalent equation in which x appears *alone* on only *one* side of the equals sign. The algebraic quantity on the other side is the *solution* for x in the original equation. This rough sequence of steps is usually the best to solve the most common algebraic equations, those linear in x:

(a) *Eliminate all fractions* by multiplying both sides of the equation by the lowest common denominator of all fractions in the equation.

(b) *Collect* only those algebraic expressions containing the desired symbol x on *one* side of the equation, by adding suitable expressions to both sides.

(c) *Factor* x from these expressions and divide both sides of the equation by the resulting coefficient, thus leaving x alone on one side.

(d) *Substitute* any numerical values for symbols in the solution for x, and simplify this expression.

(e) *Check* the solution by substituting it for x everywhere in the original equation and checking that the equation is then satisfied.

The fail-safe rule (6d) is recommended as an additional guarantee of success in carrying out these steps.

6h. Eliminating a Common Symbol from Two Equations. Two algebraic equations containing a common quantity x may always be manipulated to give a combined single equivalent equation from which x has been eliminated. The simplest steps for this procedure are:

(a) Solve *one* of the equations for x.

(b) Substitute this solution for x everywhere in the *other* equation, which becomes the final result.

The fail-safe rule (6d) is recommended.

SECT.

C SELF-ASSESSMENT TEST

(Total time: 60 minutes)

ELEMENTARY ARITHMETIC

(6 minutes)

Calculate the value (to three decimal places) of these decimal numbers:

1. $(5.60) \times (0.320) = ?$
2. $1.50/0.020 = ?$
3. $2.03 - 0.596 = ?$
4. Express 2/7 as a decimal: $2/7 = ?$

Find the value (in simplest fractional form) of:

5. $\frac{5}{6} + \frac{3}{20} - \frac{5}{12} = ?$
6. $\frac{1}{6} \div \frac{3}{10} = ?$
7. In a certain restaurant, the price of a pizza is proportional to the area of the pizza. The "large" pizza (of radius 7.5 inches) costs $4.50. The "small" pizza (of radius 5 inches) has an area 4/9 times the area of the large pizza. How much does the small pizza cost?

POWERS

(3 minutes)

Calculate the numerical value (an integer, a decimal, or a fraction in simplest form) of these numbers:

8. $(-0.3)^2 = ?$
9. $2^{-3} = ?$

10. $(2^{-2} \times 2^3)^2 = ?$

11. $\dfrac{2^3}{2^{-2}} = ?$

12. $\left(\dfrac{9}{16}\right)^{1/2} = ?$

SCIENTIFIC NOTATION

(8 minutes)

13. Arrange these numbers in order of increasing value: (e.g., as in the series $1, 2, 3, 4, 5, 6$): 3×10^2, 0, -10^{-3}, 10^{-3}, -10^3, 0.3×10^4 Calculate the value (in scientific notation) of these numbers:

14. $(-4 \times 10^4)(2 \times 10^{-2})(3 \times 10^{-2}) = ?$

15. $(4.0 \times 10^4) - (2.0 \times 10^5) = ?$

16. $(-2.0 \times 10^2)^3 = ?$

17. $(1.6 \times 10^7)^{1/2} = ?$

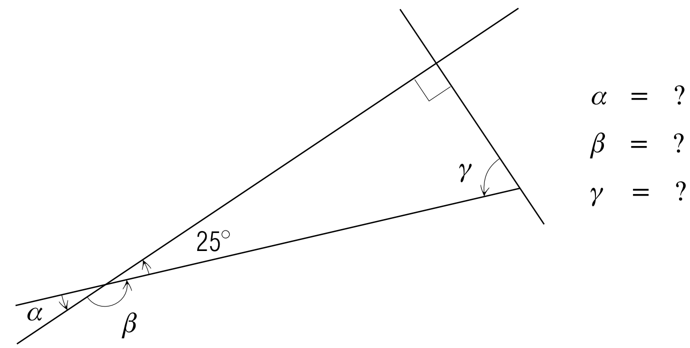
18. $\dfrac{(4.0 \times 10^3)}{(-2.0 \times 10^{-2})} = ?$

GEOMETRY

(10 minutes)

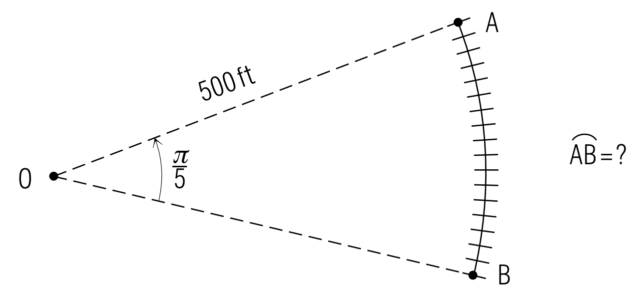
► Use the value $\pi = 3.142$ whenever necessary.

19. Find the values (in degrees) of the angles α, β, and γ in this figure:

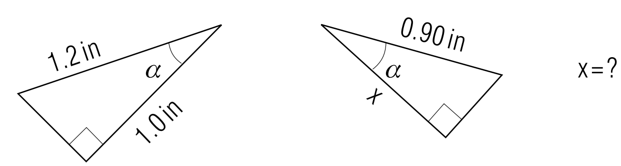


20. Which is the larger angle, 3 radians or 180°?

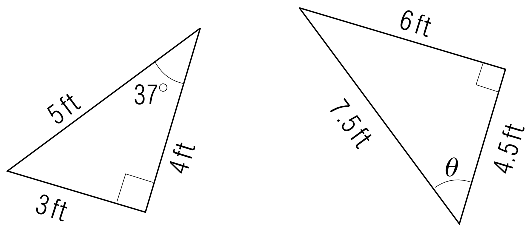
21. The following diagram shows a railroad curve in the form of a circular arc $\widehat{AB}$ about a point O. The radius of the arc is 500 feet, and the angle between the radii $\overline{OA}$ and $\overline{OB}$ is $\pi/5$ radians. What is the distance (to the nearest foot) along the track from A to B?



22. Consider the pair of similar triangles shown below. What is the length x of the indicated side?

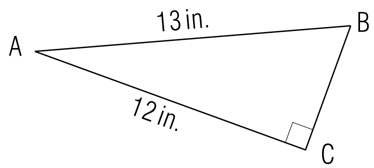


23. Consider the following pair of similar right triangles. What is the value (in degrees) of angle θ?



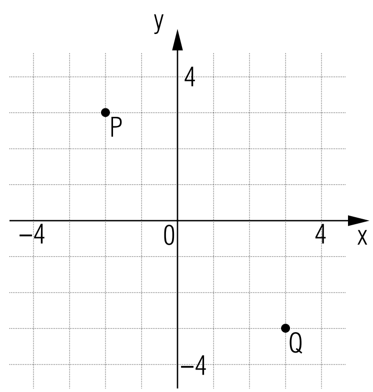
$\theta = ?$

24. In right triangle ABC, what is the length of side BC?



$\overline{BC} = ?$

25. In the Cartesian reference frame shown below, find the coordinates of points P and Q.



P: x = ? y = ?

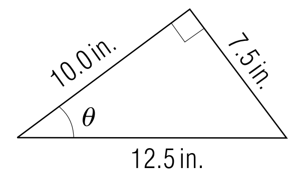
Q: x = ? y = ?

TRIGONOMETRY

(8 minutes)

► In the following problems, use the trig functions on your calculator.

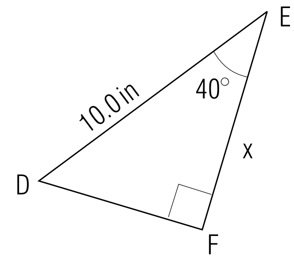
26. Find the values (to two decimal places) of $\cos\theta$ and $\tan\theta$ for the angle θ in this right triangle:



$\cos\theta = ?$

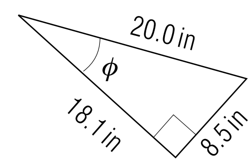
$\tan\theta = ?$

27. What is the length of side x in right triangle DEF shown below?



x = ?

28. Determine the value (to the nearest degree) of angle ϕ in this right triangle:



$\phi = ?$

29. Determine the values of these quantities:

$\cos 0^\circ = ?$

$\cos \pi/2 = ?$

$\cos \pi = ?$

$\cos 270^\circ = ?$

$\cos 2\pi = ?$

ALGEBRA

(25 minutes)

30. Determine the numerical value of each of these expressions when $x = -2$:

    $(2 - x) = ?$

    $x^0 = ?$

    $4x^2 = ?$

    $x^{-1} = ?$

31. Simplify this expression to a single algebraic fraction in simplest form:

    $$\frac{1}{R+x} - \frac{1}{R} = ?$$

32. Simplify the following expression to the indicated form (i.e., fill in the blanks with the coefficient and powers): $\dfrac{4m^2R^2}{\dfrac{2}{m}R^{n+1}} = (\ \ )m^{(\ )}R^{(\ )}$

33. Some children drop pebbles into a well and listen for the splash in the water 50 feet below. The distance d (in feet) which a pebble falls from their hands in a time of t seconds is given by

    $d = 5t^2$

    How far above the water is a pebble that has fallen for 2 seconds?

34. Which of the following values of B satisfies the equation $\dfrac{A}{B} = 0.1$?

    (a) $B = 0.1A$, (b) $B = 10A$, (c) $B = A + 10$,

    (d) none of these values

35. Is the value $x = 7/3$ a solution of the equation $9x - 2 = 6x + 5$? Check by substitution. yes or no?

Solve these equations for h:

36. $5h - 1 = 14 - \dfrac{5}{3}h\,;\qquad h = ?$

37. $\dfrac{(h-7)}{4} + \dfrac{(h+2)}{3} = h\,;\qquad h = ?$

Solve these equations for f (in simplest form):

38. $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{f}\,;\qquad f = ?$

39. $fx = e - 3gf\,;\qquad f = ?$

Calculate the numerical value of a in the following equations when the other variables have the indicated values:

40. $2b = vt + \dfrac{1}{2}at^2\,;\qquad a = ?$

    $b = 1.5$

    $v = 10$

    $t = 3$

41. $\dfrac{q}{a} + \dfrac{Q}{a+d} = 0\,;\qquad a = ?$

    $q = 1$

    $Q = -2$

    $d = 5$

42. If $E = \dfrac{1}{2}mv^2$ and $p = mv$, express E in terms of m and p alone (i.e., eliminate v):

    $E = ?$

43. $y = ay^2 + b\,;\qquad y = ?$

    $a = 0.40$

    $b = 0.50$

SECT.

D SELF-ASSESSMENT TEST ANSWERS

1. 1.792
2. 75.000
3. 1.434
4. 0.286
5. 17/30
6. 5/9
7. \$2.00
8. 0.09
9. 1/8 or 0.125
10. 4
11. 32
12. $\pm(3/4)$ or 3/4 or ± 0.75 or 0.75
13. -10^3, -10^{-3}, 0, 10^{-3}, 3×10^2, 0.3×10^4
14. -2.4×10^1 or -24
15. -1.6×10^5
16. -8×10^6 or -8.0×10^6
17. $\pm 4 \times 10^3$ or 4×10^3 or $\pm 4.0 \times 10^3$ or 4.0×10^3
18. -2×10^5 or -2.0×10^5
19. $\alpha = 25°$, $\beta = 155°$, $\gamma = 65°$
20. 180°
21. $\overset{\frown}{AB} =$ 314 ft. (to the nearest foot)
22. $x = 0.75$ in.
23. $\theta = 53°$

24. $\overline{BC} =$ 5in. or 5.0 in.
25. P: $x = -2$, $y = 3$; Q: $x = 3$, $y = -3$
26. $\cos\theta = 0.80$ $\tan\theta = 0.75$
27. $x = 7.66$ in.
28. $\phi = 25°$ (to the nearest degree)
29. $\cos 0° = 1.0$ or 1, $\cos(\pi/2) = 0$, $\cos\pi = -1.0$ or -1, $\cos 270° = 0$, $\cos 2\pi = 1.0$ or 1
30. $(2 - x) = 4$ or 4.0, $x^\circ = 1$ or 1.0, $4x^2 = 16$ or 16.0, $x^{-1} = -(1/2)$ or -0.5
31. $\dfrac{-x}{R(R+x)}$ or $-\dfrac{x}{R(R+x)}$
32. $(2)m^{(3)}R^{(1-n)}$ or $(2)m^{(3)}R^{(-n+1)}$
33. 30 feet
34. b
35. yes
36. $h = 9/4$ or $2\dfrac{1}{4}$ or 2.25
37. $h = -13/5$ or $-2\dfrac{3}{5}$ or -2.6
38. $\dfrac{xy}{(x+y)}$
39. $\dfrac{e}{(x+3g)}$
40. $a = -6$
41. $a = 5$
42. $E = p^2/2m$ or $\dfrac{m}{2}\left(\dfrac{p^2}{m^2}\right)$ or $\dfrac{m}{2}\left(\dfrac{p}{m}\right)^2$
43. 0.69 and 1.81.

SECT.

E SELF-ASSESSMENT TEST SOLUTIONS

(Total time: 60 minutes)

ELEMENTARY ARITHMETIC

1. $(5.6) \times (0.32) = ?$

   Solution:

```
   5.6
  0.32
  ----
   112
  168_
  ----
 1.792
```

   1.792

   The number of digits to the right of the decimal point in the product is equal to the sum of the number of digits to the right of the decimal point in the two numbers being multiplied.

2. $1.5/0.02 = ?$

   Solution: First simplify by multiplying by 1 in the form (100/100). This changes both the numerator and denominator to integers. Then divide the integers.

```
    75
 2|150
   14_
    10
    10
```

   75.0

3. $2.03 - 0.596 = ?$

   Solution:

```
   2.030
 - 0.596
   -----
   1.434
```


   1.434

4. Express 2/7 as a decimal. $2/7 = ?$

   Solution:

```
    .2857
 7|2.0000
   14___
    60__
    56__
     40_
     35_
      50
```

   The answer should be correct to three decimal places. .2857 is closer to .286 than to .285.

   .286

► Retest for 1-4: do Supplementary Problems 1 through 4.

5. $\dfrac{5}{6} + \dfrac{3}{20} - \dfrac{5}{12} = ?$

   Solution: The fractions must first be rewritten as fractions with the same denominator. 60 is a multiple of each denominator 6, 20, and 12, so each fraction can be rewritten with denominator 60. To do this, each fraction is multiplied by 1 in the form (b/b).

   $$\frac{5}{6} + \frac{3}{20} - \frac{5}{12} = \frac{5}{6}\left(\frac{10}{10}\right) + \frac{3}{20}\left(\frac{3}{3}\right) - \frac{5}{12}\left(\frac{5}{5}\right) = \frac{50}{60} + \frac{9}{60} - \frac{25}{60} = \frac{50+9-25}{60} = \frac{34}{60}$$

   This is not in simplest fraction form because both 34 and 60 have 2 as a factor.

   $$\frac{17\cdot 2}{30\cdot 2} = \left(\frac{17}{30}\right)\left(\frac{2}{2}\right) = \frac{17}{30}\cdot 1 = \frac{17}{30}$$

   17/30

6. $\dfrac{1}{6} \div \dfrac{3}{10} = ?$

   Solution:

   $$\frac{1}{6} \div \frac{3}{10} = \frac{1/6}{3/10}$$

A multiple of each denominator is 30, so multiply the fraction by $1 = \left(\frac{30}{30}\right)$.

$$\frac{1/6}{3/10} = \frac{(1/6)}{(3/10)}\left(\frac{30}{30}\right) = \frac{5}{9}$$

$\boxed{5/9}$

► Retest for 5-6: do Supplementary Problems 5 through 7.

7. In a certain restaurant, the price of a pizza is proportional the area of the pizza: The "large" pizza (of radius 7.5 inches) costs $4.50. The "small" pizza (of radius 5 inches) has an area 4/9 times the area of the large pizza. How much does the small pizza cost?

   Solution: If price is proportional to area, then changing area by multiplying it by 4/9 implies that the price will also change by a factor of 4/9. Therefore the price of the smaller pizza equals (4/9) times the price of the larger pizza:

   $$\frac{4}{9} \times 4.50 = 4 \times \frac{4.50}{9} = 4x(.50) = \$2.00$$

   Notice that it was not necessary to use the radii of the two pizzas.

   $\boxed{\$2.00}$

► Retest for 7: do Supplementary Problem 8.

POWERS

8. $(-0.3)^2 = ?$

   Solution:

   $$(0.3)^2 = (-0.3)(-0.3) \text{ since } a^m = \underbrace{a \cdot a \cdot a \cdots a}_{m factors}$$

   $$= .09 \text{ because for any numbers } c \text{ and } d\ (-c)(-d) = cd\ .$$

   $\boxed{.09}$

9. $2^{-3} = ?$

   Solution: Use the definition $a^{-m} = 1/a^m$.

   $2^{-3} = 1/2^3 = 1/8$.

   $\boxed{1/8}$

10. $(2^{-2} \times 2^3)^2 = ?$

    Solution: Parentheses indicate that the operations inside should be performed first.

    $$(2^{-2} \times 2^3)^2 = (2^{-2+3})^2 \text{ because } a^m a^n = a^{m+n}.$$
    $$= (2^1)^2 = 2^2 \text{ since } (a^m)^n = a^{mn}.$$
    $$= 4.$$

    $\boxed{4}$

11. $\frac{2^3}{2^{-2}} = ?$

    Solution:

    $$\frac{2^3}{2^{-2}} = 2^{3-(-2)} \text{ because } \frac{a^m}{a^n} = a^{m-n}$$

    $$= 2^{3+2} \text{ since } c - (-a) = c + a$$

    $$= 2^5 = 32$$

    $\boxed{32}$

12. $\left(\frac{9}{16}\right)^{1/2} = ?$

    Solution:

    $$\left(\frac{9}{16}\right)^{1/2} = \frac{3}{4} \text{ because } \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}\ .$$

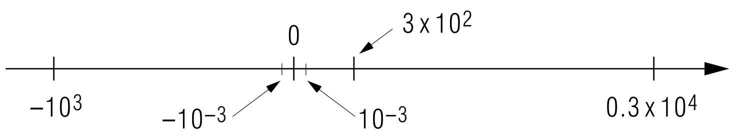
    $\boxed{3/4}$

► Retest for 8-12: do Supplementary Problems 9 through 15.

SCIENTIFIC NOTATION

13. Arrange these numbers in order of increasing value: (e.g., as in the series 1, 2, 3, 4, 5, 6): 3×10^2, 0, -10^{-3}, 10^{-3}, -10^3, 0.3×10^4.

Solution:



On the number line, zero is placed at the origin. 10^{-3} and -10^{-3} have equal magnitudes, so they are an equal distance from the origin, and they lie on the opposite sides of the origin. -10^3 has a larger magnitude than -10^{-3}, so it lies farther from the origin, in this case to the left. 0.3×10^4 is positive and has larger magnitude than 10^{-3}. It lies to the right of 10^{-3}, 3×10^2 is positive. $3\times10^2 = 300$. $0.3\times10^4 = 3000$. 3×10^2 is smaller than 0.3×10^4 but larger than 10^{-3}.

On the number line larger numbers lie to the right of the smaller numbers.

$\boxed{-10^3,\ -10^{-3},\ 0,\ 10^{-3},\ 3 \times 10^2,\ 0.3 \times 10^4}$

► Retest for 13: do Supplementary Problem 16.

14. $(-4 \times 10^4)(2 \times 10^{-2})(3 \times 10^{-2}) = ?$

Solution: First use the commutative and associative properties to group the numbers $-4, 2$, and 3 separately from the powers of ten.

$$\begin{aligned}(-4 \times 10^4)(2 \times 10^{-2})(3 \times 10^{-2}) &= (-4 \times 2 \times 3)(10^{4+(-2)+(-2)})\\ &= -24 \times 10^{4-2-2} = -24 \times 10^0\\ &= -24 \text{ since } a^0 = 1.\end{aligned}$$

A number in scientific notation consists of two factors. The left factor is a decimal number with one non-zero digit to the left of the decimal point. The right factor is an integral power of ten. -24 is written in scientific notation: -2.4×10.

$\boxed{-2.4 \times 10}$

15. $(4.0 \times 10^4) - (2.0 \times 10^5) = ?$

Solution: First group the powers of 10 separately. done by writing 10^5 as 10×10^4 and grouping 10 with 2.0 . $2.0\times10^5 = 2.0\times(10\times10^4) = (2.0 \times 10) \times 10^4 = 20 \times 10^4$. Then use the distributive property to rewrite the entire expression.

$$\begin{aligned}(4.0 \times 10^4) - (2.0 \times 10^5) &= (4.0 \times 10^4) - (20 \times 10^4)\\ &= (4.0 - 20) \times 10^4 = -16 \times 10^4\\ &= -1.6 \times 10^5\end{aligned}$$

$\boxed{-1.6 \times 10^5}$

Example: $(4.2 \times 10^{-3}) + (3.8 \times 10^{-4}) = ?$

First rewrite 10^{-3} as 10×10^{-4}

$$\begin{aligned}(4.2 \times 10^{-3}) + (3.8 \times 10^{-4}) &= (4.2 \times 10 \times 10^{-4}) + (3.8 \times 10^{-4})\\ &= (42 + 3.8) \times 10^{-4}\\ &= 45.8 \times 10^{-4}\\ &= 4.58 \times 10^{-3}\end{aligned}$$

$\boxed{4.58 \times 10^{-3}}$

16. $(-2.0 \times 10^2)^3 = ?$

Solution: Use the rule for powers: $(a^m b^n)^p = a^{mp} b^{np}$.

$$\begin{aligned}(-2.0 \times 10^2)^3 &= (-2.0)^3 \times 10^6\\ &= -8.0 \times 10^6\end{aligned}$$

The last step uses $(-2.0)^3 = (-2.0)(-2.0)(-2.0) = (4.0)(-2.0) = -8.0$.

$\boxed{-8.0 \times 10^6}$

17. $(1.6 \times 10^7)^{1/2} = ?$

Solution: In simplifying (1.6×10^7), one might first try the following procedure:

$$(1.6 \times 10^7)^{1/2} = (1.6)^{1/2} \times (10^7)^{1/2}$$
$$= (1.6)^{1/2} \times 10^{7/2} .$$

This expression is not in scientific notation because the right factor is not an integral power of ten. (Also notice that $(1.6)^{1/2}$ is *not* .4 because $(.4)^2 = .16$ *not* 1.6). To avoid a non integral power of ten, use the associative property to group one or more factors of 10 with the left factor so that the remaining power of 10 has an exponent exactly divisible by 2.

$1.6 \times 10^7 = (1.6 \times 10) \times 10^6 = 16 \times 10^6$

$(16 \times 10^6)^{1/2} = (16)^{1/2} \times (10^6)^{1/2} = 4 \times 10^3$.

$\boxed{4 \times 10^3}$

Example: $(2.7 \times 10^{-8})^{1/3}$

To simplify this expression one must rewrite the number inside the parentheses so that the exponent of 10 is exactly divisible by 3. One way to do this is to replace 10^{-8} by 10×10^{-9} and group the factor 10 with 2.7.

$2.7 \times 10^{-8} = 2.7 \times (10 \times 10^{-9}) = 27 \times 10^{-9}$

Then use the property of powers: $(a^m b^n)^p = a^{mp} b^{np}$.

$(27 \times 10^{-9})^{1/3} = (27)^{1/3} \times (10^{-9})^{1/3} = 3 \times 10^{-3}$

Since $3^3 = 27, 27^{1/3} = 3$.

18. $\dfrac{(4.0 \times 10^3)}{(-2.0 \times 10^{-2})} = ?$

Solution:

$$\frac{(4.0 \times 10^3)}{(-2.0 \times 10^{-2})} = \left(\frac{4.0}{-2.0}\right) \times \left(\frac{10^3}{10^{-2}}\right) = -2.0 \times 10^{3-(-2)}$$

$$= -2.0 \times 10^{3+2} = -2.0 \times 10^5 \ .$$

$\boxed{-2.0 \times 10^5}$

Example: $\dfrac{(7.5 \times 10^{-5})}{(2.5 \times 10^{-3})} = ?$

$$\frac{(7.5 \times 10^{-5})}{(2.5 \times 10^{-3})} = \left(\frac{7.5}{2.5}\right) \times 10^{-5-(-3)}$$

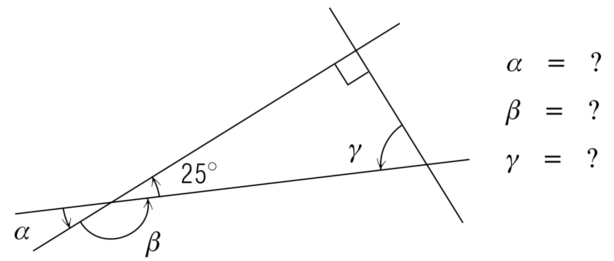
$$= 3.0 \times 10^{-5+3} = 3.0 \times 10^{-2} \ .$$

$\boxed{3.0 \times 10^{-2}}$

► Retest for 14-18: do Supplementary Problems 17 through 21.

GEOMETRY

19. Find the values (in degrees) of the angles α, β andγ in this figure:



Solution: When two lines intersect, opposite angles are equal and the sum of adjacent angles is 180° .

$\alpha = 25°$

$\beta + 25° = 180°$

$\beta = 155°$

The sum of the angles in a triangle is 180°.

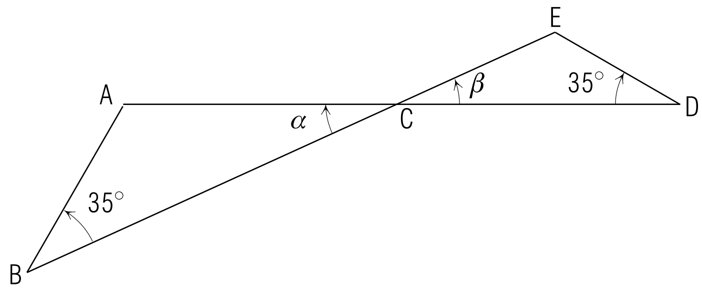
$90° + 25° + \gamma = 180°$

$\gamma = 180° - 90° - 25°$

$\gamma = 65°$

$\boxed{\alpha = 25°, \beta = 155°, \gamma = 65°}$

Example: Are triangles ABC and EDC (in the drawing below) similar triangles?



The angles α and β are equal because when two lines intersect the opposite angles are equal. The two 35° angles are equal. If one triangle has two angles which are equal to two angles in another triangle, the triangles are similar.

► Retest for 19: do Supplementary Problem 22.

20. Which is the larger angle, 3 radians or 180° ?

Solution: To compare two quantities one must first put them in the same units. Here one must express 3 radians in degrees or 180° in radians. $360° = 2\pi$ rad because there are 360° or 2π rad in a full circular angle.

$180° = \pi \text{ rad} = 3.14 \text{ rad}$

$\boxed{180° = 3.14 \text{ rad} > 3 \text{ rad}}$

Example: Express 30° in radians.

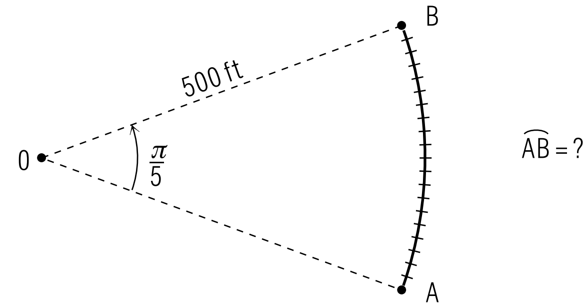
There are 360° or 2π rad in a full circular angle, therefore $360° = 2\pi$ or $1 = \dfrac{2\pi \text{ rad}}{360°}$.

$30° = 30° \left(\dfrac{2\pi \text{ rad}}{360°}\right)$ since any number can be multiplied by 1 without changing its value.

$$30° = \frac{30 \times 2}{360°}\pi = \frac{1}{6}\pi$$

$\boxed{\pi/6}$

21. The following diagram shows a railroad curve in the form of a circular arc $\overset{\frown}{AB}$ about a point O. The radius of the arc is 500 feet, and the angle between the radii $\overline{OA}$ and $\overline{OB}$ is $\pi/5$radians. What is the distance (to the nearest foot) along track from A to B?



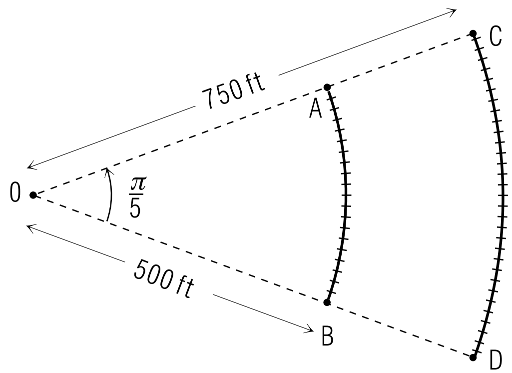
Solution: The radian measure of the angle BOA is defined as the length of arc $\overset{\frown}{AB}$ divided by the length of the radius, 500.

$$\frac{\pi}{5} = \frac{\overset{\frown}{AB}}{500}$$

$$\overset{\frown}{AB} = 500\frac{\pi}{5} = 100\pi = 314\,\text{ft}$$

$\boxed{314\,\text{ft}}$

Example: Later the railroad needs a second section of track along an arc with the same center, but at a radius of 750 ft How much track is needed for this section, $\overset{\frown}{CD}$?

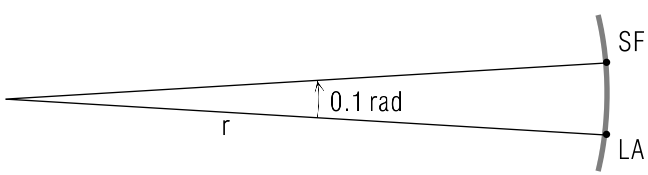


The definition of radian measure, used in exercise 20, applies to *any* arc with center at O, intersecting the sites of the angle. In particular it applies to $\widehat{CD}$. Therefore,

$$\frac{\pi}{5} = \frac{\widehat{CD}}{750}$$

$$\widehat{CD} = 750\ \frac{\pi}{5} = 150\pi = 471\,\text{ft}$$

Example: If two radii were drawn from the center of the earth to San Francisco and to Los Angeles, the angle between them would be about 0.1 radians. The distance between San Francisco and Los Angeles (measured along the surface) is about 400 mi Using this information, estimate the radius of the earth.



The surface distance, 400 mi, between San Francisco and Los Angeles, divided by the radius of the earth r gives the measure of the angle in radians, that is 0.1.

$$400/r = 0.1\ .$$

Solving this equation for r:

$$400 = 0.1r$$

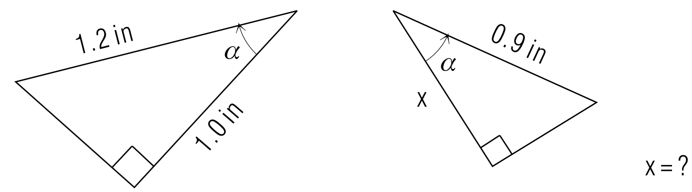
$$400(10) = 0.1r(10)$$

$$4000 = r$$

$\boxed{4000\,\text{mi}}$

► Retest for 20-21: do Supplementary Problems 23 and 24.

22. Consider the pair of similar triangles shown below. What is the length x of the indicated side?



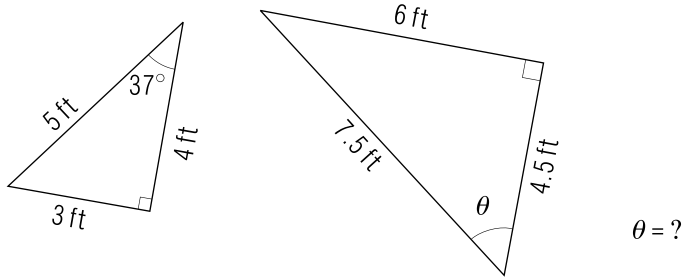
Solution: If one acute angle in a right triangle is equal to one acute angle in another right triangle, then the two triangles are similar. Since the triangles are similar, the ratios of their corresponding sides are equal. The sides of length x and 1.0 in are corresponding sides because they each lie between a right angle and an acute angle α. The sides of length 0.9 in and 1.2 in are corresponding sides because each is a hypotenuse.

$$\frac{x}{1.0} = \frac{0.9}{1.2}$$

$$x = \frac{0.9}{1.2}\left(\frac{10}{10}\right) = \frac{9}{12} = \frac{3}{4} = .75\ .$$

$\boxed{.75\,\text{in}}$

23. Consider the following pair of similar right triangles. What is the value (in degrees) of angle θ?



Solution: Compare the longest sides of each triangle. These sides have lengths 7.5 ft and 5 ft Multiplying 5 ft by 1.5 gives the result 7.5 ft A similar relation is true for the shortest sides:

(3 ft)(1.5) = 4.5 ft

and for the remaining side :

(4 ft)(1.5) = 6 ft

The two triangles are similar because there exists a number (1.5) such that if the length of any side of one triangle is multiplied by 1.5, the result is the length of the corresponding side in the other triangle.

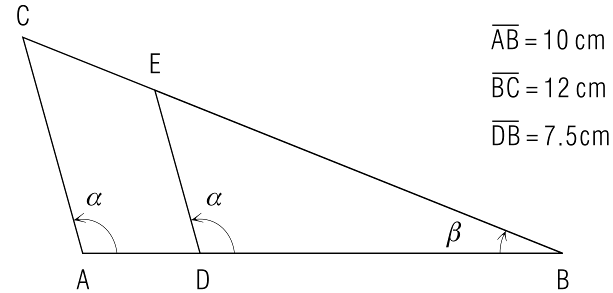
In the left triangle $37° + 90° + \beta = 180°$ because the sum of the angles in a triangle is 180°.

$\beta = 53°$

θ and β are corresponding angles because each lies between the hypotenuse and the shortest leg. If two triangles are similar, the corresponding angles are equal. Therefore $\theta = \beta = 53°$.

53°

Example: In the figure below, what is the length of $\overline{EB}$ in cm?



The triangle ABC is similar to triangle DBE because two angles of one (α and β) are equal to two angles of the other. In two similar triangles the ratios of corresponding sides are equal.

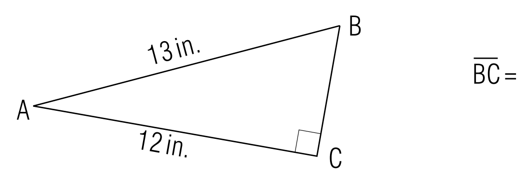
$$\frac{\overline{DB}}{\overline{AB}} = \frac{\overline{EB}}{\overline{CB}}$$

$$\frac{7.5}{10} = \frac{\overline{EB}}{12}$$

$$\overline{EB} = (.75)(12) = \left(\frac{3}{4}\right)(12) = 9$$

9 cm

24. In a right triangle ABC, what is the length (in inches) of side $\overline{BC}$?



Solution: $\overline{BC}^2 + 12^2 = 13^2$ using the Pythagorean Theorem

$$\overline{BC}^2 = 13^2 - 12^2$$

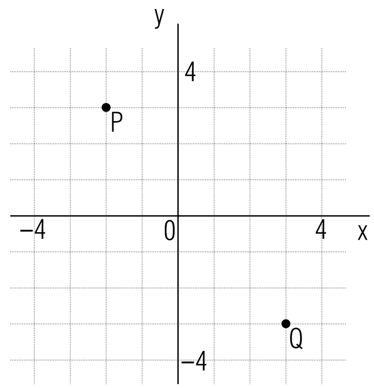
$$= 169 - 144 = 25$$

$\overline{BC} = 5$ because $5^2 = 25$

5 in

► Retest for 22-24: do Supplementary Problems 25 through 27.

25. In the Cartesian reference frame shown below, find the coordinates of points P and Q.



P: x = ? y = ?

Q: x = ? y = ?

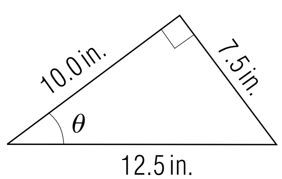
Solution: Draw lines through P and Q which are perpendicular to the x and y axes. The x and y coordinates of P are given by the intersection of these lines with the x and y axes.

$P : x = -2, y = 3$. Similarly for $Q : x = 3, y = -3$.

► Retest for 25: do Supplementary Problems 28 and 29.

TRIGONOMETRY

26. Find the values (to two decimal places) of $\cos\theta$ and $\tan\theta$ for the angle θ in this right triangle:



$\cos\theta = ?$

$\tan\theta = ?$

Solution: These calculations are direct applications of the definitions of sine, cosine, and tangent as particular ratios of the sides of any right triangle containing angle θ (see B-5a). Remember that, since the sides used in the ratios are identified by their relationship to angle θ, it is essential to identify first which side is which. In this case,

$$\cos\theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{10.0\,\text{in}}{12.5\,\text{in}}$$

or

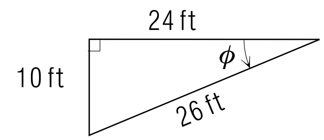
$\boxed{\cos\theta = 0.80}$

Similarly,

$$\tan\theta = \frac{\text{length of side opposite to } \theta}{\text{length of side adjacent to } \theta} = \frac{7.5\,\text{in}}{10.0\,\text{in}}$$

$\boxed{\tan\theta = 0.75}$

Example: Find the value of sin ϕ (to two decimal places) for the angle ϕ in this triangle:

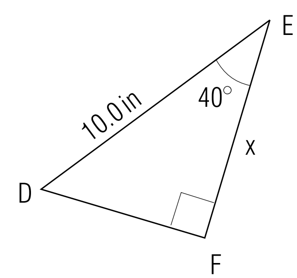


$$\sin\phi = \frac{\text{length of side opposite to } \phi}{\text{length of hypotenuse}} = \frac{10\,\text{ft}}{26\,\text{ft}}$$

$\boxed{\sin\phi = 0.38}$

► Retest for 26: do Supplementary Problems 30 and 31.

27. What is the length of side x in right triangle DEF shown below?



x = ?

Solution: If one side and one *acute* angle θ of a right triangle are known, either of the unknown sides can be found. The method is to look up the known *ratios* of side lengths for *any* right triangle containing angle $\theta(\sin\theta, \cos\theta,$ and $\tan\theta)$. The value of the known side length can then be used to solve for either unknown side. Which ratio to use depends on which side is known and which is wanted. In this case, the hypotenuse is known and the side adjacent to the 40° angle is wanted, so we can use

$$\cos 40° = \frac{\text{length of side adjacent to } 40°}{\text{length of hypotenuse}} = \frac{x}{10.0\,\text{in}}$$

Using the value from a calculator,

$$\cos 40° = (0.766) = \frac{x}{10.0\,\text{in}}\,.$$

Multiplying by (10.0 in):

$$x = (0.766)(10.0\,\text{in})$$

or

$$\boxed{x = 7.66\,\text{in}}$$

Similarly, we might also use the *complementary* angle:

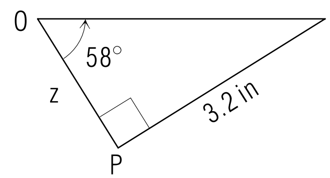
$$\sin 50° = \frac{\text{length of side opposite } 50°}{\text{length of hypotenuse}} = \frac{x}{10.0\,\text{in}}$$

$$\sin 50° = (0.766) = \frac{x}{10.0\,\text{in}}$$

$$x = (0.766)(10.0\,\text{in})$$

$$\boxed{x = 7.66\,\text{in}}$$

Example: What is the length of side z in right triangle OPQ?



Since the length of the hypotenuse is not known, we can only use the tangent. For example:

$$\tan 58° = \frac{\text{length of side opposite to } 58°}{\text{length of side adjacent to } 58°} = \frac{3.2\,\text{in}}{z}$$

$$z(\tan 58°) = 3.2\,\text{in}$$

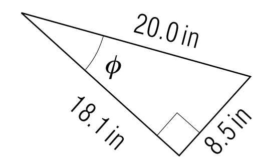
$$z = \frac{(3.2\,\text{in})}{(\tan 58°)} = \frac{(3.2\,\text{in})}{(1.600)}$$

$$z = 2.0\,\text{in}$$

Note that we could have used the tangent of 32° (the complementary angle) equally well.

► Retest for 27: do Supplementary Problems 32 and 33.

28. What is the value (to the nearest degree) of angle ϕ in the following right triangle?



$\phi = ?$

Solution: If any two sides of a right triangle are known, either of the acute angles can be found. The method is to calculate the appropriate ratio of the two sides (the sine, cosine, or tangent of the chosen angle, depending on which sides are known) and then to use a calculator to find the corresponding value for the angle. In this case, for example:

$$\sin\phi = \frac{\text{length of side opposite } \phi}{\text{length of hypotenuse}} = \frac{8.5\,\text{in}}{20.0\,\text{in}}$$

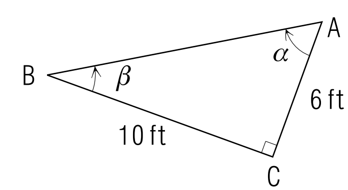
or

$$\sin\phi = 0.425$$

Now use the calculator to find the inverse sine:

$\boxed{\phi = 25°}$ (to the nearest degree)

Example Find the angles α and β in right triangle ABC:



Since only the lengths of the legs of the triangle are known, we are forced to use the tangent of either α or β to begin:

$$\tan\beta = \frac{\text{length of side opposite to } \beta}{\text{length of side adjacent to } \beta} = \frac{6\,\text{ft}}{10\,\text{ft}}$$

$\tan\beta = 0.60$

Using the inverse tangent function on a calculator,

$\beta = 31°$ and therefore $\alpha = 90° - \beta = 59°$

► Retest for 28: do Supplementary Problem 34.

29. Determine the values of these quantities:

$\cos 0° = ?$

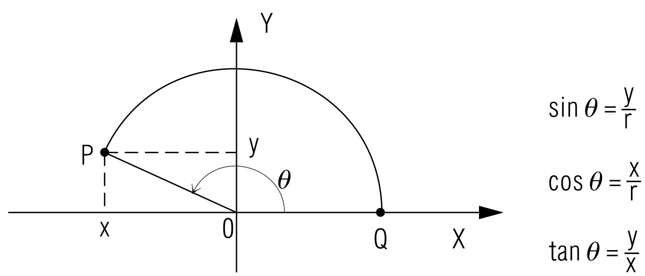
$\cos \pi/2 = ?$

$\cos \pi = ?$

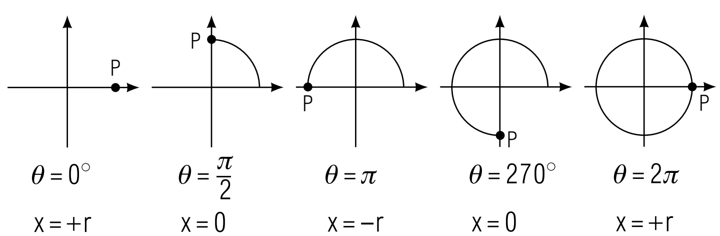
$\cos 270° = ?$

$\cos 2\pi = ?$

Solution: The values of sine, cosine, and tangent for any angle θ are defined in this figure (see B-5b):



In the present case, all of the angles correspond to arcs ending at a point P on one of the coordinate axes. Therefore, the coordinate x appearing in the definition of the cosine is either $+r, 0$, or $-r$. Specifically:



Therefore, the desired quantities are:

$$\boxed{\cos 0° = \frac{+r}{r} = +1}$$

$$\boxed{\cos \frac{\pi}{2} = \frac{0}{r} = 0}$$

$$\boxed{\cos \pi = \frac{-r}{r} = -1}$$

$$\boxed{\cos 270° = \frac{0}{r} = 0}$$

$$\boxed{\cos 2\pi = \frac{+r}{r} = +1}$$

Notice that the radius r of the circular arc is always a positive number, so the sine has the same sign as the coordinate y of point P, and the cosine has the same sign as the coordinate x of point P.

► Retest for 29: do Supplementary Problem 35.

ALGEBRA

30. Find the numerical value of each of these expressions when $x = -2$:

$(2 - x) = ?$

$x^0 = ?$

$4x^2 = ?$

$x^{-1} = ?$

Solution: An algebraic expression prescribes a sequence of arithmetic operations to be applied to real numbers, some of which are represented by symbols. If a numerical value is assigned to every symbol in an expression, the numerical value of the whole expression can be evaluated by substituting the numbers for each symbol and *carefully* carrying out the indicated operations according to the rules of arithmetic (see B-6b and B-6d). In the present case, this procedure gives these results:

$(2 - x) = (2 - (-2)) = 2 + 2$

So

$\boxed{(2-x) = +4 \text{ when } x = -2}$

For any number $x, x^0 = 1$ (see B-3), so

$\boxed{x^0 = 1 \text{ when } x = -2}$

Next,

$4x^2 = 4(-2)^2 = 4(+4)$

So

$\boxed{4x^2 = +16 \text{ when } x = -2}$

Finally, for any number x (see B-2a and B-2c),

$$x^{-1} = \frac{1}{x^1} = \frac{1}{x}$$

Therefore, when $x = -2$,

$$x^{-1} = \frac{1}{(-2)}$$

Or

$\boxed{x^{-1} = -(1/2) \text{ when } x = -2}$

► Retest for 30: do Supplementary Problems 36 and 37.

31. Simplify the following to a single algebraic fraction in simplest form:
$$\frac{1}{R+x} - \frac{1}{R} = ?$$
Solution: See the Solutions following Problem 32 (below).

32. Simplify the following expression to the indicated form (i.e., fill in the blanks with the coefficient and powers):
$$\frac{4m^2R^2}{\frac{2}{m}R^{n+1}} = (\quad)m^{(\ )}R^{(\ )}$$

Solutions for 31 and 32: Both of these problems require manipulating an algebraic expression to obtain an equivalent expression (see B-6b) in a particular "simpler" form. (Note that this process is not the same as solving an equation, since there is no equation to begin with.) To carry out this task, we can perform any indicated operation in the expression or make substitutions, trying at each step to come closer to the final form of the expression. The order of the steps is not crucial, but they should be performed one at a time, and the result should be checked (see the fail-safe rule, B-6d). In most cases, however, this sequence of steps works well (see B-6c):

(a) *Combine* all fractions, using a common denominator, into a single fraction (see B-1g).
(b) *Simplify* this fraction by removing expressions equal to zero or one, by combining and factoring terms, and by reducing the result to simplest form (see B-1h).
(c) *Combine and simplify powers* of the same quantities (see B-2a through B-2e).

We shall follow these steps in the solution of problems 31 and 32.

Soluton for Problem 31: Combining fractions, using the common denominator $R(R+x)$ [step (a)]:

$$\begin{aligned}\frac{1}{R+x} - \frac{1}{R} &= \left(\frac{R}{R}\right)\frac{1}{(R+x)} - \left[\frac{(R+x)}{(R+x)}\right]\frac{1}{R} \\ &= \frac{R}{R(R+x)} - \frac{(R+x)}{R(R+x)} \\ &= \frac{R-(R+x)}{R(R+x)}\end{aligned}$$

Simplifying by removing $R - R$, [step (b)]:

$$\begin{aligned}\frac{1}{R+x} - \frac{1}{R} &= \frac{R-R-x}{R(R+x)} \\ &= \frac{-x}{R(R+x)}\end{aligned}$$

Since there are no powers to combine in this expression, step (c) is unnecessary. The final result is therefore

$$\boxed{\frac{1}{R+x} - \frac{1}{R} = \frac{-x}{R(R+x)}}$$

If the manipulations are correct, these two expressions will have the same numerical value for *any* values of R and x.

Agreement for one arbitrary set of values (say, $R = +2$ and $x = -3$) provides a good quick check (see B-6d):

$$\frac{1}{R+x} - \frac{1}{R} =? \frac{-x}{R(R+x)}$$

$$\frac{1}{2+(-3)} - \frac{1}{2} =? \frac{-(-3)}{2(2+(-3))}$$
$$(-1) - \frac{1}{2} =? \frac{3}{2(-1)}$$
$$-\frac{3}{2} = -\frac{3}{2}$$

The result is *probably* correct.

► Retest for 31: do Supplementary Problems 38 and 39.

Solution for Problem 32: Since only one fraction is present in the original expression, we can begin with step (b), following the advice in B-1h:

$$\frac{4m^2R^2}{\frac{2}{m}R^{n+1}} = \left(\frac{m}{m}\right)\frac{4m^2R^2}{\frac{2}{m}R^{n+1}} = \frac{4m^3R^2}{2R^{n+1}} = \frac{2m^3R^2}{R^{n+1}}$$

Combining the remaining powers, using B-2b [step (c)]:

$$\frac{4m^2R^2}{\frac{2}{m}R^{n+1}} = 2m^3R^{2-(n+1)} = 2m^3R^{(1-n)}$$

The final result is therefore

$$\boxed{\frac{4m^2R^2}{\frac{2}{m}R^{n+1}} = 2m^3R^{(1-n)}}$$

Again, a quick check with some values ($n = -1, m = 2, R = -3$) can help assure a correct result:

$$\frac{4m^2R^2}{\frac{2}{m}R^{n+1}} =? 2m^3R^{(1-n)}$$

$$\frac{4(2)^2(-3)^2}{\frac{2}{2}(-3)^{(-1+1)}} =? 2(2)^3(-3)^{(1-(-1))}$$
$$\frac{4(4)(9)}{(1)(-3)^0} =? 2(8)(-3)^2$$
$$144 = 144$$

► Retest for 32: do Supplementary Problem 40.

33. Some children drop pebbles into a well and listen for the splash in the water 50 feet below. The distance d (in feet) which a pebble falls from their hands in a time of t seconds is given by $d = 5t^2$ How far above the water is a pebble that has fallen for 2 seconds?

Solution: To solve problems stated in words and algebraic symbols, it is usually best to begin by translating systematically every relationship into algebraic equations. The readily-accessible information about the problem is then written down precisely in algebraic form, ready to be used. This approach does not guarantee a successful solution, but it does guarantee a good start (and with many problems, that is enough). Three steps are involved in the translation((see B-6e):

(a) Choose a different (but easily recognized and remembered) algebraic symbol for *every distinct* numerical quantity.

(b) Write as many equations as possible directly from the verbal statements.

(c) Finally, search the situation described by the verbal statements for any new relationships that are not directly stated. Then write equations equivalent to these relationships.

Once these equations are written and the desired quantity identified, it can often be obtained directly by solving the equations (see B-6f through B-6h). In the present case, starting with the numerical quantities in the problem [step (a)]:

$D =$ depth of well

$d =$ distance fallen by pebble

$t =$ time of fall

$h =$ height of pebble above water

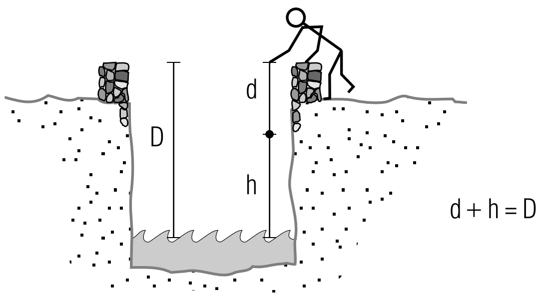
The relations following directly from the verbal statements are:

$D = 50$ feet

$d = 5t^2$

$t = 2$ seconds

We can easily find the distance d fallen by the pebble. But we want a *different* quantity, the height h of the pebble above the water. We need another relation, because none that we have involve h. Using step (c):



Solving for h (see F-7 and F-8):

$h = D - d$

$= D - 5t^2$

$= 50$ feet $-5(2)^2$ feet

$= 50$ feet -20 feet

Therefore, the solution is

$\boxed{h = 30 \text{ feet}}$

► Retest for 33: do Supplementary Problem 41.

34. Which of the following values of B satisfies the equation $\frac{A}{B} = 0.1$?

(a) $B = 0.1A$

(b) $B = 10A$

(c) $B = A + 10$

(d) none of these values.

Solution: See the Solutions following Problem 35 (below).

35. Is the value $x = 7/3$ a solution of the equation $9x - 2 = 6x + 5$? Check by substitution.

Solutions for Problems 34 and 35: A value is a *solution* for a symbol x in an equation (or the value for x is said to *satisfy* the equation) if the two sides are both the same algebraic quantity (number, symbol, or expression) when the value is substituted for x. Therefore, to check a proposed solution value, it is sufficient to substitute it for x in the equation and show that the two sides are in fact equal quantities. We shall follow this route with problems 34 and 35.

Solution for Problem 34: The equation in this case is $\frac{A}{B} = 0.1$. Checking the first proposed solution, $B = 0.1A$:

$0.1 =?\ \frac{A}{(0.1A)}$

$0.1 =?\ \frac{1}{0.1}$

$0.1 \neq 10$

Therefore, $B = 0.1A$ is *not* the solution. Checking the second proposed solution, $B = 10A$:

$0.1 =?\ \frac{A}{10A}$

$0.1 =?\ \frac{1}{10}$

$0.1 = 0.1$

Therefore, we need look no farther:

$\boxed{B = 10A}$

► Retest for 34: do Supplementary Problems 42 and 43.

Solution for Problem 35: The equation here is

$9x - 2 = 6x + 5$

Substituting the proposed solution $x = 7/3$:

$9(7/3) - 2 =?\ 6(7/3) + 5$

$3(7) - 2 =?\ 2(7) + 5.$

$19 = 19$

The value $x = 7/3$ is therefore the solution.

► Retest for 35: do Supplementary Problems 44 and 45.

36. $5h - 1 = 14 - \frac{5}{3}h \quad h = ?$

Solution: See Solutions following Problem 41.

37. $\frac{(h-7)}{4} + \frac{(h+2)}{3} = h \quad h = ?$

Solution: See Solutions following Problem 41.

38. $\frac{1}{x} + \frac{1}{y} = \frac{1}{f} \quad f = ?$

Solution: See Solutions following Problem 41.

39. $fx = e - 3gf \quad f = ?$

Solution: See Solutions following Problem 41.

40. $2b = vt + \frac{1}{2}at^2 \quad a = ?$

$b = 1.5$ $v = 10$ $t = 3$

Solution: See Solutions following Problem 41.

41. $\frac{q}{a} + \frac{Q}{a+d} = 0 \quad a = ?$ $q = +1$ $Q = -2$ $d = 5$

Solutions for Problems 36-41: Despite some apparent differences, these problems all require that a single process be carried out accurately: finding the solution of a linear algebraic equation. To solve an algebraic equation for a given symbol (say, x) the original equation is manipulated systematically to obtain an equivalent equation in which x appears *alone* on only *one* side of the equation. The algebraic quantity on the other side is the *solution* for x in the original equation (assuming, of course, the manipulations are correct).

The manipulations of the equation are restricted to applying the *same* arithmetic operations to *both* sides of the equation or replacing an expression anywhere in the equation by an equivalent expression (see B-6f). The following rough sequence of steps is usually a good path for solving any linear algebraic equation (see B-6g):

(a) *Eliminate all fractions* by multiplying both sides of the equation by the lowest common denominator of all fractions in the equation.

(b) *Collect* only those algebraic expressions containing the desired symbol x on *one* side of the equation, by adding suitable expressions to both sides.

(c) *Factor* x from these expressions and divide both sides by the resulting coefficient, thus leaving x alone on one side.

(d) *Substitute* any numerical values for symbols in the solution for x, and *simplify* this expression.

(e) *Check* the solution by substituting it for x everywhere in the original equation and checking that the equation is satisfied.

In carrying out these steps, it is usually wise to perform *one* step at a time and to*write* each step (see B-6d). The following solutions of problems 36 through 41 identify steps (a)-(e) where they apply, and follow the last bit of advice (perhaps too far).

Solution for Problem 36: The original equation, to be solved for h:

$$5h - 1 = 14 - \frac{5}{3}h$$

Multiply by 3, the lowest common denominator [step (a)]:

$$3(5h - 1) = (3)(14 - \frac{5}{3}h)$$

$$15h - 3 = 42 - 5h$$

Add 3 and $5h$ to both sides [step (b)]:

$$15h - 3 + 3 + 5h = 42 - 5h + 3 + 5h$$

$$20h = 45$$

Divide both sides by 20 [step (c)]:

$$\frac{20h}{20} = \frac{45}{20}$$

$$h = \frac{45}{20}$$

Simplify [step (d)]:

$$h = \frac{5 \cdot 9}{5 \cdot 4}$$

$$h = \frac{9}{4}$$

Check by substitution [step (e)]:

$$5\left(\frac{9}{4}\right) - 1 =?\, 14 - \frac{5}{3}\left(\frac{9}{4}\right)$$

$\frac{45}{4} - \frac{4}{4} =? \frac{56}{4} - \frac{15}{4}$

$\frac{41}{4} = \frac{41}{4}$

Hence, the correct solution is

$$\boxed{h = \frac{9}{4}}$$

Solution for Problem 37: The original equation is:

$\frac{(h-7)}{4} + \frac{(h+2)}{3} = h$

Multiply by the lowest common denominator, 12 [step (a)]:

$(12)\frac{(h-7)}{4} + (12)\frac{(h+2)}{3} = (12)h$

$3(h-7) + 4(h+2) = 12h$

$3h - 21 + 4h + 8 = 12h$

$7h - 13 = 12h$

Subtract $7h$ [step (b)]:

$7h - 7h - 13 = 12h - 7h$

$-13 = 5h$

Multiply by (1/5) [step (c)]:

$(1/5)(-13) = \left(\frac{1}{5}\right) 5h$

$-13/5 = h$

Check by substitution [step (e)]:

$\frac{(-13/5) - 7}{4} + \frac{\left(-\frac{13}{5}\right) + 2}{3} =? -\frac{13}{5}$

$\frac{\left(-\frac{13}{5}\right) - \frac{35}{5}}{4} + \frac{\left(-\frac{13}{5}\right) + \frac{10}{5}}{3} =? -\frac{13}{5}$

$\frac{(-48)}{5(4)} + \frac{(-3)}{3(5)} =? -\frac{13}{5}$

$\frac{(-12)}{5} + \frac{(-1)}{5} =? -\frac{13}{5}$

$-\frac{13}{5} = -\frac{13}{5}$

Therefore, the solution is

$$\boxed{h = -\frac{13}{5}}$$

► Retest for 36-37: do Supplementary Problems 46, 47 and 48.

Solution for Problem 38: The original equation, to be solved for f:

$\frac{1}{x} + \frac{1}{y} = \frac{1}{f}$

Multiply by the lowest common denominator, (xyf)[step (a)]:

$\frac{(xyf)1}{x} + \frac{(xyf)1}{y} = \frac{(xyf)1}{f}$

$yf + xf = xy$

Factor f and divide by its coefficient [step (c)]:

$f(y+x) = xy$

$\frac{f(y+x)}{(y+x)} = \frac{xy}{(y+x)}$

$f = \frac{xy}{(y+x)}$

Check by substitution [step (e)]:

$\frac{1}{x} + \frac{1}{y} =? \frac{1}{\left(\frac{xy}{y+x}\right)}$

$\frac{1}{x} + \frac{1}{y} =? \frac{(y+x)}{xy}$

$\frac{1}{x} + \frac{1}{y} =? \frac{y}{xy} + \frac{x}{xy}$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{x} + \frac{1}{y}$

Therefore, the solution is

$$\boxed{f = \frac{xy}{(x+y)}}$$

Solution for Problem 39:

The original equation, to be solved for f:

$$fx = e - 3gf$$

Add $(3gf)$[step (b)]:

$$fx + 3gf = e - 3gf + 3gf$$

$$fx + 3gf = e$$

Factor f, and divide by its coefficient (step (c)]:

$$f(x + 3g) = e$$

$$\frac{f(x+3g)}{(x+3g)} = \frac{e}{(x+3g)}$$

$$f = \frac{e}{(x+3g)}$$

Check by substitution [step (e)]:

$$\left(\frac{e}{x+3g}\right) x =?\, e - 3g\left(\frac{e}{x+3g}\right)$$

$$\frac{ex}{(x+3g)} =?\, e\frac{(x+3g)}{(x+3g)} - \frac{3ge}{(x+3g)}$$

$$\frac{ex}{(x+3g)} =?\, \frac{e(x+3g) - 3ge}{(x+3g)}$$

$$\frac{ex}{(x+3g)} =?\, \frac{ex + 3ge - 3ge}{(x+3g)}$$

$$\frac{ex}{(x+3g)} = \frac{ex}{(x+3g)}$$

Therefore, the solution is

$$\boxed{f = \frac{e}{(x+3g)}}$$

► Retest for 38-39: do Supplementary Problems 49, 50 and 51.

Solution for Problem 40: The original equation, to be solved for a:

$$2b = vt + \frac{1}{2}at^2$$

Multiply by the lowest common denominator, 2[step (a)]:

$$2(2b) = 2vt + 2\left(\frac{1}{2}\right)at^2$$

$$4b = 2vt + at^2$$

Subtract $(2vt)$ [step (b)]:

$$4b - 2vt = 2vt - 2vt + at^2$$

$$4b - 2vt = at^2$$

Divide by t^2 [step (c)]:

$$\frac{(4b - 2vt)}{t^2} = \frac{at^2}{t^2}$$

$$\frac{(4b - 2vt)}{t^2} = a$$

Substitute numerical values for b, v, and t, and simplify[step (d)]:

$$\frac{4(1.5) - 2(10)(3)}{(3)^2} = a$$

$$\frac{6 - 60}{9} = a$$

$$\frac{-54}{9} = a$$

$$-6 = a$$

Check by substitution (of values for a, b, v, and t),[step (e)]:

$$2(1.5) =?\, (10)(3) + \frac{1}{2}(-6)(3)^2$$

$$3 =?\, 30 + \frac{1}{2}(-54)$$

$$3 =?\, 30 - 27$$

$$3 = 3$$

Therefore the correct solution is

$$\boxed{a = -6}$$

Solution for Problem 41: The original equation, to be solved for a:

$$\frac{q}{a} + \frac{Q}{a+d} = 0$$

Multiply by the lowest common denominator, $a(a+d)$ [step (a)]:

$$a(a+d)\left(\frac{q}{a}\right) + a(a+d)\left(\frac{Q}{a+d}\right) = 0(a+d)a$$

$$(a+d)q + aQ = 0$$

$$aq + dq + aQ = 0$$

Subtract dq [step (b)]:

$$aq + dq - dq + aQ = 0 - dq$$

$$aq + aQ = -dq$$

Factor a and divide by its coefficient [step (c)]:

$$a(q+Q) = -dq$$

$$\frac{a(q+Q)}{(q+Q)} = -\frac{dq}{(q+Q)}$$

$$a = \frac{-dq}{(q+Q)}$$

Substitute values for d, Q, and q, and simplify , [step (d)]:

$$a = \frac{-(5)(1)}{(1+(-2))}$$

$$a = \frac{-5}{-1}$$

$$a = 5$$

Check by substitution (of values for a, q, Q, and d), [step (e)]:

$$\frac{(1)}{5} + \frac{(-2)}{(5+5)} =?\, 0$$

$$\frac{1}{5} - \frac{2}{10} =?\, 0$$

$$\frac{1}{5} - \frac{1}{5} = 0$$

Therefore, the correct solution is

$$\boxed{a = 5}$$

► Retest for 40-41: do Supplementary Problems 52, 53 and 54.

42. If $E = \frac{1}{2}mv^2$ and $p = mv$, express E in terms of m and p alone (i.e., eliminate v): $E = ?$

Solution: To eliminate a common quantity from two equations, the following steps are probably the simplest of many possible routes (see F-8):

(a) Solve *one* of the equations for the common quantity x.

(b) Substitute this solution for x everywhere in the *other* equation, which becomes the final result.

It is usually wise to proceed according to the fail-safe rule (B-6d) and, as these steps indicate, to use each of the two equations only *once* (otherwise, an unending circular path of trivial equations results).

In the present case, since the first equation alone contains E (which must appear in the final result) we begin by solving for v (the symbol to be eliminated) in the second equation [step (a)]:

$$p = mv$$

Dividing by m:

$$\frac{p}{m} = v$$

Substituting this solution for v in the first equation[step (b)]:

$$E = \frac{1}{2}mv^2$$

$$E = \frac{1}{2}m\left(\frac{p}{m}\right)^2$$

$$E = \frac{mp^2}{2m^2}$$

$$E = \frac{p^2}{2m}$$

To check, we can substitute some values for m and v (e.g., $m = -2$, $v = 4$) into the original equations to obtain values for E and p, and compare the value for E with that obtained by substitution in the final equation (see B-6d):

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(-2)(4)^2 = (-1)(16) = -16$$

$p = mv = (-2)(4) = -8$

From the final equation,

$$E = \frac{p^2}{2m} = \frac{(-8)^2}{2(-2)} = \frac{64}{-4} = -16$$

Therefore, it is likely that the correct result is

$$\boxed{E = \frac{p^2}{2m}}$$

► Retest for 42: do Supplementary Problem 55.

43. $y = ay^2 + b\,; \qquad y = ?$

Solution: Now use the "quadratic formula," which states that, if

$\alpha y^2 + \beta y + \gamma = 0\,,$

then:

$$y = \frac{+1 \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}\,.$$

So for our equation:

$$y = \frac{+1 \pm \sqrt{1 - 4ab}}{2a} = 0.69 \text{ or } 1.81$$

As a check, substitute back into the original equation and see that each of the two numerical solutions ($y = 0.69$ and $y = 1.81$) satisfies it.

SECT.

F SUPPLEMENTARY PROBLEMS

ELEMENTARY ARITHMETIC

Calculate the value (to three decimal places) of these decimal numbers:

1. $(3.4) \times (0.74) = ?$
2. $1.34/0.03 = ?$
3. $53.2 - 5.54 = ?$
4. Express 7/9 as a simple decimal: $7/9 = ?$

Find the value (in simplest fraction form) of:

5. $\frac{2}{3} \times \frac{5}{8} \times \frac{9}{16} = ?$
6. $\frac{1}{5} + \frac{5}{12} - \frac{3}{4} = ?$
7. $\frac{2}{3} \div \frac{5}{9} = ?$

8. Jill usually buys 0.6 lbs. of meat for dinner for herself and Dick. If the meat needed for a meal is proportional to the number of people eating the meal, how much meat is needed to make dinner for five people?

POWERS

Calculate the numerical value (an integer, a decimal, or a fraction in simplest form) of these numbers:

9. $(-0.1)^3 = ?$
10. $3^{-2} = ?$
11. $(5^1 \times 5^{-2})^2 = ?$
12. $\frac{5^{-3}}{5^{-4}} = ?$

13. $\left(\frac{4}{25}\right)^{1/2} = ?$

14. $(3^2)(3^{-2}) = ?$

15. $[(2^3)(3^2)]^{-1} = ?$

SCIENTIFIC NOTATION

16. Arrange these numbers in order of increasing value (e.g., as in the series 1,2,3,4,5,6):
    $2 \times 10^{-2}, 0, -2 \times 10^{-2}, 1.0, -0.2 \times 10^{2}, -200$

Calculate the value (in scientific notation) of these numbers:

17. $(-5.5 \times 10^3)(2.0 \times 10^{-4}) = ?$

18. $(-3.4 \times 10^{-3})(2.0 \times 10^{-4}) = ?$

19. $(1.0 \times 10^3)^3 = ?$

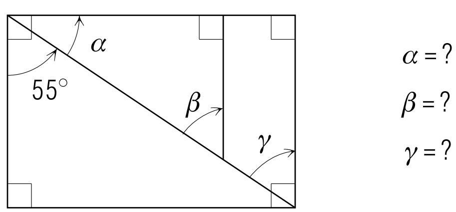
20. $(0.64 \times 10^6)^{1/2} = ?$

21. $(1.25 \times 10^5)^{1/3} = ?$

GEOMETRY

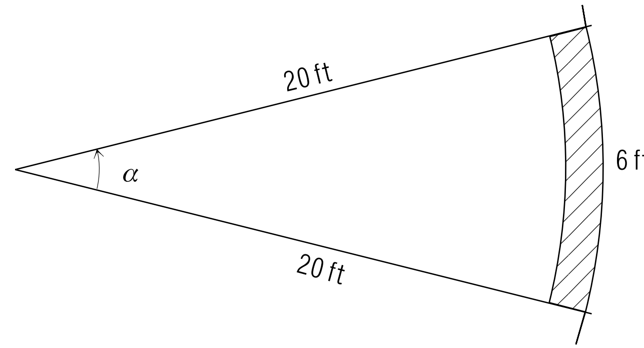
In the following examples you may use $\pi = 3.14$.

22. Find the values (in degrees) of the angles α, β, γ in the figure below.

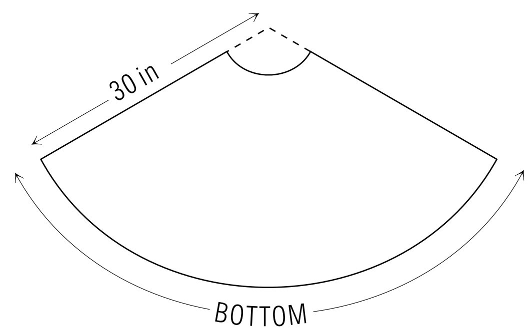


23. Wooden arches for doors are sometimes made by steaming a piece of wood and clamping it in the desired curve until it cools and dries. If a piece of wood 6 ft. in length is clamped to form a circular arc with

radius 20 ft, what is the angle α (in radians) which is spanned by the curved wood?

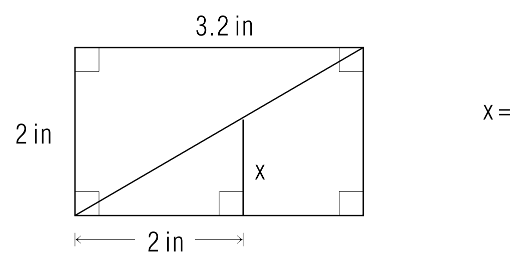


24. A skirt is made from one third of a circular piece of cloth, as shown in the drawing.

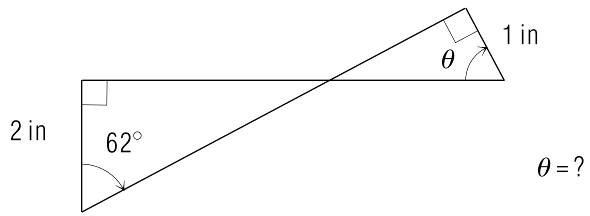


If the original circular piece of cloth had a radius of 30 in., how many inches will the skirt measure around the bottom? (Neglect seams.)

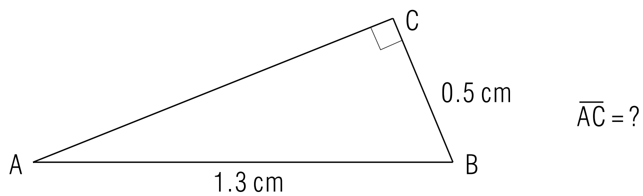
25. In the following drawing, what is the length x?



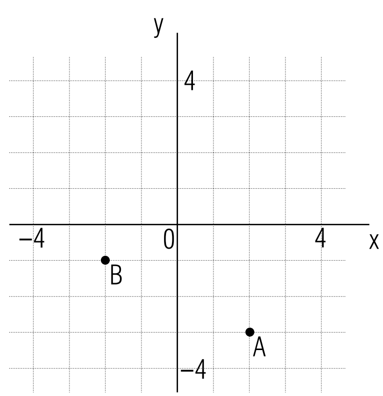
26. In the following drawing, what is the value (in degrees) of angle θ?



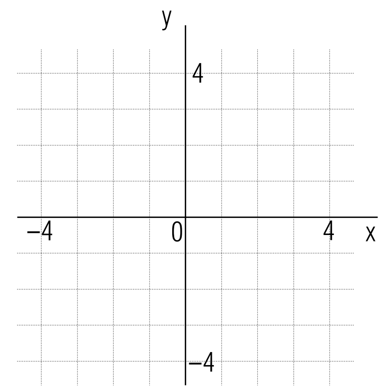
27. In right triangle ABC, what is the length of side $\overline{AC}$ in cm?



28. In the Cartesian reference frame shown below, find the coordinates of points A and B.

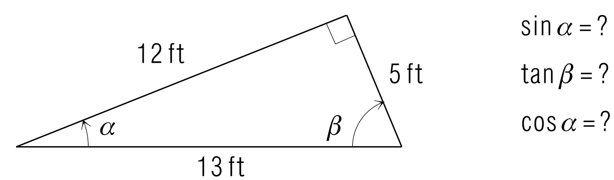


29. Place the following points A and B with respect to the coordinates given below. Point A has x coordinate 3 and y coordinate -2. Point B has x coordinate -3 and y coordinate 0.

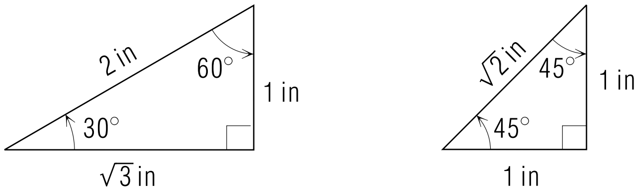


TRIGONOMETRY

30. Use the triangle below to find the value (to two decimal places) of the indicated trigonometric quantities.

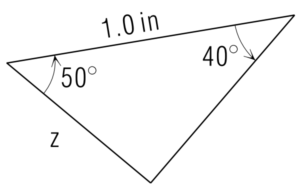


31. Use the $30° - 60°$ and $45° - 45°$ right triangles shown below to fill in the table giving the values of the sine, cosine, and tangent of $30°, 45°$, and $60°$. Leave your entries in the form of fractions.



Angle	Sine	Cosine	Tangent
30°			
45°			
60°			

32. Determine the length of side z in this triangle:

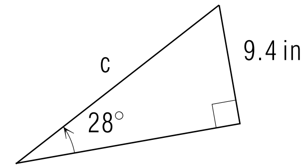


Find the answer in *two* ways:

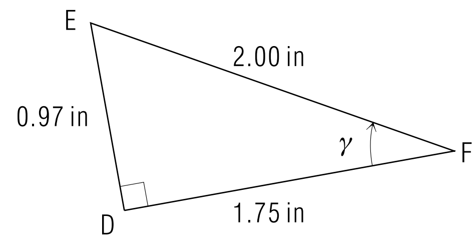
(a) using the cosine of $50°, z =$?

(b) using the sine of $40°, z =$?

33. Find the length of side c in this triangle:



34. What is the value (to the nearest degree) of angle γ in the triangle DEF?



Find the answer in two ways:

(a) by calculating $\tan\gamma$, $\gamma =$?

(b) by calculating $\cos\gamma$, $\gamma =$?

35. Without referring to any other material, give the values of each of the following quantities. (You may find a sketch useful.)

$\sin\pi/2 =$?

$\cos 270° =$?

$\sin 2\pi =$?

$\sin 3\pi/2 =$?

$\cos 180° =$?

$\cos\pi/2 =$?

$\sin 180° =$?

$\cos 2\pi =$?

ALGEBRA

36. Calculate the numerical value of the following expressions when $b = -3/2$:

$(2b+1) = ?$

$\dfrac{(b+1)}{(b-1)} = ?$

$6b^0 = ?$

$4b^2 + 1 = ?$

$(-2)b^1 = ?$

$3b^{-1} = ?$

37. Find the numerical value of these expressions when $m = 2$ and $n = 0$:

$m^n = ?$

$4^m - n^2 = ?$

$n^m = ?$

$mn = ?$

$\dfrac{n}{m} = ?$

38. Simplify the following expression to obtain a single algebraic fraction in simplest form. (Follow the steps outlined in B-6c for practice.)

$$\frac{(A+1)}{A^2} + \frac{(A-1)}{A} - \frac{A}{A^3} = ?$$

39. Perform the indicated addition and simplify the result to an algebraic fraction in simplest form:

$$\frac{4m}{(m+n)} + \frac{4n}{(m-n)} = ?$$

40. Simplify these expressions to the indicated forms:

(a) $\left(\dfrac{q}{p^3}\right)\dfrac{2p^2q^3}{1/4\left(\dfrac{q^4}{p}\right)} = (\ \ )p^{(\ )}q^{(\ )}$

(b) $\dfrac{m\left(\dfrac{2\pi R}{mT}\right)^2}{RT^3} = \dfrac{(\ \ )R^{(\ )}}{m^{(\ )}T^{(\ )}}$

41. On January 1, 1969, the population of a certain town in California was 50,000 people. For any time t years after this date, the population has been found to vary according to the formula:

$$P = 50{,}000 + 2000t + 1000t^2$$

What was the population increase in this town after a time period of $t = 2$ years?

42. Which of the following values of C satisfies the equation

$\dfrac{A}{A+C} = \dfrac{2}{3}$? Check by substitution.

(a) $C = 2A$, (b) $C = 3A$, (c) $C = \dfrac{1}{3}A$, (d) none of these.

43. Which of the following values of v satisfies the equation

$\dfrac{1}{2}mv^2 = mgh$?

(a) $v = \sqrt{gh}$, (b) $v = \sqrt{\dfrac{2gh}{m}}$, (c) $v = \sqrt{2gh}$,

(d) none of these.

44. Which of the following values for y satisfies the equation

$4y + \dfrac{5}{3} = \dfrac{9}{4} - \dfrac{2}{3}y$?

(a) $y = -\dfrac{1}{2}$, (b) $y = 2$, (c) $y = \dfrac{1}{8}$, (d) none of these.

45. Is the value $b = -1$ a solution to the equation

$2(3-b) = 4(b+1)$?

46. $\dfrac{4}{3}z - \dfrac{9}{2} = -\dfrac{10}{3} - 8z \quad z = ?$

47. $\dfrac{(z+1)}{6} - \dfrac{(z-4)}{4} = \dfrac{1}{2}z \quad z = ?$

48. $\dfrac{1}{3}(2z+1) = \dfrac{1}{9}(z-2) - \dfrac{1}{6}z - \dfrac{8}{9} \quad z = ?$

49. $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c} \quad b = ?$

50. $3by = \dfrac{3}{2} + 6bx \quad b = ?$

51. $\dfrac{a}{b} = \dfrac{c}{1-b} \quad b = ?$

52. $h - \frac{1}{3}x^2 c = bx \quad c = ?$

$h = -6$

$x = 3$

$b = -1$

53. $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E \quad k = ?$

$v = -2$

$m = 1.5$

$x = -1$

$E = 10$

54. $a = \frac{q^2}{mR^2} - g \quad m = ?$

$q = -3$

$R = 2$

$g = 10$

$a = -1$

55. If $x = vt - \frac{1}{2}at^2$ and $v - at = 0$, express x in terms of v and a alone (i.e., eliminate t).

$x = ?$

SECT.

G SUPPLEMENTARY PROBLEM ANSWERS

ELEMENTARY ARITHMETIC

1. 2.516
2. 44.667
3. 47.660
4. 0.778
5. 15/64
6. −2/15
7. 6/5
8. 1.5 lbs.

POWERS

9. −0.001
10. 1/9
11. 1/25
12. 5
13. 2/5
14. 1
15. 1/72

SCIENTIFIC NOTATION

16. $-200,\ -0.2\times10^2,\ -2\times10^{-2},\ 0,\ 2\times10^{-2},\ 1.0$

17. -1.1 or -1.1×10^0

18. -6.8×10^{-7}

19. 1.0×10^9 or 10^9

20. 8.0×10^2

21. 5.0×10^1 or 50

GEOMETRY

22. $\alpha = 35°, \beta = 55°, \gamma = 55°$

23. $\alpha = 0.3$ rad

24. 62.8 in.

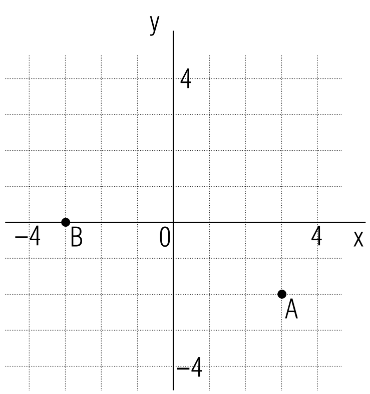
25. $x = 1.25$ in.

26. $\theta = 62°$

27. $\overline{AC} = 1.2$ cm.

28. $A: x = 2,\ \ y = -3\ \ B: x = -2\ \ y = -1$

29.



TRIGONOMETRY

30. $$\sin\alpha = \frac{5\,\text{ft}}{13\,\text{ft}} = 0.38$$
$$\tan\beta = \frac{12\,\text{ft}}{5\,\text{ft}} = 2.40$$
$$\cos\alpha = \frac{12\,\text{ft}}{13\,\text{ft}} = 0.92$$

31.

Angle	Sine	Cosine	Tangent
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

32. $$z = (1.0\text{ in.})\cos 50° = 0.64\text{ in.}$$
$$z = (1.0\text{ in.})\sin 40° = 0.64\text{ in.}$$

33. $$c = \frac{9.4\,\text{inches}}{\sin 28°} = \frac{9.4\,\text{inches}}{0.47} = 20.0\,\text{inches}$$

34. $$\tan\gamma = \frac{0.97\,\text{in}}{1.75\,\text{in}} = 0.554 \text{ so } \gamma = 29°$$
$$\cos\gamma = \frac{1.75\,\text{in}in}{2.00\,\text{in}} = 0.875 \text{ so } \gamma = 29°$$

35. $$\sin\frac{\pi}{2} = \frac{+r}{r} = +1$$
$$\cos 270° = \frac{0}{r} = 0$$
$$\sin 2\pi = \frac{0}{r} = 0$$

$$\sin\frac{3\pi}{2} = \frac{-r}{r} = -1$$
$$\cos 180^\circ = \frac{-r}{r} = -1$$
$$\cos\pi/2 = \frac{0}{r} = 0$$
$$\sin 180^\circ = \frac{0}{r} = 0$$
$$\cos 2\pi = \frac{+r}{r} = +1$$

ALGEBRA

36. $(2b+1) = -2$
$\frac{(b+1)}{(b-1)} = +\frac{1}{5}$
$6b^0 = 6$
$4b^2 + 1 = 10$
$(-2)b^1 = 3$
$3b^{-1} = -2$

37. $m^n = 1$
$4^m - n^2 = 16$
$n^m = 0$
$mn = 0$
$\frac{n}{m} = 0$

38. 1

39. $\frac{4(m^2+n^2)}{(m+n)(m-n)} = \frac{4(m^2+n^2)}{m^2-n^2}$

40. (a) $(8)P^{(0)}q^{(0)} = 8$ (b) $\frac{(4\pi^2)R^{(1)}}{m^{(1)}T^{(5)}}$

41. 8000 people

42. (d)

43. (c)

44. (c)

45. No.

46. $z = \frac{1}{8}$

47. $z = 2$

48. $z = -2$

49. $b = \frac{ac}{c-a}$

50. $b = \frac{1}{2(y-2x)}$

51. $b = \frac{a}{c+a} = \frac{1}{1+\frac{c}{a}}$

52. $c = -1$

53. $k = 14$

54. $m = 1/4$

55. $x = \frac{v^2}{2a}$