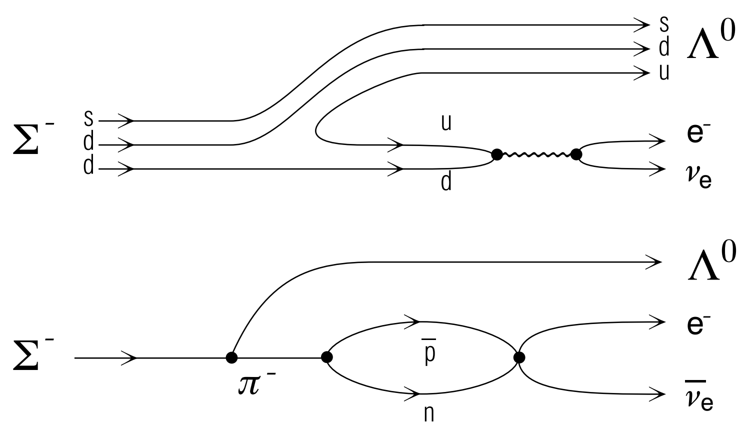
PHYSNET

MISN-0-277

PROPERTIES CONSERVED IN STRONG AND EM INTERACTIONS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PROPERTIES CONSERVED IN
STRONG AND EM INTERACTIONS
by
J. Christman

Title: **Properties Conserved in Strong and Em Interactions**

Author: J. R. Christman, Dept. of Physical Science, U. S. Coast Guard Academy, New London, CT

Version: 11/12/2001 Evaluation: Stage B1

Length: 2 hr; 0 pages

Input Skills:

1. Discuss the concept of operators, and illustrate their use.
2. Give the charge and spin of the strong-force stable particles (MISN-0-274).
3. State which properties are always conserved (MSN-0-275).

Output Skills (Knowledge):

K1. Explain how strangeness conservation or non-conservation can affect particle lifetimes.

K2. Give the definition of the charge conjugation operation and which interactions do and which do not violate charge conjugation invariance.

K3. Give the definition of the time reversal operation and state which interactions do and which do not violate time reversal invariance.

Output Skills (Rule Application):

R1. Calculate the hypercharge and strangeness of a particle, given the charge of the members of its multiplet and its baryon number.

R2. Draw diagrams showing what happens to the spin and momentum of a given single particle or of two colliding particles due to application of $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, $\mathcal{CP}$, $\mathcal{CT}$, $\mathcal{PT}$ and $\mathcal{CPT}$, whether that produces real particles or not.

R3. In general or for a given reaction, relate the possible interaction(s) to the production or non-production of particles with strangeness.

External Resources (Required):

1. See the "Readings" section of this module's *text*.

External Resources (Optional):

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1. Abstract

This module deals with the peculiar conservation laws obeyed by both the electromagnetic and strong interactions, which also obey the universal conservation laws. In a given situation, if all possible outcomes violate the laws obeyed by these two interactions, then nature is forced to resort to the weak interaction and this shows up experimentally in much longer interaction times. If the weak interaction is also not possible, then there is no interaction.

2. Readings

1. Ford, Vol. 3, Sect. 27.6 through 27.9, on reserve for you in the Physics-Astronomy Library. Ask for it as "Ford, Volume 3."
2. E. P. Wigner, "Violations of Symmetry in Physics," *Scientific American*, Dec. 1965, on reserve for you in the Physics-Astronomy Library. Ask for it as the CBI readings "Violations of . . . ."
3. Suggested: W. R. Frazer, *Elementary Particles*, if you happen to have it.

3. Multiplets

3a. Type of Property. Strangeness is a number assigned to a particle just like charge or baryon number. It is relevant for mesons and baryons.

3b. Multiplet Groupings: M, S, Q, Y. Hadrons can be arranged in small groups with all members of the group having nearly the same mass. For ease in remembering, the members of a group are called by the same name (pions, kaons, nucleons, antikaons, etc). The groups are called multiplets and the number of particles in a group is called the multiplicity of that group. All members of a multiplet have the same strangeness and it is equal to twice the average charge (in units of e) of the multiplet minus the baryon number. Twice the average charge is called the *hypercharge*

and is denoted by Y. Thus $S = Y - B$. This formula must be altered for particles that have *Charm*, another quantum number which will be introduced in a later lesson. All leptons and photons are assigned $S = 0$.

3c. Table; 9 Multiplets. Here is the baryon number (B), hypercharge (Y) and strangeness (S) of each strong-stable non-charmed meson and baryon:

Multiplet	Members	$Q_{ave.}$	Y	B	S
Pion	π^+, π^0, π^-	0	0	0	0
Kaon	K^+, K^0	1/2	1	0	1
Anti-Kaon	K^-, $\overline{K}^0$	-1/2	-1	0	-1
Eta	η^0	0	0	0	0
Nucleon	p, n	1/2	1	1	0
Lambda	Λ^0	0	0	1	-1
Sigma	Σ^+, Σ^0, Σ^-	0	0	1	-1
Xi	Ξ^0, Ξ^-	-1/2	-1	1	-2
Omega	Ω^-	-1	-2	1	-3

Antiparticles have the same magnitude of strangeness as the respective particle but opposite in sign. Note: the K^- is the anti-particle of the K^+ because the K's are mesons, which are bosons so the antiparticle is just an ordinary particle. The antiparticle of the Σ^+ is negatively charged, but it is not the Σ^-. The Σ's are baryons, which are fermions, so their antiparticles are not just ordinary particles.

4. Strangeness

4a. "Conservation," "Interaction". The total strangeness for a collection of particles is the algebraic sum of the individual strangeness numbers. Every strong and electromagnetic decay or interaction conserves total strangeness. Weak decays and interactions do not.

4b. Strangeness-Violating Decays: Nine Examples. One important consequence of strangeness conservation is that it prevents the decay of certain particles via the strong or electromagnetic interaction. These particles have the long lifetimes (10^{-10} sec) associated with weak decay rather than the shorter lifetimes associated with strong (10^{-23} sec) or electromagnetic (10^{-21} sec) decays.

Refer to the table of Note 3. The following decays are all weak and *all violate conservation of strangeness*:

$$\begin{aligned}
\mathrm{K}^0 &\rightarrow \pi^0 + \pi^0 \\
\mathrm{K}^0 &\rightarrow \pi^0 + \pi^- \\
\mathrm{K}^0 &\rightarrow \pi^0 + \mathrm{e}^- + \bar{\nu}_{\mathrm{e}} \\
\Lambda^0 &\rightarrow \mathrm{p} + \pi^- \\
\Sigma^0 &\rightarrow \mathrm{n} + \pi^0 \\
\Sigma^- &\rightarrow \mathrm{n} + \pi^- \\
\Xi^- &\rightarrow \mathrm{n} + \pi^- \\
\Xi^- &\rightarrow \Lambda^0 + \pi^- \\
\Omega^- &\rightarrow \Xi^0 + \pi^{\cdot}
\end{aligned}$$

In each case all of the *universal* conservation laws hold and there is no reason (except conservation of strangeness and, in one case, conservation of parity, to be discussed later) why these decays could not go via the stronger and faster interactions. All of the particles (initial and final) in the interactions above, except the electron and neutrino which appear in the K^0 decay, do participate in the strong interaction.

4c. Combining Consv. Laws: S, E, B. It is worthwhile to understand why the K^+, Λ^0, Ξ^0, and Ω^- cannot decay via the strong interaction or the electromagnetic interaction. The K^+ is the lightest strange particle and a combination of the energy and strangeness conservation laws forces its decay to be weak.

The K^+ typically decays according to

$$\mathrm{K}^+ \rightarrow \pi^+ + \pi^0$$

with a lifetime on the order of 10^{-8} sec, a weak decay. This decay violates conservation of strangeness.

The Λ^0 is the lightest strange *baryon* and conservation of energy, strangeness, and baryon number precludes its decay via the strong or electromagnetic interactions.

The Ξ^0, if it decayed strongly or electromagnetically, must decay to particles with a net strangeness of -2. Clearly neither two Σ's, nor two Λ's, nor a Σ and a Λ, nor a Σ and a K, nor a Λ and a K, meet energy conservation requirements. Two K's have less total mass and conserve strangeness and charge. However the products must include a baryon and all baryons

have too much mass to join the kaons as decay products. The Ξ^0 must decay weakly. In fact, the most probable decay is

$$\Xi^0 \rightarrow \Lambda^0 + \pi^0$$

with a lifetime of 3.0×10^{10} sec. This decay violates conservation of strangeness.

That the Ω^- also decays weakly can be predicted by invoking conservation of strangeness, energy and baryon number.

4d. A Strange Case: Sigma decay. The decay $\Sigma^0 \rightarrow \Lambda^0 + \gamma$ does not violate conservation of strangeness and is *not* weak. The decays of the charged sigmas are weak but their strong decay would not violate any of the conservation laws discussed so far. These decays will be discussed later.

4e. A Contrast: S vs. $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$. The remaining conservation laws of this section will be stated in a form which is different from that of the previous laws. That is, we shall not assign numbers to the particles which are interacting but rather we shall make the statement that two interactions have or do not have certain identical characteristics.

5. Charge Conjugation

5a. The $\mathcal{C}$ Operator. Charge conjugation is the operation of changing a particle into its antiparticle or vice versa. That is,

$$\mathcal{C}(\pi^+) = \pi^-$$

$$\mathcal{C}(\mathrm{p}) = \overline{\mathrm{p}}$$

$$\mathcal{C}(\overline{\mathrm{p}}) = \mathrm{p},$$

etc. Here $\mathcal{C}$ stands for the operation of charge conjugation.

5b. Invariance Under $\mathcal{C}$: Strong, Weak. The strong and electromagnetic interactions are said to be invariant under charge conjugation. This means that the strong interaction between two particles, A and B, is precisely the same as the strong interaction between the two antiparticles $\bar{A}$ and $\bar{B}$ in the sense that these two interactions have the same strength. If, in the first interaction, A is scattered through a certain angle with a certain probability, then, in the second interaction, $\bar{A}$ is scattered through the same angle with the same probability provided, of course, the experimental conditions (initial energy, momentum, and spin) are the same. If

$A \to B + C$ via the strong interaction with a certain decay time, then $\bar{A} \to \bar{B} + \bar{C}$ with the same decay time. Analogous statements about the electromagnetic interaction are also true.

5c. $\mathcal{C}$ Non-Invariance: Weak. Violation of charge conjugation invariance occurs in weak interactions and decays. As an example, consider the two decays

$$\mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu \,,$$

and

$$\mu^+ \to \mathrm{e}^+ + \nu_\mathrm{e} + \bar{\nu}_\mu \,,$$

where both muons are in states with spin in the same direction, for example. Experimentally, it is found that the electrons from the μ^- tend to leave the decay site preferentially in the direction opposite to the spin direction of the μ^- while the positrons from μ^+ tend to leave preferentially in the same direction as the spin of the μ^+. The decay is not invariant under charge conjugation.

6. Parity

6a. The $\mathcal{P}$ Operator, on x, p, L, S. The parity operator, denoted $\mathcal{P}$, reverses the components of all true vectors. For example, the parity operation on radius and momentum vectors produces:

$$\mathcal{P}(\vec{r}) = \mathcal{P}(x\hat{x} + y\hat{y} + z\hat{z}) = -x\hat{x} - y\hat{y} - z\hat{z} = -\vec{r}\,.$$

$$\mathcal{P}(\vec{p}) = \mathcal{P}(p_x\hat{x} + p_y\hat{y} + p_z\hat{z}) = -p_x\hat{x} - p_y\hat{y} - p_z\hat{z} = -\vec{p}\,.$$

Thus a radius vector gets reflected through the coordinate-space origin while any other true vector, like $\vec{p}$, gets reversed in coordinate space. Note that angular momentum is the product of two vectors so it does *not* get reversed by the parity operator:

$$\mathcal{P}(\vec{L}) = \mathcal{P}(\vec{r} \times \vec{p}) = (-\vec{r}) \times (-\vec{p}) = \vec{L}\,.$$

Angular momentum is not a true vector; in mathematics it is called a *pseudovector*. Spin is a form of angular momentum so it too does not change sign under the parity operation.

6b. Inv. Under $\mathcal{P}$: Strong & EM, not Weak. An elementary particle reaction is said to be parity invariant if the parity operator, acting on both sides of the reaction equation, produces another reaction which is

found, experimentally, to occur with exactly the same probability. That is, suppose we have particles A, B, C, and D which appear in this reaction:

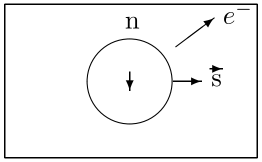
$$A + B \to C + D\,,$$

and

$$\mathcal{P}(A) = A', \;\; \mathcal{P}(B) = B', \;\; \mathcal{P}(C) = C'\,.$$

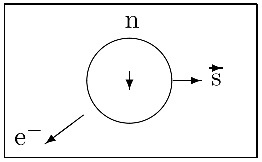
Then the interaction which produces C and D from A and B is said to be parity invariant if the two reactions, $A+B \to C+D$ and $A'+B' \to C'+D'$, are found, experimentally, to occur with equal probability.

6c. $\mathcal{P}$ and Weak Interactions, Neutrinos. Strong and electromagnetic interactions are invariant under the parity operation. The weak interaction is not. The first and most famous experiment to show this is the β decay of the neutron $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$. It is found experimentally that a neutron at rest emits electrons preferentially into the hemisphere centered on the direction of the spin:



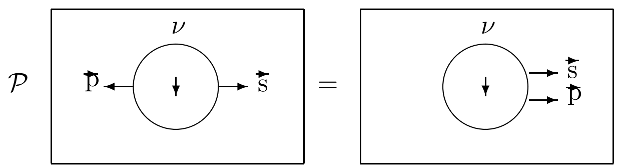
The neutron, represented by the circle, exists only before the decay while the electron exists only after the decay. The downward arrow on the "surface" of the neutron represents the direction the surface of the neutron is traveling to produce the spin vector $\vec{s}$.

Now the picture after the parity operation looks like this:



The spin of the neutron does not change but the position and momentum of the electron do. Now there is nothing physically different about the *initial conditions* in the two cases—both start with the same neutron. If parity invariance were not violated, the electrons should come out with equal probability in the hemisphere centered on the neutron's spin and in the opposite hemisphere. The fact that the electrons come out preferentially in the hemisphere centered on the spin constitutes proof that the weak interaction violates parity invariance.

6d. E **&** S **&** B **&** ℓ **&** $\mathcal{P}$ **for Weak Decays.** The neutrino by itself is a violator of parity invariance. The only neutrinos which exist are those with spins that are in the exact opposite direction to their momenta. If we operate on the neutrino with the parity operator, we get this:



which is a neutrino with its spin in the same direction as it momentum, and this does *not* exist in nature. That is, there is zero probability of finding it in nature.

Violation of parity invariance by the weak interaction is more general than the violation which must occur when a neutrino is produced. The decay

$$\Lambda^0 \to \mathrm{p} + \pi^- ,$$

is weak, does not include a neutrino, and violates parity invariance.

6e. $\mathcal{CP}$ **Invariance and Violation.** The K^0, Σ^+, Σ^-, and Ξ^- decay weakly. To show that no strong or electromagnetic decays are possible, one must invoke conservation of energy, charge, strangeness, baryon number, electron family number, and parity.

As an example, consider the strong or electromagnetic decay of the Σ^+. To conserve strangeness, which strong and electromagnetic decays require, the Σ^+ must decay either to a Λ^0 or a kaon, the only strange particles with less mass.

Consider first the decay to a Λ^0. The other decay products must have net strangeness 0, total mass less than 73.8 MeV, and charge +1. These conditions can be met only if a positron appears among the products. Other products must have net charge 0, and electron family number +1.

Only the electron's neutrino will do. Since a neutrino is required, parity invariance is violated and the decay cannot be strong or electromagnetic. The appearance of the neutrino signals the violation of parity invariance.

Now consider the possibility of decay to a $\overline{\mathrm{K}}^0$. Strangeness is conserved. Other products must have net charge +1, mass less than 695.6 MeV, and strangeness 0. In addition, at least one baryon must be involved. There is no baryon with mass less than 695.6 eV. Thus this decay cannot occur via the strong or electromagnetic interactions.

In fact, the decay most often observed is

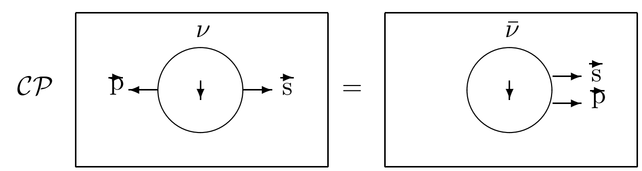
$$\Sigma^+ \to \mathrm{n} + \pi^+ ,$$

and its lifetime of 0.8×10^{-10} sec shows that it is a weak decay.

Arguments for the weak decay of the K^0, Σ^-, and Ξ^- follow the same lines.

7. Time Reversal

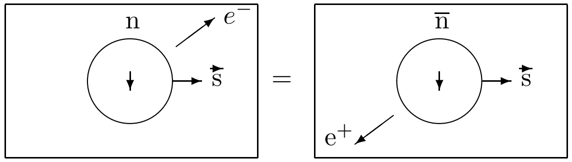
7a. The $\mathcal{T}$ **Operator: Inv. for Strong, EM.** Some weak interactions are $\mathcal{CP}$ invariant. The product of $\mathcal{C}$ and $\mathcal{P}$, $\mathcal{CP}$ is an operator which simultaneously reflects the system in the origin and changes particles to antiparticles (and antiparticles to particles). For example, $\mathcal{CP}$ operating on a neutrino produces



and this particle *does* exist in nature. In fact, the antineutrino's spin is *always* in the same direction as its momentum.

$\mathcal{CP}$ operating on neutron decay produces

$\mathcal{CP}$



This also occurs in nature, with probability equal to that of neutron decay. That is, antineutrons decay with positrons coming out preferentially in a directions opposite to the antineutrons' spins.

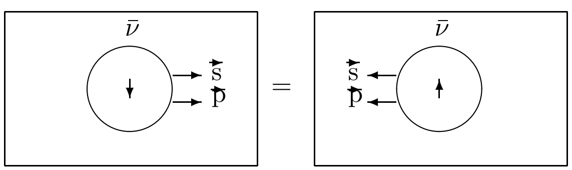
There is some evidence for $\mathcal{CP}$ violation in the decay of the kaon and it is not clear whether or not $\mathcal{CP}$ is violated in a small way in other weak interactions. For example the distribution of positrons from $\bar{n}$ decay might be slightly different from the reflection in the origin of the distribution of electrons from n decay. The difference, if any, has not been detected.

7b. $\mathcal{T}$ Inv. and Weak Decays: $\mathcal{CPT}$ Inv. The time reversal operator $\mathcal{T}$ makes time run backwards. This reverses both momentum and spin and changes ingoing particles to outgoing and outgoing to incoming. If the reaction and the time-reversed reaction take place with equal probability, then the interaction is said to be time reversal invariant.

Strong and electromagnetic interactions have been found to be time reversal invariant.

As an example, here is the effect of time-reversal on the anti-neutrino:

$\mathcal{T}$



,

which is also just an anti-neutrino (but going the other way and with its spin reversed!).

There is some question about whether or not weak interactions are time reversal invariant. The argument is indirect. Many physicists believe that all interactions must be $\mathcal{CPT}$ invariant. That is, if an interaction can occur, then the interaction obtained from it by successive operation

of the three operators also occurs with the same probability. If this is true and it is also true that $\mathcal{CP}$ is violated in weak interactions, then $\mathcal{T}$ must also be violated in just the right way to make weak interactions $\mathcal{CPT}$ invariant.

Acknowledgments

Preparation of this module was supported in part by The United States Coast Guard Academy for a Directed Studies Program. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.