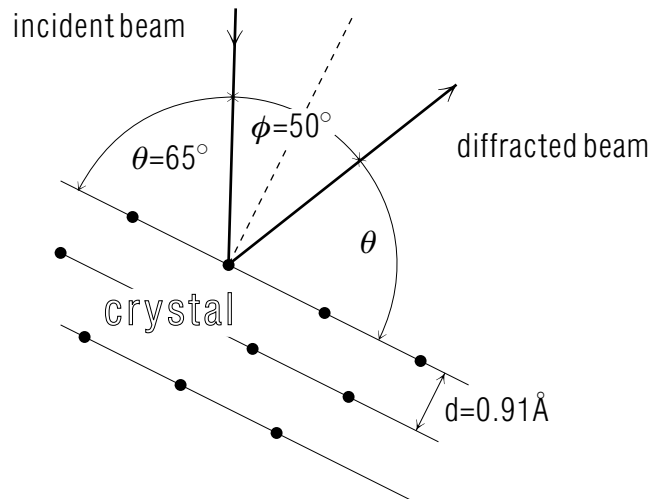


DE BROGLIE WAVES



DE BROGLIE WAVES

by

Kirby Morgan, Charlotte, Michigan

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Input Skills:

1. State the equations relating the energy and momentum of a photon to the frequency and wavelength of the light and use constants h , c , and hc , with units in eV, MeV, etc. (MISN-0-212).
2. Describe the diffraction pattern produced by a single slit (MISN-0-235) or (MISN-0-432).
3. Describe Bragg diffraction (MISN-0-237) or (MISN-0-432).
4. Predict whether a given light phenomenon can be satisfactorily described by a particle or wave model (MISN-0-246).

Output Skills (Knowledge):

- K1. Describe the Davisson-Germer electron diffraction experiment. State the relevant equations and indicate the appropriate variables with a sketch.
- K2. State the Principle of Complementarity and tell why the wave and particle theories of matter are incompatible and mutually exclusive.
- K3. Explain the significance of the wave function associated with a particle and use the diffraction of electrons by a single slit as an illustration.

Output Skills (Rule Application):

- R1. Calculate the de Broglie wavelength of a particle, given its momentum or kinetic energy.

Post-Options:

1. "The Uncertainty Relations: Description, Applications" (MISN-0-241).

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DE BROGLIE WAVES

by

Kirby Morgan, Charlotte, Michigan

1. Introduction

1a. A Dual Nature of Matter. Electrons, protons, and other sub-microscopic objects have a strange property that is not seen in the microscopic and macroscopic world. At times a single electron appears to be a very small particle-like object, moving through space on a definite trajectory, but at other times it appears to be wave-like, extended over a very large region of space, with its “path” being that of a spread-out wave. It can also exhibit such wave properties as interference and diffraction. How can a single entity act sometimes like a localized particle and at other times like a spread-out wave? The explanation for this “wave/particle duality” is provided by revisions made to the laws of mechanics in this century.

1b. Classical Mechanics Found to be Limited. In the first part of the 20th century it became apparent that the principles of Classical Mechanics, which were perfectly adequate to describe the motions of apples, people and planets, were not adequate to describe the motions of the sub-microscopic constituents of matter. The problem was that Classical Mechanics was developed on the assumption that particles can be observed without appreciably disturbing their motions. However, when dealing with the smallest constituents of matter, one finds that there is no way to measure some properties of a particle without significantly changing other properties in ways that are not completely predictable. Thus, new laws of motion were required that would describe such behavior of atomic particles as well as describe the usual phenomena of classical physics. The new and broader set of laws that developed are known collectively as “Quantum Mechanics.” These laws provide a new and astonishing explanation for the wave/particle duality of matter.

2. Wave-Particle Duality

2a. Motivation for a Duality Search. The idea that matter might exhibit both particle and wave properties was first proposed by Louis de Broglie in 1924. His hypothesis was based on the known wave-particle

duality of light and on his own brilliant insight that matter might exhibit similar properties. Using light as a guide, de Broglie worked out various consequences of his wave hypothesis.

2b. Wave/Particle Duality of Light. Electromagnetic radiation has both a wave aspect and a particle aspect.¹ We say that light exhibits “wave-particle duality,” behaving in some circumstances like a wave and in others like a particle. For example, interference and diffraction of light can be explained simply if light consists of waves.² On the other hand, phenomena such as the photoelectric and Compton effects³ can be explained simply if light consists of particle-like “photons.” A photon’s particle-like momentum p is found to be related to its wave-like wavelength λ by:

$$\lambda = h/p \quad (1)$$

where h is Planck’s Constant.

2c. The de Broglie Relation. De Broglie proposed that material particles, such as electrons, also have wavelengths given by Eq. (1). For such particles Eq. (1) is now called the “de Broglie Relation.”

An electron with a kinetic energy of 150 eV has a wavelength of 10^{-10} m. It shows interference patterns like those of x-rays that have the same wavelength and this confirms that electrons have a wave-like nature. The same has been found for other particles such as protons and neutrons.

▷ Show that an electron with a kinetic energy of 150 eV has a wavelength of 10^{-10} m. *Help: [S-1]*

2d. Confirmation: The Davisson-Germer Experiment.

De Broglie’s hypothesis that particles might exhibit wave properties was confirmed in 1927 by C. Davisson and L. Germer. They scattered a beam of electrons from a single crystal of metal and found, after using de Broglie’s relation to convert the electrons’ momenta to wavelengths, that the electron scattering matched the diffraction effects produced by x-rays (electromagnetic waves) of the same wavelength scattered from single crystals (see Fig. 1).⁴ In fact, Bragg had previously determined that

¹See “Wave-Particle Duality: Light” (MISN-0-246).

²See, for example, “Fraunhofer Diffraction” (MISN-0-235 or MISN-0-432(Sect. B)).

³See “The Photoelectric Effect” (MISN-0-213) and “The Compton Effect” (MISN-0-219), or see the Glossary.

⁴See “Diffraction Grating and X-ray Scattering From Crystals” (MISN-0-237 or MISN-0-432(Sect. B)).

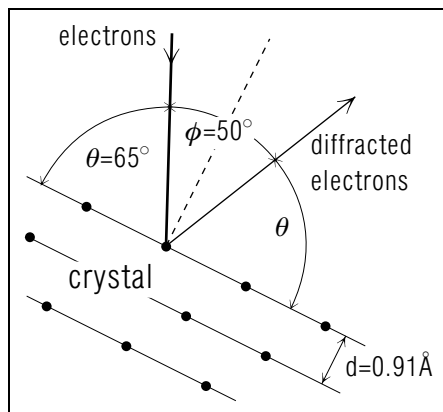


Figure 1. Electrons with 54 eV of kinetic energy diffracted by a nickel crystal have a maximum in intensity at $\phi = 50^\circ$.

the wavelength λ of x-rays was related to the angle θ of their most intense atomic-plane scattering by: $\lambda = 2d \sin \theta$, where d is the distance between the atomic planes. Davisson and Germer found that electrons obeyed the same relation.

2e. Further Confirmation. Although Davisson and Germer used electrons in their experiments, other particles such as neutrons and protons, and even atoms and molecules, are now known to sometimes exhibit wave properties. When they do, their momentum and wavelength values are in agreement with Eq. (1).

2f. Principle of Complementarity. In 1927 Niels Bohr proposed a Principle of Complementarity which states that the wave and particle aspects are both required in order to describe matter completely so they are “complementary.” The experimental arrangement is what determines which of the two aspects will dominate, if any. For example, in electron microscopes and atoms the wave properties dominate while in TV tubes and electric currents it is the particle properties. The photoelectric and Compton effects show electrons exhibiting particle-like behavior. One can switch the same electrons back and forth between the two kinds of experiments, showing that duality is inherent in each electron.

3. Wave Theory of Matter

3a. Something Must be Reinterpreted. Before wave/particle duality was established, it was thought that any entity that transports energy and momentum from one place to another is either a particle or a wave,

but that these two were incompatible concepts. For example, a particle is at a definite point in space following a well-defined trajectory, while a wave spreads out with time and even bends around corners. The overlapping of two coherent waves produces interference effects, and that is totally alien to the picture of two colliding particles. Yet, as we have seen, the same “particle” really can exhibit both wave and particle aspects. The resolution of this seeming paradox requires that we give up trying to learn the exact positions of particles, and give up thinking of waves as “physical” entities that must carry along momentum and energy.

3b. The Wave Function for a Particle. The modern, accepted resolution of the wave/particle paradox is that we can only determine the *probabilities* of finding particles in particular regions of space, and that the probabilities are determined by a wave-like equation. The input to this wave-like “Schrödinger Equation” is the potential energy function for the system being studied. The solution to the resulting equation is universally written “ ψ ” and it is called the “wave function” for the system described by the input energy function. The solution ψ , the wave function, gives us all the information we can learn about the system, including all measurable chemical and physical properties. The hard part is trying to find the solution to the Schrödinger Equation, and it can only be found with high precision for quite simple systems such as simple atoms and molecules. For other systems it can be found, if at all, with varying degrees of accuracy. However, computers and newly-invented algorithms now allow us to obtain far more accurate ψ functions than in the past.

3c. Probability Interpretation of ψ . The major significance of the wave function ψ is that its square, $|\psi|^2$, is the probability density for observing the particle which ψ describes.⁵ That is, if $\psi(x)$ is the wave function at each point x , then the probability of finding the particle between x_1 and x_2 is just the area under the curve of $|\psi(x)|^2$ between x_1 and x_2 . This means that the probabilities of finding the particle in various regions of space can be calculated precisely.

3d. Photons. The Schrödinger Wave Equation is only valid for particles traveling at speeds small compared to the speed of light, so it certainly does not apply to photons (which of course travel at the speed of light because they *are* light). The wave and particle properties of photons are

⁵Some scientists, including Einstein, continued to reject the probabilistic interpretation of Quantum Mechanics long after its general acceptance. For a discussion of this, see “Quantum Tunnelling Through a Barrier” (MISN-0-250).

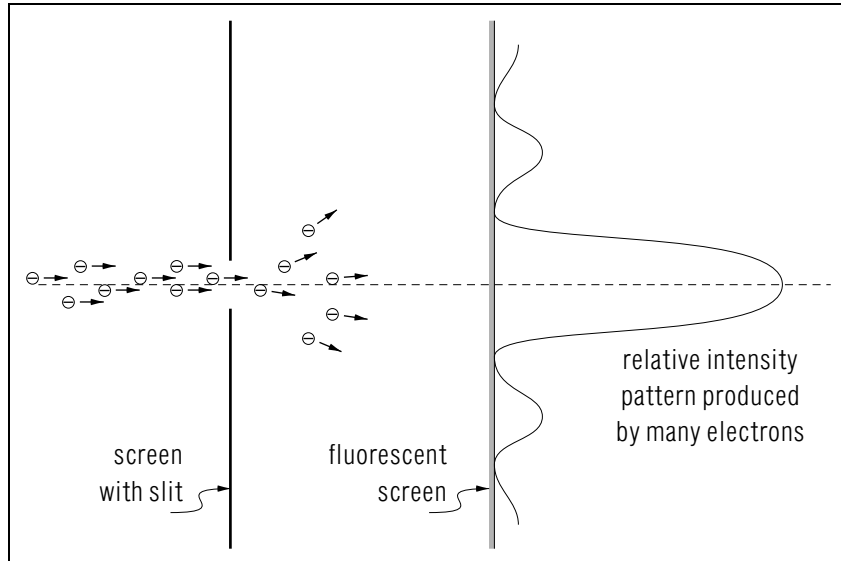


Figure 2. Diffraction pattern produced by a beam of electrons passed through a single narrow slit.

treated elsewhere,⁶ but we note here that the relationship between $|\psi|^2$ and a particle's "position" becomes, in the case of photons, a relationship between the square of the electric field at any point in space and a photon's "position."

4. Example: Electron Diffraction

4a. Diffraction of an Electron Beam. A beam of electrons passing through a single narrow slit produces the diffraction pattern shown in Fig. 2. This pattern can be made visible by letting the electrons strike a phosphorescent screen like that of a standard TV set. The properties of the diffraction pattern follow directly from wave theory and the de Broglie Relation, Eq. (1).

4b. The Intensity is Determined by the Wave Function. For electrons producing a single-slit diffraction pattern on a screen (Fig. 2), the intensity from any locality on the screen depends on the number of electrons per second that strike that locality. The number of electrons per second, in turn, is proportional to the value of $|\psi|^2$ in that locality. There

⁶See "Photons," MISN-0-212.

is no way that we can determine precisely where an individual electron will hit the screen, but $|\psi|^2$ does tell us precisely what the probability is for it arriving at any given locality. This probabilistic precision is a general characteristic of Quantum Mechanics.

Acknowledgments

Professor James Linneman made helpful suggestions. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **de Broglie relation:** the quantum mechanical equation relating a particle's wavelength to its momentum: $\lambda = h/p$. Valid for all particles, with or without mass.
- **de Broglie wavelength:** the wavelength of the probability wave associated with a particle, with or without mass, traveling with a definite momentum.
- **Compton effect:** the lowering of the frequency of light via scattering from a free electron. This effect helped establish the particle nature of light.
- **photoelectric effect:** the ejection of an electron from a metal that is illuminated by light. This effect helped establish the particle nature of light.
- **principle of complementarity:** a statement that both the wave and particle theories are needed to fully describe matter.
- **quantum mechanics:** the theory that describes much of the behavior of all particles, including atomic ones. Classical mechanics is embedded in this theory as a special case.
- **wave function ψ :** the function used to predict all the observable properties of a particle. For example, its square at some space point is the probability *density* for finding the particle in the immediate vicinity of that point. Multiplying that density by the volume of a small region around the point gives the probability of finding the particle in that volume. The word *small* has the usual calculus meaning.

- **wave-particle duality:** matter and light behave in some ways like waves, in some ways like particles. One aspect will often appear dominant, depending on the circumstances involved. The apparent conflict in description is resolved in quantum mechanics, where the wave aspects are found to be wave-like probabilistic statements about the particles.

PROBLEM SUPPLEMENT

$$\begin{aligned}
 h &= 4.13 \times 10^{-15} \text{ eV s} \\
 hc &= 1.240 \text{ eV } \mu\text{m} = 1.240 \times 10^{-6} \text{ eV m} \\
 m_e &= 0.51 \times 10^6 \text{ eV}/c^2 = 0.51 \text{ MeV}/c^2
 \end{aligned}$$

Problem 4 also occurs in this module's *Model Exam*.

NOTE: in these problems you should use these mass units: eV/c^2 , KeV/c^2 , MeV/c^2 , etc.⁷ While it is true that you can convert back and forth between kg and eV/c^2 , using $kg = 0.624 \times 10^{19} \text{ eV}c^2$, professionals use $\text{eV}c^2$ and u for atom-scale mass units and $\text{MeV}c^2$ for nucleus-scale masses. The quantity c , the speed of light, is left as a symbol since it often cancels out and then does not have to be put in as a number. Writing physical quantities in terms of it usually makes equations simpler.

1. Determine the de Broglie wavelengths of electrons with these kinetic energies: (a) 1.0 eV ; (b) $1.0 \times 10^2 \text{ eV}$; and (c) $1.0 \times 10^3 \text{ eV}$. *Help: [S-2]*
2. An electron and a photon each have a wavelength of $2.0 \times 10^{-10} \text{ m}$. Determine: (a) their momenta; (b) the kinetic energy of the electron; and (c) the energy of the photon.
3. A particular cyclotron accelerates U^{238} nuclei to a kinetic energy of 476 MeV . Find these particles' de Broglie wavelength, given that each one's mass is $2.22 \times 10^5 \text{ MeV}/c^2$.
4. Calculate the de Broglie wavelength of a proton with a kinetic energy of $1.5 \times 10^2 \text{ eV}$ (the proton mass is $938 \text{ MeV}/c^2$).

Brief Answers:

1. (If you couldn't work this problem, see the similar problem you should have worked as you went through Sect. 2c in the text. If you are having trouble with properties of photons, go back and review that material.
 - a. $\lambda = 12 \times 10^{-10} \text{ m}$
 - b. $\lambda = 1.2 \times 10^{-10} \text{ m}$

⁷For definitions of MeV , etc., see "Photons" (MISN-0-212).

c. $\lambda = 0.39 \times 10^{-10} \text{ m}$

2. a. electron and photon:

$$pc = \frac{hc}{\lambda} = \frac{1.240 \text{ eV} \times 10^{-6} \text{ m}}{2 \times 10^{-10} \text{ m}}$$

$$p = 6.2 \times 10^3 \text{ eV}/c$$

b. Electron:

$$E_k = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(6.2 \times 10^3 \text{ eV})^2}{2(0.51 \times 10^6 \text{ eV})} = 38 \text{ eV}.$$

c. Photon:

$$E = pc = hc/\lambda = \frac{1.240 \times 10^{-6} \text{ eV m}}{2 \times 10^{-10} \text{ m}} = 6.2 \times 10^3 \text{ eV}.$$

3. $\lambda = 8.53 \times 10^{-17} \text{ m}$

4. $\lambda = 2.3 \times 10^{-12} \text{ m}.$

SPECIAL ASSISTANCE SUPPLEMENT

S-1

(from TX-2c)

For non-relativistic speeds, recall that momentum p and kinetic energy E_k are related by:^a

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}.$$

Thus one can use m and either E_k or v to find p for de Broglie's relation. Solve the equation shown above for p in terms of E_k . Substitute that into the expression for the de Broglie wavelength, $\lambda = h/p$, multiply numerator and denominator by c and you will find: *Help: [S-3]*

$$\lambda = \frac{hc}{\sqrt{2m c^2 E_k}}.$$

As an example, find the wavelength of an electron with a kinetic energy of 150 eV. Use the data at the head of the Problem Supplement (note: it is a waste of time to convert to MKS units).

$$m_e = 0.51 \times 10^6 \text{ eV}/c^2,$$

hence (after multiplying both sides by c^2): *Help: [S-3]*

$$mc^2 = 0.51 \times 10^6 \text{ eV}$$

$$hc = 1.240 \text{ eV} \times 10^{-6} \text{ m}.$$

This gives:

$$\lambda = 1.00 \times 10^{-10} \text{ m}.$$

^aSee "An Overview of Particle Dynamics" (MISN-0-62)

S-2

(from PS-Problem 1)

See the similar problem you should have worked as you went through Sect. 2c in the text.

S-3

(from [S-1])

See the NOTE at the beginning of this module's *Problem Supplement*.

MODEL EXAM

$$hc = 1.240 \text{ eV } \mu\text{m}$$

1. See Output Skills K1-K3 in this module's *ID Sheet*. The actual exam may contain one or more, or none of these skills.
2. Determine the de Broglie wavelength of a proton with a kinetic energy of $1.5 \times 10^2 \text{ eV}$ (the proton mass is $938 \text{ MeV}/c^2$).

Brief Answers:

1. See this module's *text*.
2. See this module's *Problem Supplement*, problem 4.