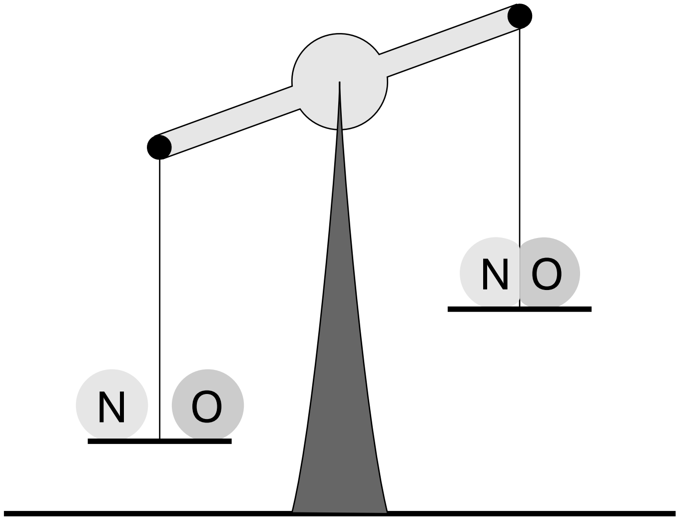
PHYSNET

MISN-0-23

RELATIVISTIC ENERGY; REACTION THRESHOLDS, BINDING ENERGIES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

RELATIVISTIC ENERGY; REACTION THRESHOLDS, BINDING ENERGIES

by

Peter Signell

1. Readings Alternatives: Choose One

Title: **Relativistic Energy; Reaction Thresholds, Binding Energies**

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Version: 4/24/2002 Evaluation: Stage B1

Length: 1 hr; 16 pages

Input Skills:

1. Analyze linear collisions using conservation of energy (MISN-0-21).
2. Expand a function around a point using Taylor's series (MISN-0-4).

Output Skills (Knowledge):

K1. Reduce the expression for relativistic kinetic energy to its non-relativistic form, starting from the general expression for Taylor's series.

Output Skills (Problem Solving):

S1. Calculate CM-frame thresholds for given $2 \Rightarrow 2$ particle reactions using Conservation of Energy and the masses of the four particles involved, and vice versa.

S2. Calculate binding energies using Conservation of Energy and the masses of the separated and conglomerate given objects, and vice versa.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison-Wesley, Reading MA (1970), or: R. Resnick, *Basic Concepts in Relativity and Early Quantum Theory*, John Wiley & Sons, New York (1972), or: R. T. Weidner and R. L. Sells, *Elementary Modern Physics*, Allyn and Bacon, Boston (1973).

   See this module's *Local Guide* for access.

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1. Readings Alternatives: Choose One

1a. Alternative 1: AF. Read in AF[1] at least Sections 12.1, 12.4, 12.6, and pages 242-3, paying attention to those parts pertaining to the Ouput Skills. We will not use the concept of "relativistic mass" in this unit. The CM frame of reference, called the "C-frame" in AF, is the regular center-of-mass frame; meaning the zero-total-momentum frame. For two particles the CM frame of reference is thus the one in which the particles have equal and opposite momenta. If the two particles have equal mass then they have equal but opposite velocities as viewed from this frame. For the purposes of this unit we ignore the "L-frame" material in AF. Do these AF Chapter 12 problems: 3-6, 8, 10.

Brief Answers:

3. 8.20×10^{-14} J, 1.51×10^{-10} J, 0.51 MeV, 940 MeV.
4. $v_p = 0.99954\,\mathrm{c}$, $p = 32.9576\,\mathrm{m\,c}$.
5. b. $v_p = 0.027\,\mathrm{c} = 8.1 \times 10^6$ m/s.
6. a. 4.03×10^{-5}.
   b. 0.0480.
   c. 0.749.
8. electron:
   a. 0.0789 MeV
   b. 0.581 MeV
   c. 0.463 MeV
   d. 1.98 MeV

   proton:

   a. 27.8 MeV
   b. 107 MeV
   c. 852 MeV
   d. 3650 MeV
10. $0.115\,\mathrm{c} = 3.45 \times 10^7$ m/s

1b. Alternative 2: RR. Read in RR[2] at least Sections 3.3, 3.4, and 3.5, including examples. Do problems 2, 3, 4, 7, 26, and 34 on p. 106-109.

Brief Answers:

2. 0.866 c, No
3. a. $0.942\,\mathrm{c} = 2.83 \times 10^8$ m/sec
   b. $621\,M_e$
   c. 212 MeV, 1.6×10^{-19} kg m/sec
4. a. $1962\,M_e$
   b. 0.99999987 c
   c. $2.961\,M_e$, 0.940 c
7. Derivation
26. 92.1 MeV
34. 29.6 MeV (neutrino), $4.15\,M_e$ (muon)

1c. Alternative 3: WSM. Read in WSM[3] at least Sections 3-1, 3-2, and 3-3, including examples. For the purposes of this module, it is not necessary to learn the derivations given in Section 3-1; only the results.

Do problems 3-2, 3-4, 3-10, p. 83.

Brief Answers:

3-2. Show it.
3-4. 0.14 c.
3-10 0.999998 c.

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[1] M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970) (see this module's Local Guide for details on obtaining this reference).

[2] R. Resnick, *Basic Concepts in Relativity and Early Quantum Theory*, Wiley(1972) (see this module's Local Guide for details on obtaining this reference).

[3] R. Weidner and R. Sells, *Elementary Modern Physics*, 3rd Edition, Allyn and Bacon (1980) (see this module's Local Guide for details on obtaining this reference).

2. Collisions

Consider the problem of particle A colliding with particle B, producing particles C and D: $A + B \to C + D$. If the total mass after an interaction is greater than before, then the mass-energy has increased during the interaction. Since overall energy must be conserved, some other form of energy must have decreased during the interaction. Usually this is kinetic energy. For such cases, conservation of energy becomes:

$$E_A + E_B = E_C + E_D\,,$$

where E_A is the total energy (mass energy plus kinetic energy) of particle A, etc. The minimum initial energy necessary for this reaction will be that which barely allows the final two particles to be created: the kinetic energy of that initial state is referred to as the threshold kinetic energy necessary for the reaction. The CM kinetic energy of the final state will be zero when the initial energy is at the threshold.

▷ Problem A: Use Conservation of Energy to determine the CM-frame threshold kinetic energy, in Joules, for the process:[4]

$$\mathrm{n}^0 + \pi^0 \to \mathrm{p}^+ + \pi^-\,,$$

$$\begin{aligned} m_{\mathrm{n}^0} &= 939.550\,\mathrm{MeV/c^2} \\ m_{\pi^0} &= 134.975\,\mathrm{MeV/c^2} \\ m_{\pi^-} &= 139.578\,\mathrm{MeV/c^2} \\ m_{\mathrm{p}^+} &= 938.256\,\mathrm{MeV/c^2}. \end{aligned}$$

3. Binding Energy

Consider the deuteron nucleus used in the example on p. 243 of AF, or Example 6 on p. 101 of RR, or Example 3-3 on p. 85 of WSM. Suppose we bombard such nuclei with a beam of γ particles which have the characteristic that they can be completely converted to energy upon absorption. Our γ's have been produced in a manner such that each one in the beam

has the same energy. We now gradually increase the beam energy, the energy of each γ in the beam, and look for the "photodisintegration" reaction,

$$\gamma + d \to n + p\,,$$

in which a γ is annihilated and its energy is absorbed by the deuteron's constituents, the neutron and proton. We find experimentally that this reaction is not observed to take place until the energy of each γ reaches 2.225 MeV. This disintegration threshold energy is called the "binding energy" of the conglomerate deuteron. At threshold the n and p are produced essentially at rest. If the beam energy of the γ's is increased further, the excess energy goes into kinetic energy of the final-state n and p.

NOTE: the binding energy is actually defined as the disintegration threshold energy as measured in the CM frame so that the n and p can be produced completely at rest. However, the γ carries so little momentum that the deuteron-at-rest frame is very close to being a zero-momentum frame.

▷ Problem B: The binding energy of the nitrogen monoxide molecule (NO) has been measured to be 5.296 eV. Use Conservation of Energy to calculate the difference between the mass of the molecule and the total mass of its constituents, in amu. Note that $m_N = 14.0067$ amu, and $m_0 = 15.9994$ amu.

Brief Answers:

A. $$\underbrace{E_{\mathrm{mass}}(\mathrm{initial})}_{(m_{\pi^0}+m_{\mathrm{n}})c^2} + \underbrace{E_k(\mathrm{initial})}_{E(\mathrm{thresh.})} = \underbrace{E_{\mathrm{mass}}(\mathrm{final})}_{(m_{\pi^-}+m_{\mathrm{p}})c^2} + \underbrace{E_k(\mathrm{final})}_{0(\mathrm{thresh.})}$$

$E_{\mathrm{thresh.}} = 5.30 \times 10^{-13}$ J.

B. $$\underbrace{E_{\mathrm{mass}}(\mathrm{initial})}_{(m_{NO})c^2} + \underbrace{E_k(\mathrm{initial})}_{E_B} = \underbrace{E_{\mathrm{mass}}(\mathrm{final})}_{(m_N+m_O)c^2} + \underbrace{E_k(\mathrm{final})}_{0(\mathrm{thresh.})}$$

$\Delta m \equiv (m_N + m_O) - (m_{NO}) = E_B/c^2 = 5.69 \times 10^{-9}$ amu

4. Reduction to Non-Relativistic Form

Expand the function[5]

$$k(x) = (1 - x)^{-1/2}$$

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[4] If this module is part of a book, conversion factors should be in an appendix. Otherwise, see the conversion factors at the beginning of this module's *Problem Supplement*.

[5] See "Taylor's Series for the Expansion of a Function About a Point" (MISN-0-4).

in a power series about the origin, where $x \equiv v^2/c^2$ and $k \equiv E/(m_0c^2)$. We find:

$$E = m_0c^2 + (1/2)m_0v^2 + (3/8)m_0(v^4/c^2) + \ldots$$

Then for $v^2 \ll c^2$, the third and higher terms can be neglected compared to the first and second. The first term is not kinetic since it is not a function of velocity. The leading kinetic term is:

$$E_k \approx (1/2)m_0v^2 \; ; \; v^2 \ll c^2 \, .$$

NOTE WELL: The reduction given in AF is insufficient.

Acknowledgments

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LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as “The readings for CBI Unit 23.” Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

$$E = km_0c^2 \qquad \text{where: } k = (1 - v^2/c^2)^{-1/2}$$

$$\text{amu} = 1.6604 \times 10^{27}\,\text{kg} = 931\,\text{MeV/c}^2$$

$$\text{MeV} = 1.6022 \times 10^{-13}\,\text{J}$$

Note: These problems are also on this module's *Model Exam*.

1. Starting from Taylor's Series for the expansion of a function about a point, reduce the expression for relativistic kinetic energy to its non-relativistic form.
2. In reactions produced by scattering protons on protons, we find that Δ's are produced,
$$\text{p} + \text{p} \to \text{p} + \Delta\,,$$
only when the total CM kinetic energy of the initial protons exceeds 298 MeV. Calculate the mass of the Δ, using Conservation of Energy.
3. The binding energy of the deuteron nucleus has been measured to be 2.225 MeV. Use Conservation of Energy to calculate its mass in amu's, given the masses of its constituents:

$$\begin{array}{rcrl} \text{neutron:} & m_\text{n} & = & 939.5527\,\text{MeV/c}^2 \\ \text{proton:} & m_\text{p} & = & 938.2592\,\text{MeV/c}^2 \end{array}$$

Brief Answers:

1. $k(x) = (1 - x)^{-1/2}$ where $x \equiv v^2/c^2$

$$k'(x) = (1/2)(1 - x)^{-3/2}$$

$$k''(x) = (3/4)(1 - x)^{-5/2}$$

$$\begin{aligned} k(x) &= k(0) + \frac{k'(0)}{1!}x + \frac{k''(0)}{2!}x^2 + \ldots \\ &= 1 + (1/2)x + (3/8)x^2 + \ldots \end{aligned}$$

$$E = k(v^2/c^2)m_0c^2 = m_0c^2 + (1/2)m_0v^2 + (3/8)m_0(v^4/c^2) + \ldots$$

Then if $v^2 \ll c^2$, neglect the third and higher terms compared to the second and lower. The first term is not kinetic, so

$$E_k \approx \frac{1}{2}mv^2, \qquad \text{for } v^2 \ll c^2\,.$$

2. $$\underbrace{E_\text{mass}(\text{initial})}_{2(m_\text{p})c^2} + \underbrace{E_k(\text{initial})}_{298\,\text{MeV}} = \underbrace{E_\text{mass}(\text{final})}_{(m_\text{p} + m_\Delta)c^2} + \underbrace{E_k(\text{final})}_{0(\text{thresh.})}$$

$$m_\Delta = 1236\,\text{MeV/c}^2$$

3. $$\underbrace{E_\text{mass}(\text{initial})}_{m_\text{d}c^2} + \underbrace{E_k(\text{initial})}_{E_B} = \underbrace{E_\text{mass}(\text{final})}_{(m_\text{p} + m_\text{n})c^2} + \underbrace{E_k(\text{final})}_{0(\text{thresh.})}$$

$$m_\text{p}c^2 + m_\text{n}c^2 = m_\text{d}c^2 + E_B$$

$$m_\text{d} = m_\text{p} + m_\text{n} - E_B/c^2 = 2.0146\,\text{amu}.$$

MODEL EXAM

$$E = km_0c^2 \qquad \text{where: } k = (1 - v^2/c^2)^{-1/2}$$

$$\text{amu} = 1.6604 \times 10^{27}\,\text{kg} = 931\,\text{MeV/c}^2$$

1. See this Output Skill K1 on this module's *ID Sheet*.
2. In reactions produced by scattering protons on protons, we find that Δ's are produced,

   $$\text{p} + \text{p} \rightarrow \text{p} + \Delta\,,$$

   only when the total CM kinetic energy of the initial protons exceeds 298 MeV. Calculate the mass of the Δ, using Conservation of Energy.
3. The binding energy of the deuteron nucleus has been measured to be 2.225 MeV. Use Conservation of Energy to calculate its mass in amu's, given the masses of its constituents:

   $$\begin{aligned} \text{neutron:} \quad m_\text{n} &= 939.5527\,\text{MeV/c}^2 \\ \text{proton:} \quad m_\text{p} &= 938.2592\,\text{MeV/c}^2 \end{aligned}$$

Brief Answers:

1. See this module's *text* and *Problem Supplement*, problem 1.
2. See this module's *Problem Supplement*, problem 2.
3. See this module's *Problem Supplement*, problem 3.