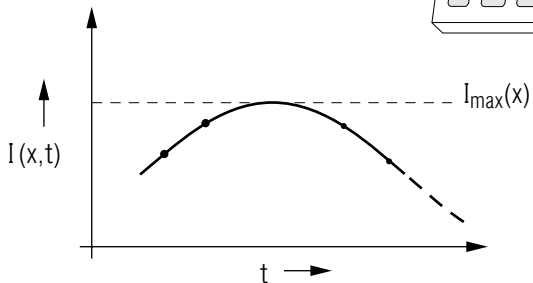
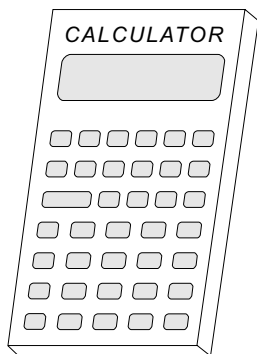
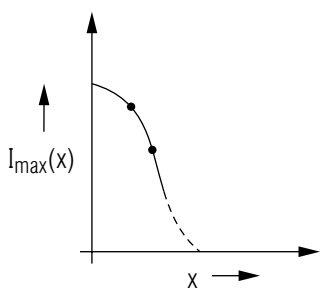


AN INTERFERENCE PROJECT



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by

J. S. Kovacs and Peter Signell
Michigan State University

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Title: An Interference Project

Author: J.S. Kovacs and P. Signell, Michigan State University

Version: 11/7/2001

Evaluation: Stage 1

Length: 1 hr; 16 pages

Input Skills:

1. Determine the wave disturbance at a given point resulting from two equal coherent wave sources (MISN-0-205).
2. Draw and annotate graphs for good scientific communication (MISN-0-401).

Output Skills (Project):

- P1. Use numerical amplitude addition to construct a significant portion of a wave interference pattern as specified in the module.

External Resources (Required):

1. A calculator with square roots and trigonometric functions, and graph paper.

Post-Options:

1. "Interference, Many Sources; Radio Interferometry" (MISN-0-231).

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1. Introduction

The basic idea of this project is to take waves from two synchronous sources, arranged so the waves from the two sources cross each others' paths, and calculate the interference pattern one detects where the waves overlap. This interference pattern will be the one produced by plotting the output of a detector that is moving along a line parallel to the line connecting the sources (see Fig.1). For sound waves the detector would be a measuring sound receiver.

2. General Goals

Calculation of the interference pattern shown by the detector illustrates how interference patterns arise. It also tests one's understanding of wave properties: wave motion, phase, intensity, and average intensity all enter into the calculation in crucial ways. There is also calculator practice in locating a zero of a non-linear function. In fact, if your calculator is programmable you can plot much more of the interference pattern than just the central peak required here. This project, the mathematical construction of wave amplitudes and interference, is a good test of your understanding of waves. If you have trouble, it is a signal that your understanding is inadequate.

3. Procedure

- Obtain a trig calculator and graph paper.
- Decide on numerical values of the apparatus dimensions shown in Fig. 1. The distances (d, L, x) can be specified in units of the wavelength of the waves produced by each source. We suggest that d , the source separation, be of the order of a wavelength. *Help: [S-1]*¹ The source-to-detector-line distance L should be at least a few wavelengths;

¹ *Help: [S-1]* refers to sequence *Help: [S-1]* in the attached Special Assistance Supplement. It provides special help, if you need it, for what was just discussed.

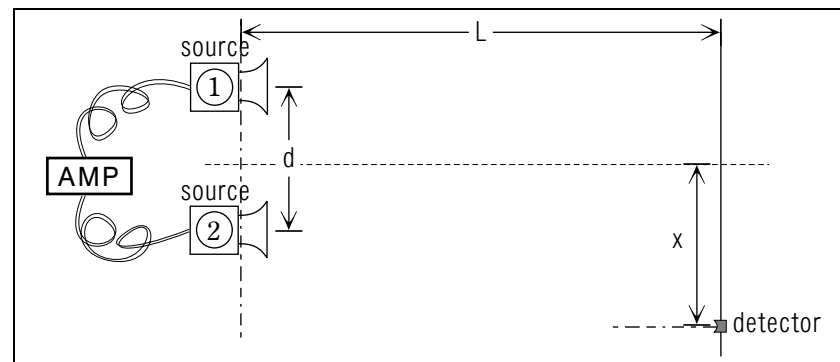


Figure 1. A two-source interference system.

then the first minimum will occur at a modest value of detector position x . Note that the interference pattern will be symmetric about $x = 0$. *Help: [S-2]*

- Choose a convenient function to describe the time and position variation of the waves that are generated by the sources (one normally picks a sine or cosine function). *Help: [S-3]*
- Using the function chosen in (c), calculate the peak intensity at $x = 0$ as well as a time at which this peak occurs. We suggest that you measure time in units of the wave's vibrational period (inverse of frequency). *Help: [S-4]*
- Using simple trig, the calculator, and interpolation, determine the first detector position x_0 that shows zero intensity at all times. *Help: [S-6]* (Assume that the amplitude of each wave is constant; that it does not fall off with distance from the source. This simplifying assumption is a good approximation when $x \gg L$, as is usually the case.)
- Choose two or more detector points x that nicely divide the interval between $x = 0$ and x_0 , the first minimum. Calculate the intensity as a function of time at each of these points, and at $x = 0$ and x_0 , graphing the results in order to find the peak intensity at each point.² *Help: [S-5]*
- Plot a graph of average intensity (time-average) vs. detector position x , including the values you obtained at $x = 0$, at x_0 , and at the two or

²You can use calculus, if you wish, to determine the times at which the peaks occur.

more points inbetween. Clearly identify x_0 as well as the other points on your graph. *Help:* [S-7]

Acknowledgments

We would like to thank field testers Bill Lane, Steve Smith, and William Francis for their strong feedback on the first version. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Appendix

A. Formal Solution (for those interested).

$$\begin{aligned} A &= A_1 + A_2 \\ &= \cos 2\pi \left(\frac{D}{\lambda} - \frac{t}{T} \right) + \cos 2\pi \left(\frac{D}{\lambda} - \frac{t}{T} \right) \quad \text{Help: [S-11]} \\ &= \alpha \cos 2\pi \left(\frac{\bar{D}}{\lambda} - \frac{t}{T} \right) \end{aligned}$$

Where:

$$\alpha = 2 \cos[\pi(D_1 - D_2)/\lambda]$$

D_1 = distance from slit #1 to position x on screen. [S-6]

D_2 = ... slit #2 ...

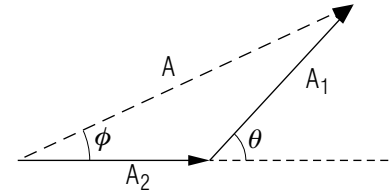
$$\bar{D} = (D_1 + D_2)/2$$

T = light wave period

Then the time-average intensity is:

$$\bar{I} = \alpha^2/2$$

B. Phasor Method (for those interested).



See side A in the sketch.

See Appendix A for def. of D_1 , D_2 .

$$A = \alpha = (2 + 2 \cos \theta)^{1/2} = 2 \cos \theta/2$$

$$\theta = 2\pi \frac{D_1 - D_2}{\lambda}$$

$$\alpha = 2 \cos \frac{\pi(D_1 - D_2)}{\lambda}$$

$$\phi = \theta/2$$

$$\begin{aligned} (\text{phase of } A) &= (\text{phase of } A_2) + \phi = (\text{phase of } A_1) - \phi \\ &= 2\pi \frac{D_2}{\lambda} + \frac{\theta}{2} = 2\pi \frac{D_2}{\lambda} + \frac{\pi(D_1 - D_2)}{2\lambda} \\ &= \frac{2\pi}{\lambda} \left(\frac{D_1 + D_2}{2} \right) \end{aligned}$$

$$\text{Hence: } A = A_1 + A_2 = \alpha \cos \left[2\pi \left(\frac{D_1 + D_2}{2\lambda} - \frac{t}{T} \right) \right]$$

Then the time-average intensity is: $\bar{I} = \alpha^2/2$ *Help:* [S-14]

SPECIAL ASSISTANCE SUPPLEMENT

[S-1] (from TX-3(b))

For example: $d = 2\lambda$; $L = 3\lambda$

[S-2] (from TX-3(b))

The physical apparatus is symmetrical about $x = 0$ so the interference pattern produced by it must have that same symmetry.

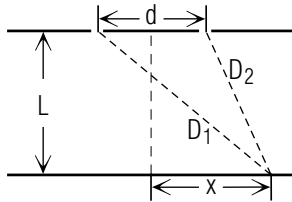
[S-3] (from TX-3(c))

We will choose (see the diagram):

$$A_{1,2} = \cos \left[2\pi \left(\frac{D_{1,2}}{\lambda} - \frac{t}{T} \right) \right].$$

We could equally well have chosen a sine and cosine function, or either of them with a phase.

Note: T is wave "period." With our cosine choice, at position x from each of the slits, the amplitudes are:



$$A_1 = \cos \left[2\pi \left(\frac{D_1}{\lambda} - \frac{t}{T} \right) \right]$$

$$A_2 = \cos \left[2\pi \left(\frac{D_2}{\lambda} - \frac{t}{T} \right) \right]$$

where:

$$\frac{D_1}{\lambda} = \left[\left(\frac{L}{\lambda} \right)^2 + \left(\frac{x + d/2}{\lambda} \right)^2 \right]^{1/2}$$

$$\frac{D_2}{\lambda} = \left[\left(\frac{L}{\lambda} \right)^2 + \left(\frac{x - d/2}{\lambda} \right)^2 \right]^{1/2}$$

[S-4] (from TX-3(d))

$x = 0$. Then:

$$\frac{D_1}{\lambda} = \frac{D_2}{\lambda} = \left[\left(\frac{L}{\lambda} \right)^2 + \left(\frac{d/2}{\lambda} \right)^2 \right]^{1/2}$$

hence $A_1 = A_2$ and the total amplitude is just twice either one. The peak value of either one is unity so the peak value of the total amplitude is 2. It occurs when each amplitude has a peak; that is, when the cosine arguments vanish. This time is:

$$\frac{t_{peak}}{T} = \frac{D_1}{\lambda} = \left[\left(\frac{L}{\lambda} \right)^2 + \left(\frac{d/2}{\lambda} \right)^2 \right]^{1/2}$$

For the [S-1] case,

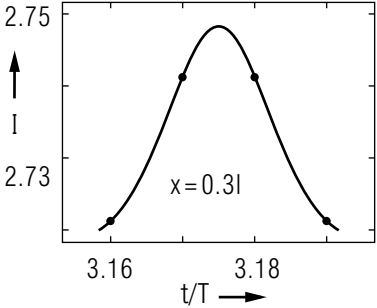
$$t_{peak} = \sqrt{10} T \approx 3.16 T$$

[S-5] (from TX-3(f))

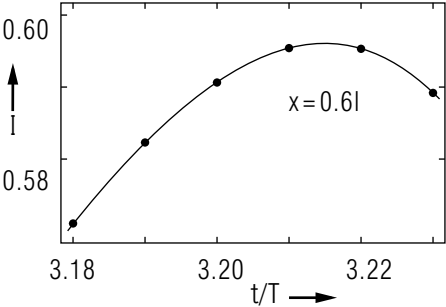
Using [S-1] plus the fact that the intensity I is the square of the total amplitude A , we find at points $x = 0.3\lambda$ and $x = 0.6\lambda$:

$$I = |A|^2 = |A_1 + A_2|^2$$

x/λ	t/T	I
0.3	3.16	2.724
	3.17	2.745
	3.18	2.746
	3.19	2.724
[S-8]		



x/λ	t/T	I
0.6	3.18	.574
	3.19	.587
	3.20	.595
	3.21	.599
	3.22	.599
	3.23	.593



Thus:

$$I_{peak}(x = 0.3\lambda) = 2.750$$

$$I_{peak}(x = 0.6\lambda) = 0.600$$

[S-6] (from TX-3(e) and Appendix A)

The null will occur when:

$$D_1 - D_2 = \lambda/2$$

so the two-source amplitudes are then:

$$\frac{D_1}{\lambda} - \frac{D_2}{\lambda} = 0.5$$

Assuming the same constants as in [S-1],

$$0.5 = \left[9 + \left(\frac{x_0}{\lambda} + 1 \right)^2 \right]^{1/2} - \left[9 + \left(\frac{x_0}{\lambda} - 1 \right)^2 \right]^{1/2}$$

Solve by interpolation (faster than algebra, but use algebra if you wish! [S-12])

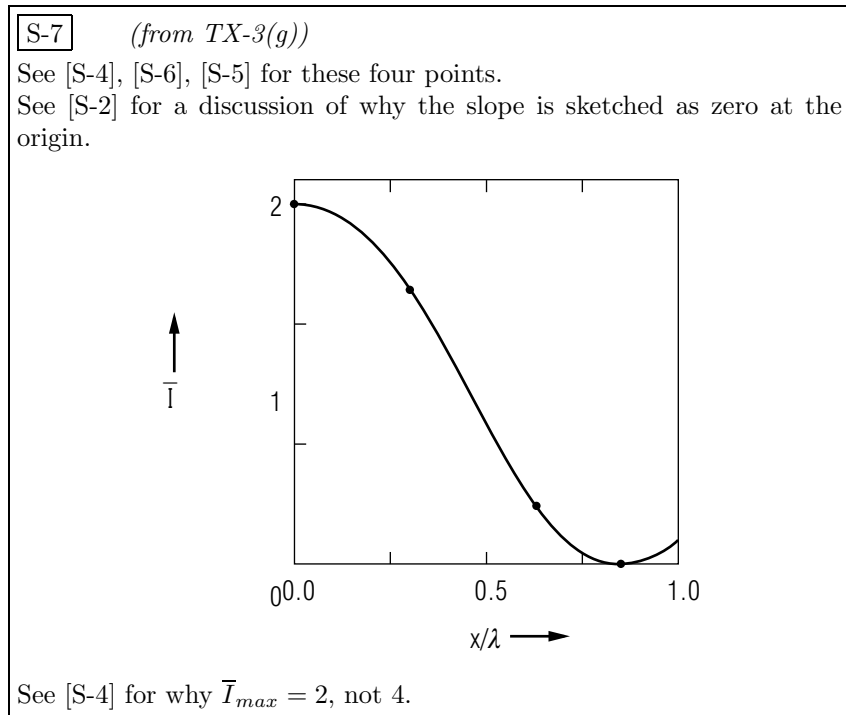
Note: RHS \equiv "Right Hand Side of the equation."

x/λ	RHS	Differences
.650	.403	
		> .030
.700	.433	
		> .029
.750	.463	
		> .029
.800	.492	
		> .029
.850	.521	

Constant differences mean that the RHS is linear in (x/λ) . We can interpolate graphically or algebraically. Algebraically:

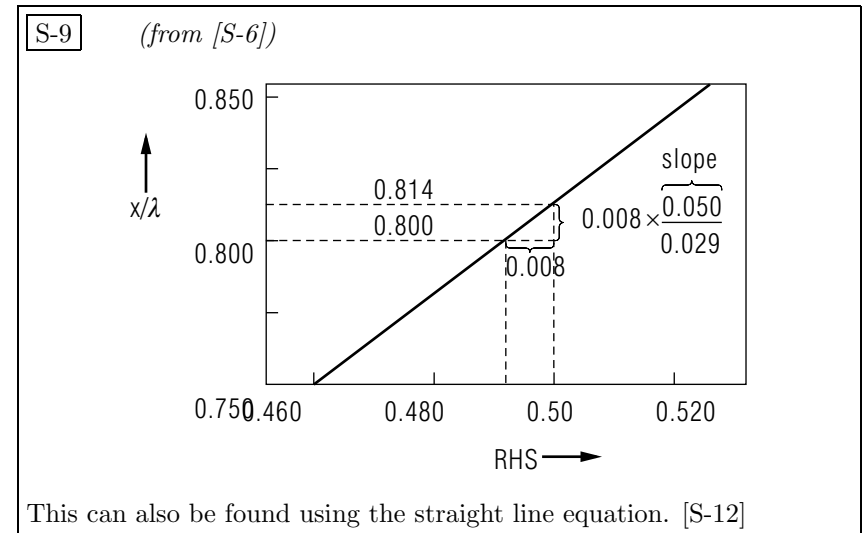
$$\frac{x_0}{\lambda} = 0.800 + \left(0.008 \times \frac{0.050}{0.029} \right) = 0.814 \text{ [S-9]}$$

Thus: $A(x = 0.814\lambda, \text{ any time}) = 0$. Graphical interpolation gives the same answer, of course. [S-13]



[S-8] (from [S-5])

The values of x/λ , d/λ , and L/λ were used to get D_1/λ and D_2/λ . These in turn were used to get A_1 and A_2 , hence I , for various times t . Should you use radians or degrees? Does it matter? [S-10]



[S-10] (from [S-8])

It sure does matter! The “ 2π ” in the equation denominators are really “ 2π radians.” Degrees would be incorrect.

[S-11] (from Appendix A)

See Input Skill #1 for the reference.

[S-12] (from [S-9])

To find x/λ brainlessly, find the equation of the straight line:

$$\left(\frac{x}{\lambda}\right) = (\text{slope}) \cdot (\text{RHS}) + (\text{intercept}).$$
 Do it by substituting two known points:

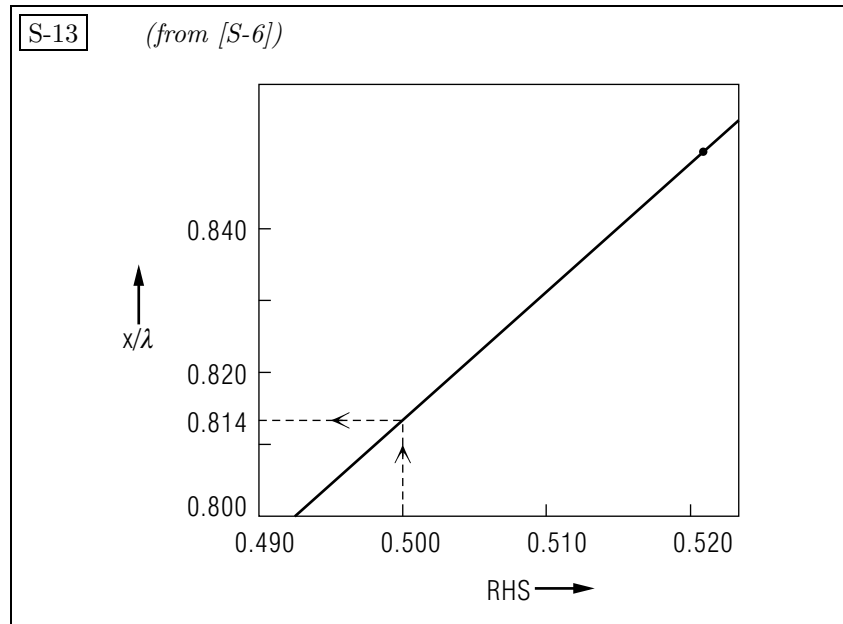
$$0.800 = (\text{slope})(0.492) + (\text{intercept});$$

$$0.850 = (\text{slope})(0.521) + (\text{intercept}).$$
 Solve those two equations in two unknowns, simultaneously, to get:

$$\text{slope} = \frac{.850 - .800}{.521 - .492} = \frac{.050}{.029};$$

$$\text{intercept} = (.800) - (.492)\left(\frac{.050}{.029}\right).$$
 Then put those into the desired RHS:

$$\left(\frac{x}{\lambda}\right) = \left(\frac{.050}{.029}\right) \cdot (.500) + (.800) - (.492) \cdot \left(\frac{.050}{.029}\right)$$



S-14 (from TX, Appendix B)
 For an explanation of this method, see “Phasors” (MISN-0-27)

MODEL EXAM

Examinee:

On your computer output sheet(s):

- (i) Mark page numbers in the upper right corners of all sheets.
- (ii) Label all output, including all axes on all graphs.

On your Exam Answer Sheet(s), for each of the following parts of items (below this box), show:

- (i) a reference to your annotated output; and
- (ii) a blank area for grader comments.

When finished, staple together your sheets as usual, but include the original of your annotated output sheets just behind the Exam Answer Sheet.

1. Your graphs and the original of your geometry sketch which shows your constants. No credit will be given if your choices for numerical values of d and/or L coincide with those used in the Special Assistance Supplement, sequence [S-1].

INSTRUCTIONS TO GRADER

If the student has submitted copies rather than originals of the computer output, state that on the exam answer sheet and **immediately stop grading the exam and give it a grade of zero.**

Note that the award of points is set up in such a way that a student will get 50% or less on the exam if the student does only computer work and no physics.