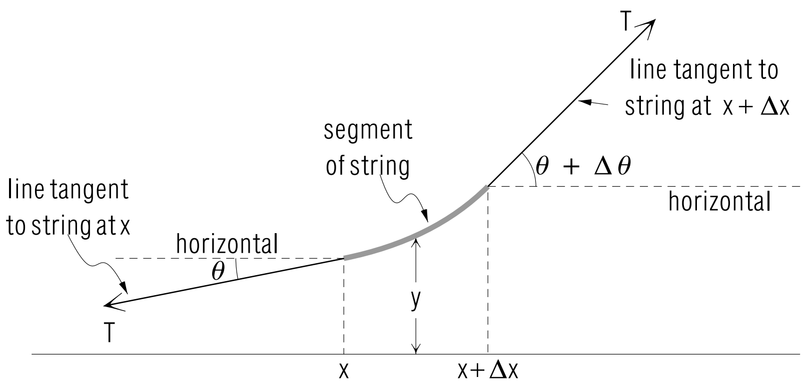
PHYSNET

MISN-0-202

SOUND WAVES AND SMALL TRANSVERSE WAVES ON A STRING



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

SOUND WAVES AND SMALL TRANSVERSE WAVES ON A STRING

by

J. S. Kovacs and O. McHarris

Title: **Sound Waves and Small Transverse Waves on a String**

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Input Skills:

1. Vocabulary: wavelength, amplitude, wave number, wave speed, traveling wave, wave equation (MISN-0-201); Hooke's law (MISN-0-26).
2. State the one-dimensional differential wave equation and its traveling wave solution (MISN-0-201).

Output Skills (Knowledge):

K1. Vocabulary: sound wave, longitudinal wave, transverse wave, compressions, rarefactions, bulk modulus (of elasticity), Young's modulus, stress, strain.

K2. Starting with Newton's second law, derive the expression relating the net force on an element of mass of a stretched string to the transverse acceleration of that string. Comparing the resultant expression to the one-dimensional wave equation, find the expression for the speed of a transverse wave in a stretched string.

K3. Determine the speed of the waves (in terms of the properties of the medium), given the differential wave equations describing: (i) transverse waves in a stretched string; (ii) longitudinal compressional waves in a solid or a gas.

Output Skills (Rule Application):

R1. For a given harmonic (sinusoidal) disturbance write down the equation representing the waveform and calculate the wavelength and frequency of the wave for: (i) transverse waves in a stretched string; and (ii) longitudinal compressional waves in a solid or a gas.

R2. Given a transverse harmonic disturbance, determine the displacement, speed, and acceleration of the waveform at any time in the wave cycle.

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1. Overview

Many physical systems are described by the wave equation and its solutions. By applying Newton's second law, the wave equations are found for transverse waves on a string, and longitudinal waves on a rod or in a gas. The speed of each type of wave is then found by inspection of the wave equation.

2. Introduction

2a. The Wave Equation and its Solutions. The solutions of the equation:

$$\frac{\partial^2 \xi}{\partial t^2} = v^2 \frac{\partial^2 \xi}{\partial x^2}\,, \tag{1}$$

are waves of displacement ξ traveling at speed v in the positive and negative x-direction.[1]

2b. Finding the Wave Equation for a Physical System. Let us look at appropriate physical systems, apply Newton's second law, and see that a wave equation results. For a physical displacement ξ we will use Newton's second law in the form:

$$F = ma = m\frac{\partial^2 \xi}{\partial t^2}\,.$$

Then whenever the net force F on the system under study is proportional to the displacement's spatial "bending function," $\partial^2\xi/\partial x^2$, we will have the wave equation, Eq. (1). The solution of this equation is the equation of motion of the system. The resulting motion should be that of a traveling wave and the velocity of this wave can be determined just by inspecting the differential equation. As examples of this procedure we will examine a transverse wave on a stretched string, a longitudinal wave in a rod of solid material, and a longitudinal wave in a gas. In each of these cases we will derive the correct differential equation for the system and then determine the wave velocity by comparison to Eq. (1).

[1]See "The Wave Equation and Its Solutions" (MISN-0-201).

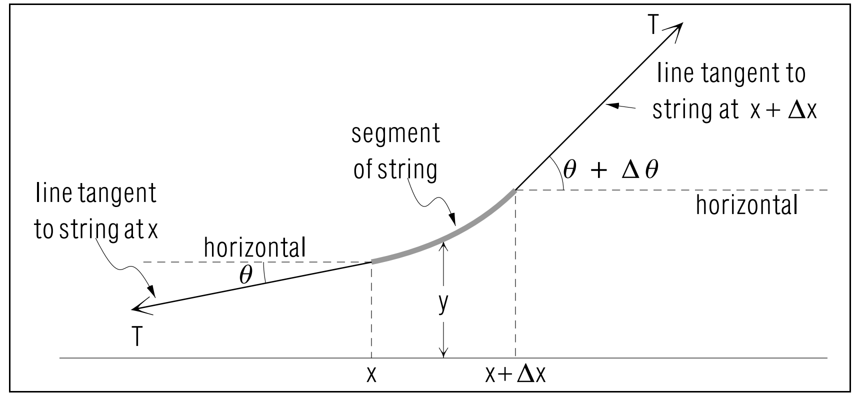

Figure 1. Geometrical descriptors for the displacement y of a small segment Δx of a stretched string.

3. Small Transverse Waves on a String

3a. Geometrical Descriptions. Above is shown a very short length Δx of a string that is vibrating transversely (in the figure, up and down). The string is stretched along the x-axis and the wave propagates along it in the x direction. However, the individual particles of the string move, parallel to the y-axis, at right angles to the direction of the wave's motion. The equilibrium position of the string is at $y = 0$, and the displacement y is assumed to be small, as are the angles θ and $\Delta\theta$.[2]

3b. Net Force on a Segment. Neglecting the force of gravity, the two forces on the segment Δx are the tension T in the string on the right hand end (the tangent at that point) pulling the segment up and to the right and the tension T in the string on the left hand end pulling it down and to the left. Due to the shape of the wave, the tangents to the two ends of the segment are at different angles to the x-axis. At the right hand end of the segment:

$$F_y = T\sin(\theta + \Delta\theta) \qquad \text{and} \qquad F_x = T\cos(\theta + \Delta\theta)\,,$$

while at the left hand end:

$$F'_y = -T\,\sin\theta \qquad \text{and} \qquad F'_x = -T\cos\theta\,.$$

[2]For small displacements the restoring force on the string varies linearly with displacement and hence produces simple harmonic motion. This makes the motion of the string easily soluble mathematically.

Since both θ and $\Delta\theta$ are small, $\cos\theta$ and $\cos(\theta+\Delta\theta)$ are both essentially equal to 1 and there is negligible net force in the x-direction.[3] For small θ, $\sin\theta$ on the other hand is essentially equal to θ, and the net force in the y-direction is:[4]

$$F_y = T\,\Delta\theta\,.$$

3c. Applying Newton's Second Law. If the linear density (mass per unit length) of a stretched string is μ, a segment with length Δx has a mass $\Delta m = \mu\Delta x$ and a transverse acceleration $a = \partial^2 y/\partial t^2$ (see Fig. 1). Notice that for "a" we must use partial derivatives of y with respect to t since the displacement y is a function of both x and t. Application of Newton's second law then gives us (with F from the previous section):

$$T\,\Delta\theta = \mu\,\Delta x \partial^2 y/\partial t^2\,.$$

This implies:

$$T\frac{\Delta\theta}{\Delta x} = \mu\frac{\partial^2 y}{\partial t^2}\,. \qquad (2)$$

We can now let Δx and $\Delta\theta$ shrink until $\Delta\theta/\Delta x$ becomes $\partial\theta/\partial x$ so Eq. (2) becomes:

$$T\frac{\partial\theta}{\partial x} = \mu\frac{\partial^2 y}{\partial t^2}\,. \qquad (3)$$

3d. Getting the Wave Equation. For a wave equation, Eq. (1), we need $\partial^2 y/\partial x^2$ rather than the $\partial\theta/\partial x$ of Eq. (3). Let us therefore rewrite $\partial\theta/\partial x$ in terms of y and x. These quantities are related by:

$$\tan\theta = \text{ slope of curve } = \partial y/\partial x\,.$$

Differentiating with respect to x gives us:

$$\frac{\partial}{\partial x}\tan\theta = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)\,,$$

$$\frac{1}{\cos^2\theta}\frac{\partial\theta}{\partial x} = \frac{\partial^2 y}{\partial x^2}\,,$$

[3] $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$; For small θ, the θ^2 and higher terms can be neglected.

[4] $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$

where we now have the second derivative we need. Remembering that $\cos\theta \approx 1$ for small θ, we have finally:

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}\,. \qquad (4)$$

This is the wave equation for any small traveling wave on a stretched string. Comparing to Eq. (1) we see that the speed of the wave is:

$$v = \sqrt{T/\mu}\,. \qquad (5)$$

3e. Physical Solutions. The wave equation contains symbols whose values must come from the physical problem at hand. The wave's speed comes from the properties of the medium in which the wave propagates, as illustrated in Eq. (5) with tension and mass-per-unit-length for a string. The wave's shape, whether sinusoidal or something else, and its frequency, depend as well on the driving force (on the manner in which the string is made to keep vibrating) and on the damping properties of the string.

4. Longitudinal Waves on a Rod

4a. Overview. Applying Hooke's law and Newton's second law to longitudinal compression in a solid rod, we can derive the equation for acoustic waves in the rod. Such a compression occurs when, for example, a rod is struck on one end, displacing the individual particles of the rod in the direction of the rod's length and causing a displacement wave to travel down the rod in the same direction as the motion of the particles.

4b. Stress, Strain, Young's Modulus. In general, when a rod is subjected to a force in the direction of its length and acting over its cross section—when, for example, it is vertical and holds up the roadbed of a bridge or some other weight suspended from its end—Hooke's Law applies. This law states, in general, that in an elastic medium:[5]

$$\frac{\text{stress}}{\text{strain}} = \text{ constant}\,, \qquad (6)$$

where stress is the applied force per unit cross sectional area and strain is the rod's fractional deformation due to the stress.

$$\text{stress } = F/A\,.$$

[5] Our rod will remain "elastic" as long as it is not stretched too much in proportion to its length.

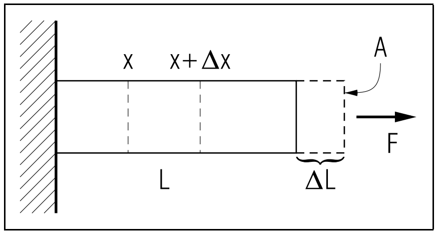

Figure 2. Geometrical descriptions for a rod undergoing longitudinal compression and/or extension.

In the case of a rod, the stress causes a change in the length of the rod. The strain is thus the fractional change in length. Writing L as the original length of the rod and ΔL is its elongation due to the stress on it, the strain is $\Delta L/L$. For a rod, the "constant" of Eq. (6) is called Young's modulus and is designated by the letter Y; it is a property of the material of which the rod is made. Thus for elastic longitudinal deformations of our rod:

$$Y = (F/A)/(\Delta L/L)\,,$$

and hence:

$$F = YA\,\Delta L/L\,.$$

4c. Force on a Segment: Static Case. In order to derive a wave equation for a rod, let us analyze what happens to a small segment of the rod. Consider a segment with unstretched length Δx and cross section A, where A is also the cross section for the rod as a whole (see Fig. 2). In a static situation, such as that of a spring pulling on the end of the rod, the force is constant along the rod, and the segment of length Δx is stretched by its proportionate amount $\Delta\xi$ such that the $\Delta\xi$'s summed over all the Δx's of the rod equals the total elongation ΔL. In this case, Hooke's law for the segment gives us:

$$F = YA\,\Delta\xi/\Delta x\,.$$

We now let Δx shrink until that expression becomes:

$$F = YA\frac{d\xi}{dx}\,. \qquad (7)$$

4d. Force on a Segment: Dynamic Case. When a wave is traveling along the rod, the situation is not static and the forces at the two ends of any particular segment are not equal. The acceleration of the segment

at any particular time t is due to the net force on it at that time:

$$\begin{aligned} dF_{\rm net} &= F(x+dx) - F(x) \\ &= \frac{dF}{dx}\,dx \\ &= YA\frac{d^2\xi}{dx^2}\,dx. \qquad \text{(fixed time)}\,. \end{aligned}$$

We must remember what $\xi(x,t)$ is, though. It is the displacement, from equilibrium, of the individual particles in the rod as the wave travels along the rod. As such, it is a function of both x and t. Thus to get rid of the "fixed time" label in Eq. (8) we can use partial derivatives:

$$dF = YA\frac{\partial\xi}{\partial x}\,dx\,.$$

4e. Applying Newton's Second Law. We apply Newton's Second Law to a small segment of a rod that has volume density ρ and cross-sectional area A. The small segment has mass dm and length dx:

$$dm = \rho\,dV = \rho A\,dx\,.$$

The acceleration of the rod particles at x and t is:

$$a(x,t) = \frac{\partial^2\xi}{\partial t^2}\,.$$

Then Newton's second law, for the segment of length dx at x and t, gives us:

$$YA\frac{\partial^2\xi}{\partial x^2}\,dx = \rho A\,dx\frac{\partial^2\xi}{\partial t^2}\,,$$

which can be written:

$$\frac{\partial^2\xi}{\partial t^2} = \frac{Y}{\rho}\frac{\partial^2\xi}{\partial x^2}\,. \qquad (8)$$

This is the wave equation so by comparison to Eq. (1) speed for a longitudinal wave on a rod is:

$$v = (Y/\rho)^{1/2}\,. \qquad (9)$$

5. Longitudinal Waves In a Gas

5a. Stress, Strain, Bulk Modulus of Elasticity. Longitudinal waves in a gas, such as sound waves through the air, obey Hooke's law:

$$\frac{\text{stress}}{\text{strain}} = \text{ constant}\,. \tag{10}$$

This equation also applies to a long thin rod where a stress causes a one-dimensional strain (a change in length). Actually, a more thorough analysis of that case shows that the rod's cross section decreases slightly as the length increases: the rod's volume stays approximately constant. A gas does not act the same way at all since it is compressible and its volume changes with pressure. Here the appropriate constant in Eq. (10) is the gas's bulk modulus of elasticity, K, usually defined by:[6]

$$K = -\frac{dP}{dV}\,V\,.$$

5b. The Wave Equation. The equation of motion for a volume element of a gas is mathematically more complicated than that for a segment of a rod. However, in advanced texts it is shown that the wave equation for a longitudinal wave in a gas is like that for a longitudinal wave in a rod, except that K replaces Y and the mass density is specifically designated as that at equilibrium:

$$\frac{\partial^2\xi}{\partial t^2} = \frac{K}{\rho_0}\frac{\partial^2\xi}{\partial x^2}\,. \tag{11}$$

For a gas, then, the wave speed is

$$v = \sqrt{K/\rho_0} \tag{12}$$

5c. Pressure, Density, and Heat Capacity Ratio. Using the definition of K, we can write the wave velocity in terms of gas properties that we have seen before. The first gas-related equation you are likely to recall is the equation of state for an ideal gas,[7] $PV = nRT$. This

[6]Some authors write K in terms of density rather than volume.

[7]"For an ideal gas, the simplest model of intermolecular forces is assumed: there are no interactions between the molecules unless their centers happen to coincide, in which case they bounce off one another like hard spheres, and the molecules are assumed to be point masses. For such a model, the equation of state, that is, the relation between the measurable quantities (pressure, volume, temperature), can be derived in a straightforward way once temperature and average pressure are defined." (Quoted from *Temperature And Pressure of an Ideal Gas: The Equation Of State*, MISN-0-157.)

makes it look as if we could solve for P, differentiate with respect to V, and end up with a value for K. Historically, this was the first approach. However, it gives the wrong value for the wave speed and the reason is this: it implicitly assumes that the temperature T is constant. That is, it assumes that as the wave travels through the gas, heating it in the wave-peak compressions and cooling it in the wave-trough rarefactions, the gas is able to instantaneously exchange heat with its surroundings and stay at the same temperature. Actually, a wave usually travels through a gas so rapidly that there is no time for heat transfer. Thus the appropriate relationship is the one for adiabatic conditions,

$$P\,V^\gamma = \text{ constant}\,,$$

where γ is the ratio of heat capacities:

$$\gamma = \frac{C_p}{C_v}\,.$$

Differentiating this equation gives us:

$$dP\,V^\gamma + \gamma\,P\,V^{\gamma-1}\,dV = 0\,.$$

Then:

$$V\frac{dP}{dV} = -\gamma P\,,$$

and hence:

$$K = \gamma\,P\,.$$

Thus the wave speed in a gas becomes

$$v = (\gamma P/\rho_0)^{1/2}\,. \tag{13}$$

5d. Dependence on Temperature. Starting from the expression for wave speed as a function of pressure, Eq. (13), we can use the equation of state of an ideal gas to find the speed as a function of temperature. That is, we can substitute

$$PV = nRT$$

into Eq. (13) and get:

$$v = \left(\frac{\gamma nRT}{V\rho_0}\right)^{1/2} = \left(\frac{\gamma nRT}{m}\right)^{1/2}\,.$$

We can put this in a form useful for determining sound speeds in different gases by writing it in terms of $M = m/n$, the mass of one mole of the gas:

$$v = \left(\frac{\gamma RT}{M}\right)^{1/2} . \tag{14}$$

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **bulk modulus (of elasticity)**: the ratio of the increase in pressure to the decrease in the fractional volume of a fluid.
- **compressions**: the portions of the phase of a longitudinal acoustical wave when the pressure of the medium is higher than the equilibrium pressure.
- **longitudinal wave**: a wave in which the wave displacement from equilibrium is parallel to the wave velocity (example: sound waves).
- **rarefactions**: the portions of the phase of a longitudinal acoustical wave when the pressure of the medium is lower than the equilibrium pressure.
- **sound wave**: a longitudinal acoustical wave traveling through an elastic medium.
- **strain**: the fractional deformation of the dimension or dimensions of a material subjected to a stress. If only one dimension is relevant, the strain is given by the ratio of the change in length to the undeformed length. If the entire material volume is altered, the strain is given by the ratio of the change in volume to the undeformed volume.
- **stress**: applied force per unit cross-sectional area. For a fluid this is the same as pressure.

- **transverse wave**: a wave in which the wave displacement from equilibrium is perpendicular to the wave velocity (example: water waves).
- **Young's modulus**: the ratio of stress to strain for a deformation of length.

PROBLEM SUPPLEMENT

$$\frac{T}{\mu}\frac{\partial^2\xi}{\partial x^2} = \frac{\partial^2\xi}{\partial t^2} \qquad \frac{Y}{\rho}\frac{\partial^2\xi}{\partial x^2} = \frac{\partial^2\xi}{\partial t^2} \qquad \frac{\gamma P}{\rho_0}\frac{\partial^2\xi}{\partial x^2} = \frac{\partial^2\xi}{\partial t^2}$$

Note 1. Work the following problems *in order*, completely finishing each one successfully before going on to the next.

Note 2: Get appropriate velocity formulas by simply looking at the wave equations shown above.

Note 3: Some of the Answers have references to *help* sequences in this module's *Special Assistance Supplement*.

1. A guitar string of length 2.00 ft has a mass of 0.700 grams. When mounted on a guitar the string is placed under a tension of 20.0 N. Determine the speed of transverse waves traveling in the string when plucked.

2. A stretched steel wire under a tension of 1.50×10^4 N is attached at one end to an oscillator with a period of 4.00×10^{-4} sec. Given that the diameter of the wire is 0.3572 cm and that the density of steel is $7.850 \times 10^3\,\mathrm{kg/m^3}$, determine the speed of propagation, frequency, and wavelength of transverse waves traveling along the wire. Write the equation for the waveform of the wave.

3. In Westerns, the "Indians" frequently detected an approaching train by placing their ears to the railroad track and listening to the transmitted sound of the train wheels in contact with the track. Calculate the speed of sound in the steel track, treating the problem as compressional waves in a simple rod. Determine the speed of sound in air at 75 °F (23.9 °C). How do the two speeds compare?

Steel:	**Air**:
$Y = 1.95 \times 10^{11}\,\mathrm{N/m^2}$	$\gamma = 1.4$
$\rho = 7.850 \times 10^3\,\mathrm{kg/m^3}$	$M = 29.8$ grams/mole

$R = 8.31\,\mathrm{J/(K\,mole)}$

4. A copper and an aluminum rod, each of cross section $0.75\,\mathrm{cm^2}$, are welded together end-on-end to form one continuous length of metal rod. Longitudinal waves are excited in the copper rod by a vibrating source with a frequency of 675 Hz. If the wavelength of longitudinal waves in the copper portion of the rod is 5.25 m, determine the wavelength of the waves in the aluminum.

Copper:	**Aluminum**:
$Y = 11.2 \times 10^{10}\,\mathrm{N/m^2}$	$Y = 6.9 \times 10^{10}\,\mathrm{N/m^2}$
$\rho = 8.93\,\mathrm{grams/cm^3}$	$\rho = 2.7\,\mathrm{grams/cm^3}$

5. Scuba divers must use exotic mixtures of gases instead of air when diving to great depths in order to avoid nitrogen narcossis and oxygen poisoning. Such breathing mixtures typically consist mainly of helium as an "inert" gas, and a small amount of oxygen. However, when communicating with the surface, the divers' voices sound high-pitched and squeaky because sound travels faster in helium than in air (which is mostly nitrogen). Calculate the frequency of "middle C" in an environment of pure helium assuming the note is 256 Hz in air. Note that the wavelength is the same in both media since it depends only on the geometry of the voice box producing the sound. Also, the temperature is assumed to be the same in both environments.

Helium:	**Air**:
$\gamma = 5/4$	$\gamma = 7/5$
$M = 4.00$ grams/mole	$M = 29.8$ grams/mole

6. A stereo speaker cone is attached to a long tube in such a way as to generate longitudinal waves traveling down the air-filled tube. The speaker cone is made to oscillate harmonically as described by the equation:
$$y = (0.010\,\mathrm{cm})\sin\left[(8.0 \times 10^2\,\pi\,\mathrm{sec}^{-1})\,t\right]$$
where y is the horizontal displacement of the speaker surface as a function of time. Assuming the equilibrium pressure of the air is atmospheric pressure, determine the speed of waves travelling in the tube and write the equation for the waveform (for air, $\gamma = 1.4$, $\rho_0 = 1.33 \times 10^{-3}\,\mathrm{grams/cm^3}$, 1 atmosphere $= 1.01 \times 10^5\,\mathrm{N/m^2}$).

7. The density of Aluminum is $2.7 \times 10^3\,\mathrm{kg/m^3}$ and $Y_{Al} = 0.70 \times 10^{11}\,\mathrm{N/m^2}$. A thinly-drawn aluminum wire of length 10.00 m and cross-sectional area of $5.0\,\mathrm{mm^2}$ is held under a tension of 2.0×10^2 N.

   a. Compare the velocity of propagation of transverse mechanical waves and longitudinal sound waves in this wire.

   b. Which propagates faster?

c. Can both velocities be made the same?

8. The molecular mass of oxygen is 16 times the molecular mass of hydrogen, while the molecular mass of krypton (a noble gaseous element) is 42 times the molecular mass of hydrogen. Compare the speed of sound in oxygen and krypton at a given temperature. Oxygen is diatomic while krypton is monatomic so the relevant γ's are: $\gamma_{O_2} = 1.4, \gamma_{Kr} = 1.6$.

Brief Answers:

1. $v = 132\,\mathrm{m/s}$. *Help: [S-2]* *Help: [S-6]*

2. $v = 437\,\mathrm{m/s}$; *Help: [S-1]* *Help: [S-2]* *Help: [S-5]* *Help: [S-6]*
   $\nu = 2.5 \times 10^3\,\mathrm{Hz}$;
   $\lambda = 0.175\,\mathrm{m}$;
   $y = y_0 \sin\left[(35.9\,\mathrm{m}^{-1}\,x \pm (1.57 \times 10^4\,\mathrm{s}^{-1})\,t + \theta_0\right]$

3. $v_{\mathrm{steel}} = 4.98 \times 10^3\,\mathrm{m/s}$; *Help: [S-2]* *Help: [S-6]*
   $v_{\mathrm{air}} = 3.40 \times 10^2\,\mathrm{m/s}$ *Help: [S-4]* *Help: [S-6]*
   The speed of sound is 14.4 times larger in steel than in air.

4. $\lambda = 7.5\,\mathrm{m}$ *Help: [S-2]* *Help: [S-7]*

5. $\nu_{\mathrm{helium}} = 662\,\mathrm{Hz}$; nearly 2 octaves higher. *Help: [S-2]*

6. $v = 327\,\mathrm{m/s}$; *Help: [S-2]* *Help: [S-6]*
   $\xi = 0.010\,\mathrm{cm}\,\sin\left[(7.7\,\mathrm{m}^{-1}\,x - (8.00 \times 10^2\,\pi\,\mathrm{s}^{-1})\,t + \theta_0\right]$

7. a. $v_{\mathrm{mechanical}} = 121\,\mathrm{m/s}$, $v_{\mathrm{sound}} = 5.09 \times 10^3\,\mathrm{m/s}$.

   b. sound.

   c. A tension of $3.5 \times 10^5\,\mathrm{N}$ would equalize the speeds. The tensile strength of aluminum is less than that so the aluminum would pull apart.

8. $v_{\mathrm{O}} = 1.52\,v_{\mathrm{Kr}}$. [NOT 1.07, NOT 1.62] *Help: [S-3]*

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS-Problem 2)*

The problem statement says **transverse** waves in a **wire**, not *longitudinal* waves in a *rod* or a *gas*. Use the right wave equation.

S-2 *(from PS-Problems 1, 2, 3, 4, 5, 6)*

Non-MKS units (e.g., "feet," "grams," "cm,") must be converted to MKS units.

S-3 *(from PS-Problem 8)*

$M_O/M_{Kr} = (M_O/M_H)/(M_{Kr}/M_H)$ which is *given* to you as 16/42.

S-4 *(from PS-Problem 3)*

$\mu = 1.148 \times 10^{-3}\,\mathrm{kg/m}$.

S-5 *(from PS-Problem 2)*

$\mu = 0.0787\,\mathrm{kg/m}$. If I buy 1 m of the wire I get 0.0787 kg of wire. If I buy 2 m of the wire I get 0.1574 kg of wire. That is what "linear density" μ is all about.

S-6 *(from PS-Problem 3)*

Kindly read Note 2 at the head of the *Problem Supplement* and do what it says to do.

S-7 *(from PS-Problem 4)*

$v_{al} = 5.06 \times 10^3\,\mathrm{m/s}$. Think about what happens physically at the interface between the two metals. What quantity is preserved across the interface? Wavelength? Velocity? Frequency?

MODEL EXAM

$$\frac{T}{\mu}\frac{\partial^2\xi}{\partial x^2} = \frac{\partial^2\xi}{\partial t^2} \qquad \frac{Y}{\rho}\frac{\partial^2\xi}{\partial x^2} = \frac{\partial^2\xi}{\partial t^2} \qquad \frac{\gamma P}{\rho_0}\frac{\partial^2\xi}{\partial x^2} = \frac{\partial^2\xi}{\partial t^2}$$

1. See Output Skills K1-K3 in this module's *ID Sheet*. One or more of these skills, or none, may be on the actual exam.

2. The density of Aluminum is $2.7 \times 10^3\,\text{kg/m}^3$ and $Y_{Al} = 0.70 \times 10^{11}\,\text{N/m}^2$. A thinly-drawn aluminum wire of length 10.00 m and cross-sectional area of $5.0\,\text{mm}^2$ is held under a tension of $2.0 \times 10^2\,\text{N}$.

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Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 7.

3. See this module's *Problem Supplement*, problem 8.