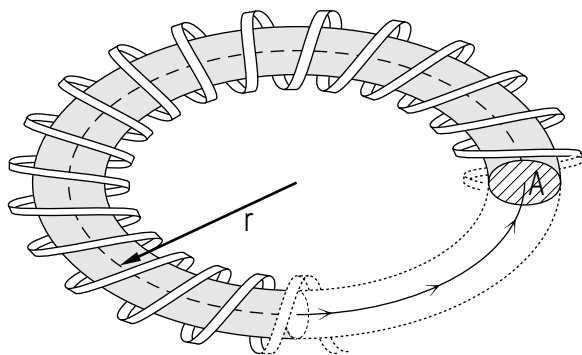


## SELF-INDUCTANCE AND INDUCTORS



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by  
Peter Signell  
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**Input Skills:**

1. Vocabulary: solenoid, toroid, inductance, induced voltage, induced current, induced magnetic field, Faraday-Henry law, Lenz's law (MISN-0-142).
2. Use Ampere's law to determine the magnetic field due to a long straight current (MISN-0-138).

**Output Skills (Knowledge):**

- K1. Vocabulary: coaxial cable, henry, inductor, self-inductance.
- K2. Write down Ampere's Law and from it derive the self-inductance of a toroidal solenoid, explicitly justifying each step.
- K3. Write down Ampere's Law and from it derive the self-inductance per unit length of a coaxial cable, explicitly justifying each step.
- K4. Starting from the relation between power, voltage and current in a steady state circuit, derive the energy stored in the electric field of an inductor.
- K5. Describe the flow of energy: (a) when the current through an inductor is increased; (b) when the current through an inductor is very gradually decreased; and (c) when the current through an inductor is quickly decreased.

**Post-Options:**

1. "Two Element DC-Driven LRC Circuits" (MISN-0-151).
2. "Velocity of a Signal in a Coaxial Cable" (MISN-0-150).

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# SELF-INDUCTANCE AND INDUCTORS

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## 1. Introduction

A solenoid or a toroid, sometimes of miniature size, is used in electronic circuits to: (1) slow the rate of change of electric current; (2) “tune” a circuit to a particular oscillational frequency; or (3) control the speed of transmission of signals. In these applications one is making use of the fact that a change in the current going through the device produces a change in the associated magnetic field and that in turn induces a current that opposes the change in the original current. A device used in that manner is called an “inductor” and the strength of its change-opposing character is called its “self-inductance” or simply its “inductance.” This (self) inductance is measured in the S.I. unit called the “henry,” so a circuit designer may specify an inductor of, say, 35 milli-henries to achieve one of the three above-mentioned aims.

## 2. Self-Inductance $L$

**2a. The Definition of  $L$ .** “Inductance” in general includes potentials and currents induced in one conductor by a time-changing current in another conductor, but “self-inductance” refers to potentials and currents that are induced in a single conductor by its own time-changing current. A device used for this purpose usually has the shape of a solenoid or a toroid.

A current flowing through an inductor of course sets up a magnetic field so changes in the current produce changes in the magnetic field. Such changes produce an induced voltage drop in the inductor, a voltage drop that opposes the change in the current. The magnitude of the induced voltage is proportional to the time-rate-of-change of the current, as we have seen,<sup>1</sup> so we write:

$$V_{ind} = L \frac{dI}{dt}. \quad (1)$$

<sup>1</sup>See “Magnetic Inductance,” MISN-0-142.

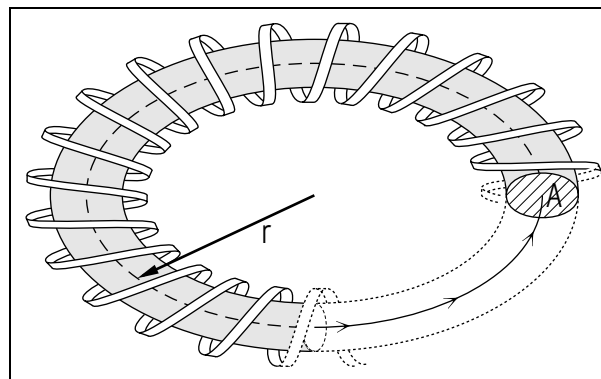


Figure 1. A toroidal solenoid.

Here  $L$  is a proportionality constant that depends on the geometry of the inductor and the inductor’s material: it is called the inductor’s *inductance*. An inductor’s inductance can be much enhanced by placing its loops of wire around a magnetic material such as iron.

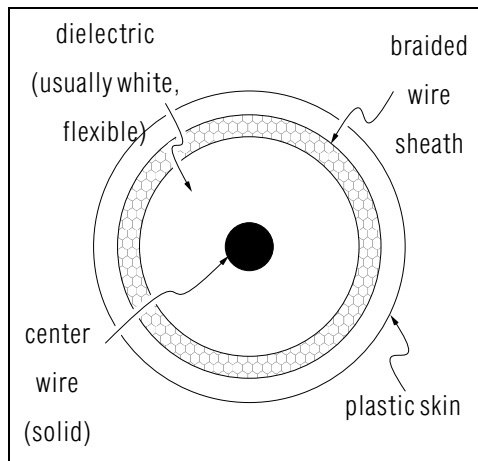
**2b.  $L$  When the Flux is Known.** If one knows the flux through the surface bounded by a circuit, then the self-inductance can be determined by integrating Eq. (1) to get:

$$\Phi = L I. \quad (2)$$

For example, if the flux enclosed by a loop is  $0.026 \text{ T m}^2$  when the current in the loop is  $1.3 \text{ A}$ , then the inductance of the loop is  $L = 20 \text{ mH}$ . If the inductor is altered to contain 15 successive loops glued together, then the flux met by the current will be increased by a factor of 15 and so will  $L$ .

**2c.  $L$  for a Toroidal Solenoid.** To find the self-inductance of a toroidal solenoid, we use Ampere’s law and we draw the integration loop as a circle of radius  $r$  from the center of the toroid (see Fig. 1). If  $r$  is less than the inner radius of the toroid, there is zero current going through any surface bounded by the integration loop. If  $r$  is between the inner and outer radii of the toroid the the current going through the enclosed surface is  $NI$  where  $N$  is the number of turns of wire carrying the current  $I$ . If  $r$  is greater than the outer radius of the toroid, there is again zero *net* current going through any surface bounded by the integration loop (the current going one direction through the surface is exactly canceled by the current going the other way through the surface). Then by Ampere’s law, the magnetic field inside the solenoidal loops is:

$$B(r) = \frac{2 k_m N I}{r}. \quad (3)$$



**Figure 2.** Cross-section of a coaxial cable.

If the radius of each loop is much smaller than the toroidal radius, then  $r$  will vary little over the cross-sectional area  $A$  of each loop and we can take  $r$  as a constant. To get the flux in a single loop of the winding, we need merely multiply the  $B$  by the loop area  $A$ . Then for the  $N$  windings in the inductor:

$$\Phi = N A B = \frac{2 k_m N^2 I A}{r}. \quad (4)$$

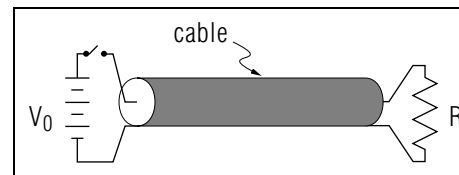
Then from Eq. (2):

$$L = \frac{2 k_m N^2 A}{r}. \quad (5)$$

Typical numbers for a circuit toroid will give an extremely small inductance unless the flux is enhanced through the use of an iron core inside the solenoidal loops. This enhancement may be by a factor of five thousand or more.

### 3. $L/\ell$ for a Coaxial Cable

**3a. Physical Description of a Coaxial Cable.** Physically, a coaxial cable looks like a fat round wire: a good example is the cable that feeds television programs to TV sets from a “cable company.” If you were to cut through such a cable you would see a central conducting wire surrounded by a dielectric material (usually flexible white plastic). This insulating material is surrounded by a cylindrical sheath woven from conducting wires (woven to make it flexible). That outer conductor is, in turn, covered by a thin skin of flexible insulating plastic (see Fig. 2). Current flows



**Figure 3.** Longitudinal view of a coaxial cable in a hypothetical circuit.

down one conductor and exactly the same amount of current flows in the opposite direction in the other conductor. One can imagine a voltage source connected between the inner and outer conductors at one end of the cable and a resistor connected between them at the other end, as in Fig. 3. We will assume that the current traveling the inner wire is along its surface, at a radius  $r_i$  (a good assumption for high-frequency waves). We will call the radius of the outer conductor  $r_o$ .

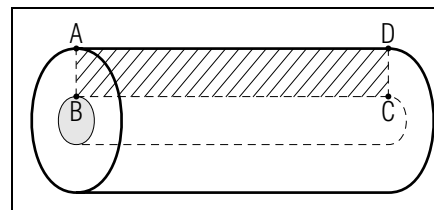
**3b. Getting  $B$ .** For self-inductance we need flux per unit current, and for flux we need the magnetic field. For a coaxial cable the entire magnetic field is between the two conductors. That is easily seen because by Ampere’s law there is zero net current crossing a cross-sectional area larger than the outside conductor (remember that the currents in the two conductors are equal but opposite in direction). For the region between the two conductors, Ampere’s law immediately shows that the magnetic field is simple that of the inner conductor. For a long straight wire it is:<sup>2</sup>

$$B = \frac{2 k_m I}{r},$$

and of course the direction of the field is everywhere perpendicular to the outward cylindrical radius from the inner wire.

**3c. Getting  $L/\ell$ .** To get the flux in our coaxial cable we must integrate the component of the magnetic field normal to a surface bounded by the loop of electrical current. To make it easy we choose a surface that is

<sup>2</sup>Apply Ampere’s law in your head or see “Magnetic Fields from Currents,” MISN-0-138.



**Figure 4.** A longitudinal view of the coaxial cable showing the integration surface  $ABCD$  for obtaining the flux.

radial, running between the two conductors and along the length of the cable (see Fig. 4). Now the element of flux at a particular radius is just the value of  $B$  there times the element of area at that radius:

$$d\Phi = (B) dA = \left( \frac{2k_m I}{r} \right) \ell dr.$$

where  $\ell$  is the length of the cable. Integrating,

$$\Phi = \int d\Phi = \int_{r_i}^{r_o} \left( \frac{2k_m I}{r} \right) \ell dr = 2k_m I \ell \ln(r_o/r_i).$$

Finally, then, the self-inductance per unit length of cable is:

$$L/\ell = \frac{\Phi/\ell}{I} = 2k_m \ln(r_o/r_i). \quad (6)$$

The total inductance for any particular piece of cable can be obtained by multiplying  $L/\ell$  by the piece's length.

## 4. Inductive Energy In a Circuit

**4a. Energy Needed to Set up a Current.** When a circuit switch is closed, starting a flow of current through a circuit, an inductor in the path of the current resists the rise of the current from zero by developing a counter-voltage drop (a voltage rise). The source of voltage in the circuit must push the current past this voltage rise, doing work equal to the voltage rise times the amount of charge pushed through it. The power expended (the energy per unit time) is just the current (the charge per unit time) times the self-induced voltage:

$$P = I L \frac{dI}{dt}.$$

Using  $P = dE/dt$  we can easily integrate both sides of:

$$dE = I L dI$$

to get:

$$E = \frac{1}{2} L I^2. \quad (7)$$

This is the energy that must be expended by the circuit's energy source in order to raise the circuit current from zero to the value  $I$ .

**4b. The Energy is Recoverable.** The energy expended in setting up a current in an inductor is recoverable if the circuit's voltage source is removed. Assuming there is still a complete circuit without the voltage source, the inductor will keep the current flowing until the energy of Eq. (7) has been completely dissipated in the circuit's resistances or perhaps transferred to a capacitance for storage there.

**4c. Location of the Energy.** We describe the energy of Eq. (7) as being stored in the magnetic field of the inductor. That stored energy was zero in the beginning of our example, when the circuit current was zero so the inductor's magnetic field was zero. As the current, hence the inductor's magnetic field, increased the energy in the magnetic field increased as the circuit's voltage source supplied energy to the field. If the current became steady, the inductor's magnetic field became steady along with it and there was no longer a transfer of energy from the voltage source to the inductor's field. However, the energy stored in the inductor's magnetic field stays stored there.

**4d. Energy Flow When a Current is Stopped.** Suppose a steady current is flowing in a circuit containing an inductor, and then one opens a switch so current can presumably no longer flow: what happens to the energy stored in the inductor's magnetic field? The answer is that as the switch is opened the current will drop quickly, creating a large induced voltage that ionizes the air across the switch gap and thus causes an electric arc. The energy stored in the inductor's magnetic field is thus dissipated in chemical and heat energy in breaking down the air in the switch gap and in burning the contact points in the switch and in burning anything else in the vicinity. In fact, the stored energy in a large inductor can be extremely dangerous to anyone attempting to stop the current in a hurry.

## Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

## Glossary

- **co-axial cable:** a circuit element that looks like a long fat plastic-covered wire, containing in successive cylindrical layers: an inner solid-wire conductor, a surrounding flexible dielectric, and an outer braided-wire sheath. The circuit's current goes down one conductor (solid or sheath) and back the other.
- **henry:** the SI unit of self-inductance, abbreviated H and defined to be an ohm-second. Thus:  $H \equiv \Omega s = V s A^{-1}$ .
- **inductor:** a circuit element whose purpose is to provide self-inductance, an electrical circuit analog of mechanical inertia (mass). An inductor is usually in the shape of a solenoid or a toroid. The inductance of an inductor depends on the geometry of the inductor and the magnetic susceptibility of the materials of which the inductor is constructed. Inductors in electronic circuits typically are in the mH range.
- **self-inductance:** the negative of the induced voltage around a loop divided by the time-rate-of-change of magnetic flux through any surface bounded by that loop:  $L = -V_{induced}/\dot{\Phi}$ . The minus sign shows that the induced voltage opposes the change in the flux. The SI unit of inductance is the henry (which see).

## MODEL EXAM

1. See Output Skills K1-K5 in this module's *ID Sheet*. As usual, "Vocabulary" means defining the words as well as being able to use them properly.

### Brief Answers:

1. See this module's *text*.