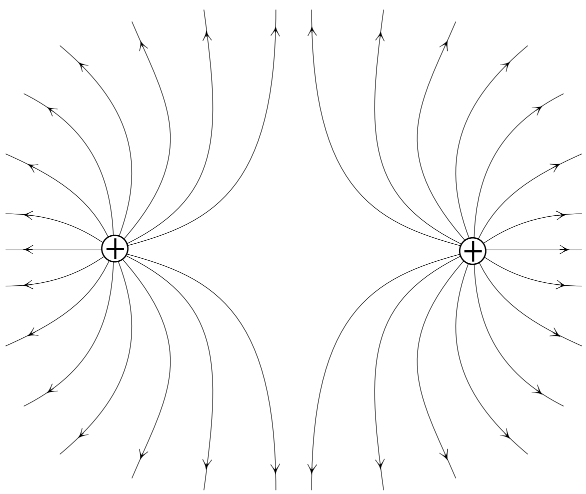
PHYSNET

MISN-0-115

# POINT CHARGE: FIELD AND FORCE



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

POINT CHARGE: FIELD AND FORCE

by

J. S. Kovacs and Kirby Morgan

Title: **Point Charge: Field and Force**

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**Input Skills**:

1. Vocabulary: gravitational field (MISN-0-108); point charge, electric force constant $k_e$ (MISN-0-114).
2. State Coulomb's law (MISN-0-114).
3. Express a vector in terms of unit vectors associated with a fixed coordinate system (MISN-0-2).
4. Solve two-dimensional kinematics problems (MISN-0-8).
5. For a given vector, construct a unit vector with the same direction (MISN-0-9).

**Output Skills (Knowledge)**:

K1. Vocabulary: action-at-a-distance, electric field, electric field vector (electric field strength), electric force, field, lines of force, non-uniform electric field, uniform electric field.

K2. State the expression for the electric field at a specified point in space due to a fixed point charge. Indicate in a sketch the direction of this field vector at the point.

K3. State how lines of force are related to the electric field.

**Output Skills (Problem Solving)**:

S1. Calculate the electric field at a specified point in space due to a given spatial distribution of charged point particles.

S2. Calculate the force on a charged point particle at a point where you know the electric field. Also determine the motion of a charged particle in a region where the electric field is known.



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## 1. Introduction

The electrostatic force between two point charges can be thought of from two points of view. According to Coulomb's law, the particles exert a force on each other even though they are not in direct contact; this is the action-at-a-distance concept. An alternative approach is the field concept where one charge sets up an electrostatic field at each point in space and the second particle feels a force due to the field. These two points of view may seem to be equivalent and so the introduction of the field concept appears to be superfluous; but it isn't. For example, when the charged particles move at relativistic speeds, the force will not be transmitted instantaneously as required by the action-at-a-distance viewpoint because of the finite speeds with which signals can propagate. The field concept survives such difficulties and, furthermore, opens the way to an understanding of more difficult topics such as the propagation of electromagnetic signals and the properties of light. In this module we confine our attention to defining the electric field vector and finding the force on a particle in the presence of an electric field.

## 2. The Electric Field

**2a. The Field Concept.** The field concept enables us to avoid considering action-at-a-distance forces. Coulomb's law states that two charged particles exert forces on one another in spite of the fact that they are not in physical contact; that's action-at-a-distance. The alternative is to view the situation in two steps. First, one of the particles establishes an electric field such that, associated with every point in space, there is an electric field vector. The other particle, placed at one of the points in space, interacts via its own charge with the electric field at the point. The force this particle feels is a contact-like force, exerted on it by the field at the particle's location. At another point, where the field has a different magnitude and direction, the particle would feel a different force. Another charged particle placed at the same point might have a different force exerted on it.





**2b. Definition of $\vec{E}$.** The electric field vector or electric field strength, $\vec{E}$, at a point in space is defined in such a way that the force on a charged particle placed at that point is given by

$$\vec{F} = q\vec{E}\,, \tag{1}$$

where $q$ is the charge of the particle. If the field $\vec{E}$ varies from point to point, it can be looked at mathematically as a vector function of position and written as

$$\vec{E}(x,y,z) = E_x(x,y,z)\hat{x} + E_y(x,y,z)\hat{y} + E_z(x,y,z)\hat{z}\,,$$

where $E_x$, $E_y$, $E_z$ are the components of $\vec{E}$, and $x$, $y$, $z$ are the cartesian coordinates of the point relative to some coordinate system. This is frequently written as $\vec{E}(x,y,z) = \vec{E}(\vec{r})$ where $\vec{r}$ is the vector from the origin of the coordinate system to the point at which the electric field is to be determined.

**2c. The Electric Force.** The force $\vec{F} = q\vec{E}$ on a charged particle which is placed at a point designated by a vector $\vec{r}$, is independent of how the electric field $\vec{E}$ comes about. Whether $\vec{E}$ is the electric field due to a single point charge (located at some point in space other than $\vec{r}$, where $q$ is) or whether it is the electric field due to some distribution of a large number of discrete charges, the force on charge q placed at point $\vec{r}$ is given by Eq. (1).

**2d. Uniform Electric Field.** For what sort of simple, physically realizable situations can we write down a function which represents the electric field for that system? The simplest function is a constant, i.e. the field is uniform in magnitude and direction. A uniform electric field is such that a charged particle placed at any point in the region of that field feels the same force. Such a uniform electric field can be achieved between two large parallel planes of equal but opposite uniform distributions of charge.[1]

**2e. Charge Motion in a Uniform Electric Field.** The equation of motion for an electric charge in an electric field is given by

$$m\vec{a} = \vec{F} = q\vec{E}\,, \tag{2}$$

or

$$\vec{a} = \frac{q\vec{E}}{m}\,. \tag{3}$$

[1] See "Gauss's Law Applied to Cylindrical and Planar Charge Distributions" (MISN-0-133).

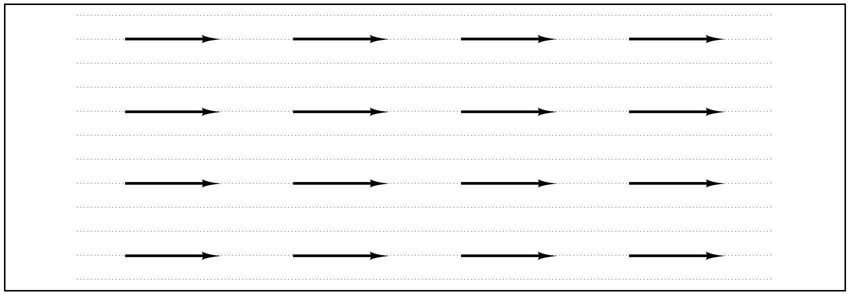


**Figure 1.** A uniform electric field.

The velocity and position of the particle (of charge $q$ and mass $m$) can be found by integrating Eq. (3). If the electric field is uniform, $E$ is a constant and $a$ is a constant also. In this case for motion parallel to the field.

$$v = v_0 + \frac{q}{m} E t \,, \tag{4}$$

and

$$x = x_0 + v_0 t + \frac{q}{2m} E t^2 \qquad \textit{Help: [S-1]} \,. \tag{5}$$

**2f. Comparison to Gravitational Field.** The constant electric field has as an analog the gravitational field near the surface of the earth. Just as the gravitational field interacts with the particle's mass, the electric field interacts with the charge of the particle.[2] The force on a particle of mass $m$ in a gravitational field $\vec{g}$ is given by $\vec{F} = m\vec{g}$ while the force on a particle of charge $q$ in an electric field $\vec{E}$ is $\vec{F} = q\vec{E}$. Near the surface of the earth the gravitational field varies very little, but for greater distances the field varies inversely as the square of the distance from the center of the earth. The latter situation is analogous to the electric field of a point charge.

## 3. Field of a Point Charge

**3a. Derivation From Coulomb's Law.** The field of a point charge can be determined from the definition of the electric field [Eq. (1)] and Coulomb's law.[3] With a particle of charge $q'$ placed at the origin of the coordinate system, the force on another particle with charge $q$, placed at the end of the position vector $\vec{r}$, is given by

$$\vec{F} = k_e \frac{qq'}{r^2} \hat{r} \,, \tag{6}$$

directed along the unit vector $\hat{r}$, either away from or toward the origin depending upon the sign of the product $qq'$. Combining this with Eq. (1) gives

$$\vec{E}(\vec{r}) = k_e \frac{q'}{r^2} \hat{r} \,. \tag{7}$$

for the electric field at the position $\vec{r}$ due to the charge $q'$.

[2] The magnetic field, on the other hand, interacts with a charged particle in a more complicated way, with its charge and velocity. See "Force on a Charged Particle in a Magnetic Field" (MISN-0-122).

[3] See "Coulomb's Law" (MISN-0-114).





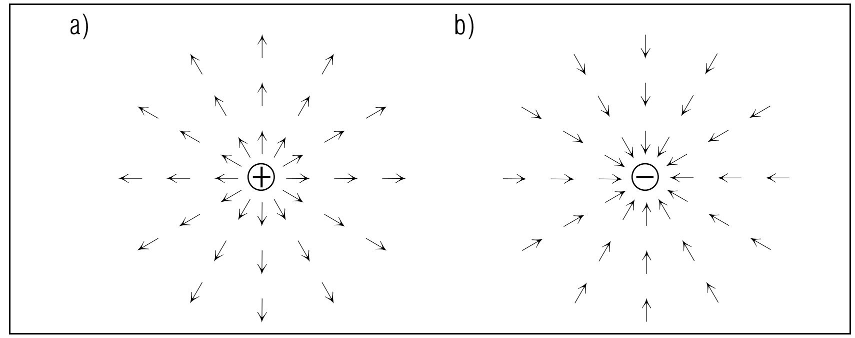



**Figure 2.** The Field of a Point Charge: (a) a positive charge, (b) a negative charge.

**3b. Characteristics of the Field.** Many quantitative and qualitative aspects of the field of a point charge can be ascertained from Eq. (7). First of all, the electric field vector points away from $q'$ if it is positive, and toward it, if it is negative. Moreover, the field's magnitude is not the same at all points since it decreases inversely as the square of the distance from the origin. The field due to a point charge is therefore an example of a non-uniform field. What are the dimensions of the electric field vector? From the definition, Eq. (1), it can be seen that $E$ has the dimensions newtons per coulomb (N/C).

**3c. Field Due to More Than One Charge.** To find $\vec{E}$ at a given point for a group of charges, we must calculate $\vec{E}_i$ due to each charge $q_i$ as if it were the only charge present and then add the results vectorially:

$$\vec{E}_{\text{Total}} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n = \sum_{i=1}^{n} \vec{E}_i \,.$$

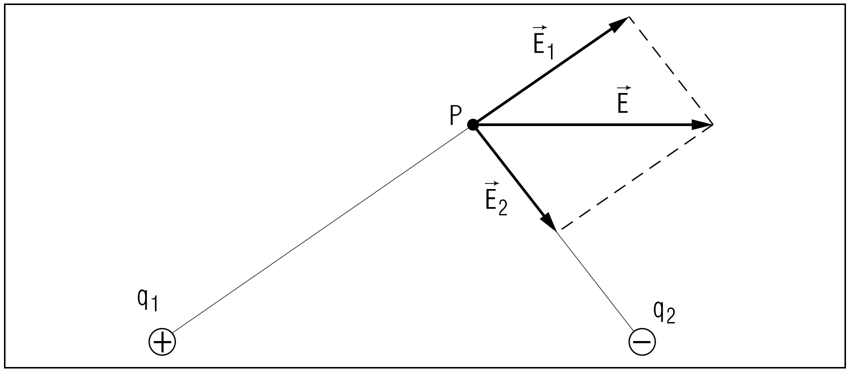



**Figure 3.** The net electric field due to two charges.

For example, Fig. 3 shows the resultant electric field $\vec{E}$ at a point $P$ due to two point charges of opposite sign.

**3d. Lines of Force.** Instead of sketching the electric field vector at selected points (as was done in Figs. 1 and 2) another convenient way to visualize the field is to imagine "lines of force." The (imaginary) lines of force are related to the electric field vector in these ways:

1. The tangent to the line of force gives the direction of $\vec{E}$ at that point.
2. The number of lines of force drawn per unit cross-sectional area is proportional to the magnitude of $\vec{E}$.

The lines of force are shown in Fig. 4 for two oppositely charged parallel planes and a positive point charge. Figure 5 shows the lines of force present for the combination of two unlike charges and two like charges.

## 4. Calculating the Field

**4a. Statement of the Problem.** Three equivalent, alternative methods can be used for determining the electric field at an arbitrary point in space due to a point charge at another arbitrary point in space (see Fig. 6). Notice that the displacement $\vec{R}$ from the charge $Q$ to the point $P$ is now the relevant quantity in the determination of the electric field, and not the position vector $\vec{r}$ of the point $P$ with respect to the origin of





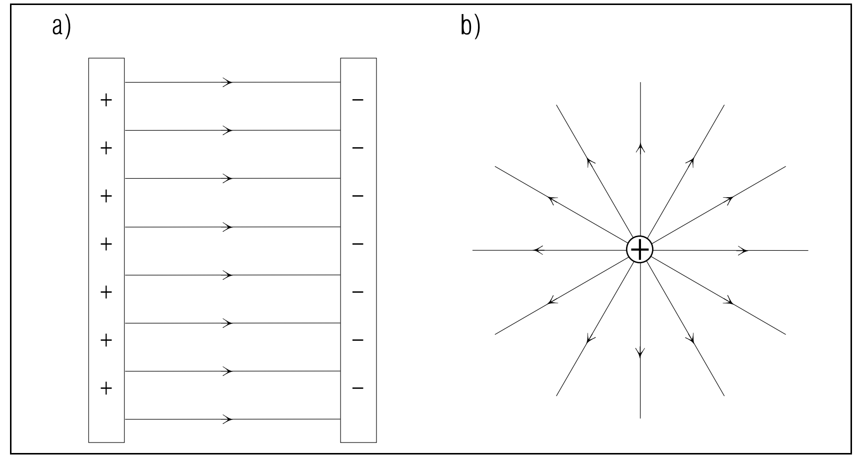



**Figure 4.** Lines of force for: (a) oppositely charged parallel planes; (b) positive point charge.

coordinates.

**4b. Method 1: Cartesian Components of $\vec{R}$.** The electric field at the point $P$ is given by

$$\vec{E} = k_e \frac{Q}{R^2} \hat{R}\,, \tag{8}$$

where $\hat{R}$ is a unit vector pointing from the charge $Q$ toward the point $P$, and $R$ is the distance from the charge to point $P$ as shown in Fig. (7). The direction of the unit vector $\hat{R}$ needs to be expressed in terms of the

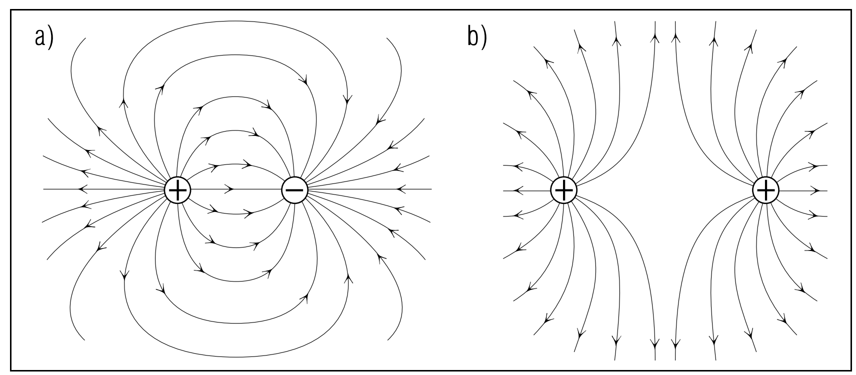



**Figure 5.** Lines of force for two charges: (a) opposite charges; (b) like charges.

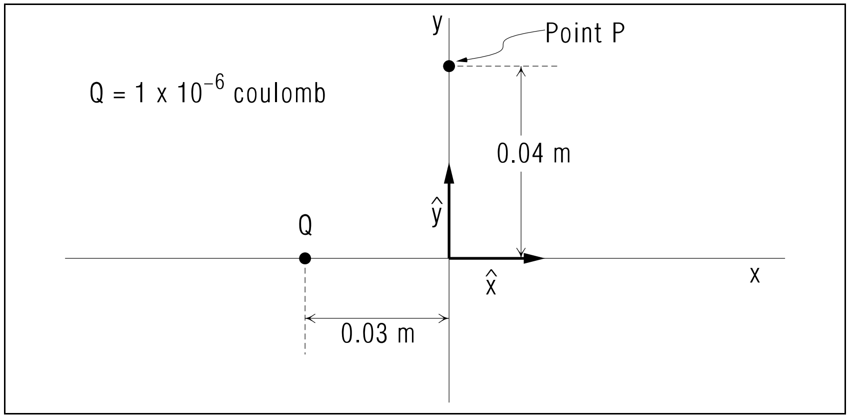



**Figure 6.** A point charge $Q$ is at point $x = -0.03\,\text{m}$ on the $x$-axis. The electric field is to be determined at point $P$ on the $y$-axis.

unit vectors $\hat{x}$ and $\hat{y}$ which are associated with the coordinate system.[4]

[4]The unit vector $\hat{R}$ itself is not generally useful. If there were several point charges in this region, and the resultant field at $P$ were needed, each charge would have associated with it its own unit vector $\hat{R}$ and these couldn't be easily combined unless each was referred to a fixed reference system such as that determined by $\hat{x}$ and $\hat{y}$.

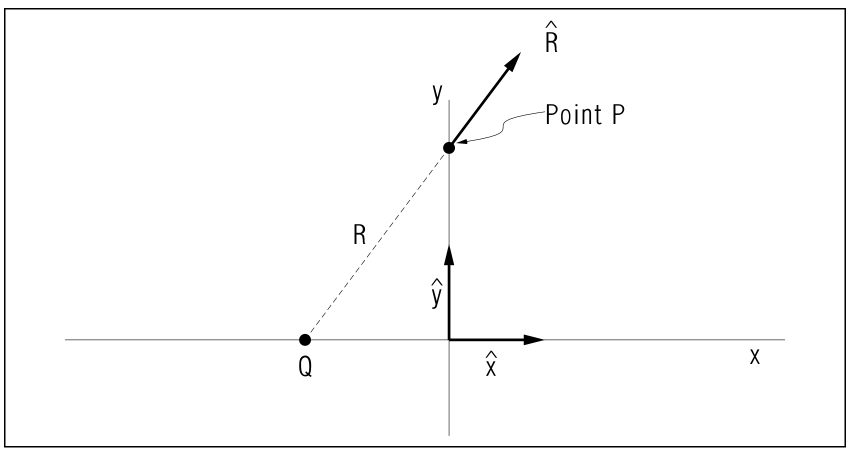



**Figure 7.** The distance $R$ from the source of the field charge $Q$, to the point $P$ where the electric field is to be determined using Eq. (8).





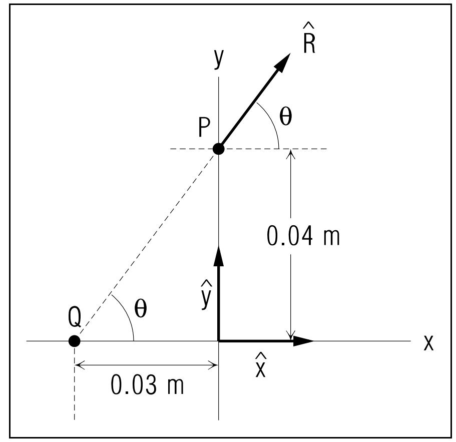



**Figure 8.** The direction of the field at $P$ due to the point charge $Q$ is given by the angle $\theta$ that the field vector $\vec{E}$ makes with the $x$-axis.

Thus (refer to Figs. 6 and 7):

$$\vec{R} = (0.03\,m)\hat{x} + (0.04\,m)\hat{y}\,, \tag{9}$$

and

$$\vec{R}\cdot\vec{R} = R^2 = (0.05\,m)^2\,. \tag{10}$$

The unit vector $\hat{R}$ can be expressed by dividing $\vec{R}$ by its magnitude:

$$\hat{R} = \frac{\vec{R}}{R} = \frac{0.03}{0.05}\hat{x} + \frac{0.04}{0.05}\hat{y}\,. \tag{11}$$

Thus the field $E$ at point $P$ can be written:

$$\vec{E}_P = (3.6\times10^6\,\text{N/C})\hat{R} = (2.16\times10^6\hat{x} + 2.88\times10^6\hat{y})\,\text{N/C}\,. \tag{12}$$ *Help: [S-2]*

$\vec{R}$ in component form (in general) is:

$$\vec{R} = R_x\hat{x} + R_y\hat{y} + R_z\hat{z}\,. \tag{13}$$

(For the specific situation of Fig. 6, $R_x = 0.03\,\text{m}$, $R_y = 0.04\,\text{m}$, $R_z = 0$). Thus:

$$\vec{E} = k_e\frac{Q}{R^2}\frac{\vec{R}}{R} = k_e\frac{Q}{R^2}\left[\left(\frac{R_x}{R}\right)\hat{x} + \left(\frac{R_y}{R}\right)\hat{y} + \left(\frac{R_z}{R}\right)\hat{z}\right]. \tag{14}$$

**4c. Method 2: Magnitude and Direction.** An alternative way of expressing $\vec{E}$ at the point $P$ would be in terms of its magnitude and an angle relative to some fixed direction. As indicated in Fig. 8, the angle $\vec{R}$ makes with the $x$-axis can be determined from the right triangle involving

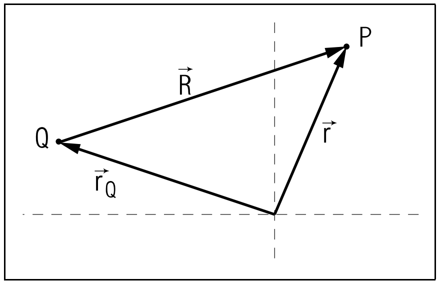



**Figure 9.** Vector diagram illustrating the relation between the vectors $\vec{r}$, $\vec{r}_Q$, and $\vec{R}$.

$\vec{R}$ and the $x$- and $y$-components of $\vec{R}$. The angle $\theta$ can be found from its tangent

$$\tan\theta = \frac{0.04m}{0.03m} = \frac{4}{3}\,, \tag{15}$$

$$\theta = \tan^{-1}(4/3) = 53.1^\circ\,. \tag{16}$$

$\vec{E}$ is thus $3.6 \times 10^6$ N/C at $53.1^\circ$ with respect to the positive $x$-axis (measured counter-clockwise). The field can then be decomposed into $x$ and $y$ components:

$$\vec{E} = E_x\hat{x} + E_y\hat{y} = E\cos\theta\hat{x} + E\sin\theta\hat{y}\,.$$

**4d. Method 3: General Expression.** A general expression for the field at any point $P$ due to a charge $Q$ located at any other point can be derived in terms of the cartesian coordinates of $P$. Rewriting the expression for the electric field (Eq. (8)):

$$\vec{E} = k_e\frac{Q}{R^2}\hat{R} = k_e\frac{Q}{R^2}\frac{\vec{R}}{R}\,, \tag{17}$$

where $R\hat{R} = \vec{R}$, the vector from $Q$ to point $P$. We now define these vectors (see Fig. 9):

$\vec{r}_Q \equiv$ position vector from origin to charge $Q$

$\vec{r} \equiv$ position vector from the origin to the point where we wish to know the field.

$\vec{R} \equiv$ displacement vector from $Q$ to point $P$

From Fig. 9 we can see that:

$$\vec{r} = \vec{r}_Q + \vec{R}\,,$$





or

$$\vec{R} = \vec{r} - \vec{r}_Q\,, \tag{18}$$

so that $\vec{R}$ has these components:

$$R_x = x - x_Q; \qquad R_y = y - y_Q; \qquad R_z = z - z_Q\,, \tag{19}$$

and the magnitude of $\vec{R}$ is given by:

$$R = \left(R_x^2 + R_y^2 + R_z^2\right)^{1/2} = \left[(x-x_Q)^2 + (y-y_Q)^2 + (z-z_Q)^2\right]^{1/2}. \tag{20}$$

Thus Eq. (17) can be written:

$$\vec{E} = k_e\left(\frac{Q}{R^2}\right)\hat{E}\,. \tag{21}$$

Here:

$$\hat{E} = \left(\frac{x-x_Q}{R}\right)\hat{x} + \left(\frac{y-y_Q}{R}\right)\hat{y} + \left(\frac{z-z_Q}{R}\right)\hat{z}\,,$$

with $R$ defined in Eq. (20). Notice that this expresses the field in terms of the coordinates of the location of the charge and the coordinates of the location of some point in space. For our example in Fig. 6, all components are zero except $x_Q$ and $y$, so that

$$\vec{E} = k_e\frac{Q}{R^2}\left[\frac{-x_Q}{R}\hat{x} + \frac{y}{R}\hat{y}\right]. \tag{22}$$

Here: $x_Q = -0.03$ m (note the sign); $y = +0.04$ m; $R = +0.05$ m.

Substitution yields the previous result given in Eq. (12). *Help: [S-3]*

## Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

## Glossary

- **action-at-a-distance**: a classical-mechanics viewpoint in which two particles exert a force on each other even though they are not in direct contact. In quantum mechanics, such a force is transmitted from one particle to the other by the exchange of virtual particles called *quanta*.

- **electric field**: the vector quantity associated with a point in space, representing the force per unit charge experienced by a test charge placed in the field.
- **electric field vector**: the vector representing the magnitude and direction of the electric field at a given point in space.
- **electric force**: the force on a particle due solely to the electric field at the particle's location.
- **field**: a quantity that has a value at each point in space.
- **lines of force**: (fictional) lines drawn to visualize the electric field; the direction of the field is tangent to the lines at all points and the magnitude of the field is proportional to the density of the lines.
- **non-uniform electric field**: an electric field that has a different direction and magnitude at different points in space.
- **uniform electric field**: an electric field that has the same direction and magnitude at all points in a region of space.

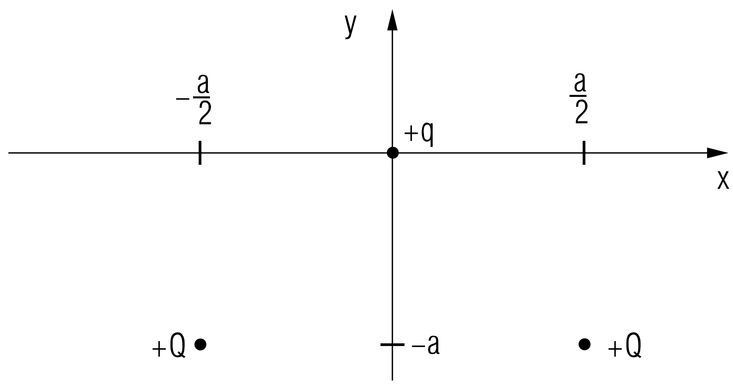




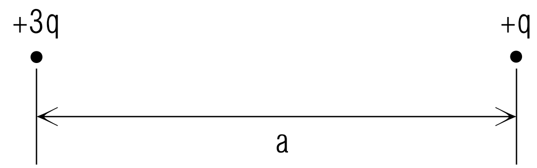
## PROBLEM SUPPLEMENT

Note: Problems 15 and 16 also occur in this module's *Model Exam*.

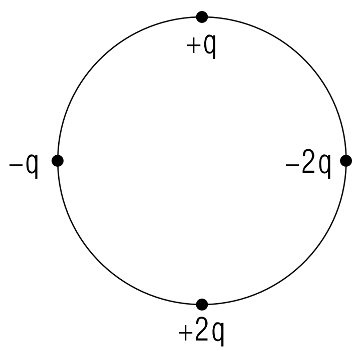
1. What is the magnitude and direction of $\vec{E}$ at a point midway between two equal and opposite charges of magnitude $2.5 \times 10^{-7}$ C which are 16 cm apart? What force (magnitude and direction) would act on an electron ($-1.60 \times 10^{-19}$ C) placed there?
2. Two point charges with charge $+3.0 \times 10^{-7}$ C and $+7.5 \times 10^{-8}$ C are 15 cm apart.
   a. What magnitude of electric field does each produce at the position of the other?
   b. What magnitude of force acts on each?
3. Three charges are arranged as shown below. Find the magnitude and direction of the electric field at the origin due to the other two charges and compute the force on $q$.




4. Find the point along the line joining the two charges shown below at which the electric field due to the two charges is zero ($a = 5$ cm).




5.




Find the magnitude and direction of the electric field at the center of the circle which has charges arranged on it as shown below if $q = 2.0 \times 10^{-7}\,\mathrm{C}$. The radius of the circle is 3.0 cm.

6. A positron, a particle of charge $q = +1.6 \times 10^{-19}\,\mathrm{C}$ and mass $m = 9.1 \times 10^{-31}\,\mathrm{kg}$, enters a uniform electric field $-2.0 \times 10^{-3}\,\mathrm{N/C}\,\hat{x}$ with a velocity $2.0 \times 10^{3}\,\mathrm{m/s}\,\hat{x}$. Find how far it travels before coming (momentarily) to rest. Find its velocity after $1.0 \times 10^{-5}\,\mathrm{s}$.





**Brief Answers**:

1. $|\vec{E}| = 7.0 \times 10^{5}\,\mathrm{N/C}$, toward the negative charge.

   $|\vec{F}| = 1.1 \times 10^{-13}\,\mathrm{N}$, toward the positive charge.

2. a. $E_1 = 1.2 \times 10^{5}\,\mathrm{N/C}$

   $E_2 = 3.0 \times 10^{4}\,\mathrm{N/C}$

   b. $F_1 = 9.0 \times 10^{-3}\,\mathrm{N}$

   $F_2 = 9.0 \times 10^{-3}\,\mathrm{N}$

3. $\vec{E} = (0.36)(4)(k_e)\dfrac{Q}{a^2}\,\hat{y}$

   $\vec{F} = (0.36)(4)(k_e)\dfrac{Qq}{a^2}\,\hat{y}$ *Help: [S-6]*

4. $\vec{E} = 0$ at 3.2 cm to the right of the 3q charge. *Help: [S-5]*

5. $E = 2.8\times 10^{6}\,\mathrm{N/C}$, $\theta = 45^\circ$ with respect to positive $x$-axis. *Help: [S-4]*

6. $x = 5.7 \times 10^{-3}\,\mathrm{m}$ *Help: [S-7]*

   $\vec{v} = -1.5 \times 10^{3}\,\mathrm{m/s}\,\hat{x}$

**Supplementary Problems**:

7. A point charge $Q_4$ is located at point $x = a$, $y = 0$, $z = 0$. Write down the expression for the electric field at point $x = -2a$, $y = 0$, $z = 0$. *Answer: 7*

8. A charge $Q_1 = -3.00 \times 10^{-6}\,\mathrm{C}$ is located at the origin of the coordinates while a charge $Q_2 = +4.00 \times 10^{-6}\,\mathrm{C}$ is located at $x = +0.300\,\mathrm{m}$, $y = z = 0.000\,\mathrm{m}$. Find the resultant electric field at the point, $x = 0.600\,\mathrm{m}$, $y = z = 0.00\,\mathrm{m}$. *Answer: 10*

9. Do the same for the point $x = +0.10\,\mathrm{m}$, $y = z = 0.00\,\mathrm{m}$ *Answer: 15.*

10. Using the results of Problems 8 and 9, find the force at each of the points in these problems on a $+2.0 \times 10^{-8}\,\mathrm{C}$ charge: Problem 8 *Answer:* ; Problem 9 *Answer: 13.*

11. Is the force on this particle constant? *Answer: 2* This force will cause the particle to accelerate. Will the acceleration be constant? *Answer: 16* (Can you use the same kinematic equations relating position, velocity, and acceleration as you do for a particle in the constant gravity field near the surface of the earth? *Answer: 19*

12. Repeat Problem 10 for a charge of $-2.0 \times 10^{-8}\,\mathrm{C}$. *Answer:* , *Answer: 18.*

13. A uniform electric field $\vec{E} = (+5.0 \times 10^7\,\mathrm{N\,C^{-1}})\hat{z}$ exists in a region of space.

    a. What is the force that $+3.0 \times 10^{-6}\,\mathrm{C}$ charge feels when placed at a point in this region? *Answer: 15*

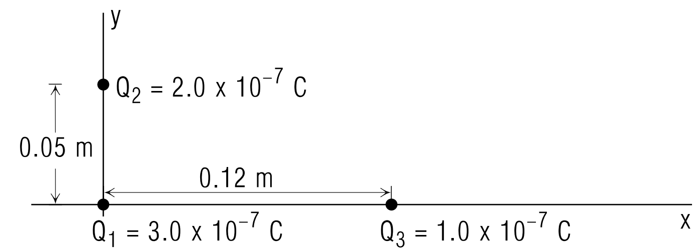
    b. What force does a $-3 \times 10^{-6}\,\mathrm{C}$ charge feel in this same region? *Answer: 14*

14. Repeat Problem 11, applying it to the particle of Problem 13. *Answer: 16*

15. Three charges are arranged as shown:








    a. Find the electric field at the point $x = 0.120\,\mathrm{m}$, $y = 0.0500\,\mathrm{m}$ due to the three charges located as shown above. *Answer: 20*

    b. Find the force on charge $q_3$, due to $q_2$ and $q_1$. *Answer: 20*

16. Find the acceleration of an alpha particle (charge= $+2e$, mass= $M_\alpha$) in a uniform electric field $E\hat{y}$. If its initial velocity $v_0$ is perpendicular to the field, find the equations for its displacement as functions of time. *Answer: 21*

**Brief Answers**:

7. $\vec{F} = (6.5 \times 10^{-3}\,\text{N})\,\hat{x}$
8. No
9. $-6.5 \times 10^{-3}\,\text{N}\,\hat{x}$
10. $3.25 \times 10^{5}\,\text{N/C}\,\hat{x}$
11. $1.5 \times 10^{2}\,\text{N}\,\hat{z}$
12. No
13. $-7.2 \times 10^{-2}\,\text{N}\,\hat{x}$
14. $-1.5 \times 10^{2}\,\text{N}\,\hat{z}$
15. $-3.6 \times 10^{6}\,\text{N/C}\,\hat{x}$
16. All answers are Yes.
17. $\vec{E} = -k_e \dfrac{Q\hat{x}}{9a^2}$
18. $7.2 \times 10^{-2}\,\text{N}\hat{x}$
19. No
20. a. $\vec{E} = (2.72 \times 10^{5}\,\text{N/C})\hat{x} + (4.21 \times 10^{5}\,\text{N/C})\hat{y}$
    b. $\vec{F} = (0.0285\,\text{N})\hat{x} - (0.0041\,\text{N})\hat{y}$
21. $\vec{a} = (2eE/M_\alpha)\hat{y}$

    $x = x_0 + v_0 t$

    $y = y_0 + eEt^2/M_\alpha$





## SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-2e)*

$$a = \frac{dv}{dt} = \frac{qE}{m}$$

$$\int_{v_0}^{v} dv' = \int_0^t \left(\frac{q}{m}E\right) dt'$$

$$v - v_0 = \frac{q}{m}Et$$

$$v = \frac{dx}{dt} = \frac{q}{m}Et + v_0$$

$$\int_{x_0}^{x} dx' = \int_0^t \left(\frac{q}{m}Et'\right) dt' + \int_0^t v_0 dt'$$

$$x - x_0 = \frac{q}{2m}Et^2 + v_0 t$$

S-2 *(from TX-4b)*

$$E = k_e \frac{Q}{R^2} = (8.99 \times 10^{9}\,\text{N}\,\text{m}^2\,\text{C}^{-2})\frac{10^{-6}\,\text{C}}{0.0025\,\text{m}^2}$$

$$= 3.60 \times 10^{6}\,\text{N/C}$$

$$\vec{E}(P) = 3.60 \times 10^{6}\,(\text{N/C})\left[\left(\frac{0.03}{0.05}\right)\hat{x} + \left(\frac{0.04}{0.05}\right)\hat{y}\right]$$

$$= 2.16 \times 10^{6}\,(\text{N/C})\hat{x} + 2.88 \times 10^{6}\,(\text{N/C})\hat{y}$$

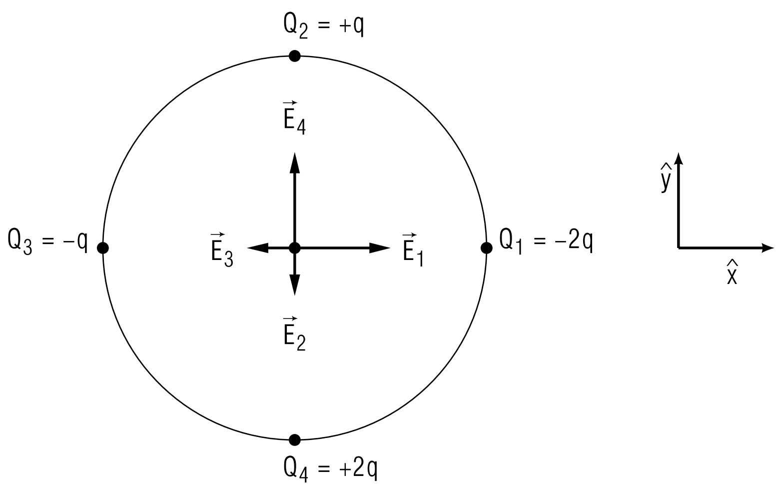
**S-3** *(from TX-4d)*

$$\vec{E} = k_e \frac{Q}{R^2}\left[\left(\frac{-x_Q}{R}\right)\hat{x} + \left(\frac{0.04}{0.05}\right)\hat{y}\right]$$

$$= (8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}})\frac{10^{-6}\,\mathrm{C}}{(0.05\,\mathrm{m})^2}\left[\left(\frac{0.03}{0.05}\right)\hat{x} + \left(\frac{0.04}{0.05}\right)\hat{y}\right]$$

$$= 2.16 \times 10^6\,(\mathrm{N/C})\hat{x} + 2.88 \times 10^6\,(\mathrm{N/C})\hat{y}$$

**S-4** *(from PS, problem 5)*




$$E_x\hat{x} = k_e\frac{-2q}{R^2}(-\hat{x}) + k_e\frac{-q}{R^2}(\hat{x}) = k_e\frac{q}{R^2}\hat{x}$$

$$E_y\hat{y} = k_e\frac{+q}{R^2}(-\hat{y}) + k_e\frac{+2q}{R^2}(\hat{y}) = k_e\frac{q}{R^2}\hat{y}$$

$$\vec{E} = (8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}})\frac{2.0 \times 10^{-7}\,\mathrm{C}}{(0.03\,\mathrm{m})^2}(\hat{x} + \hat{y})$$





**S-5** *(from PS, problem 4)*

$$k_e\frac{q_1}{x^2} = k_e\frac{q_2}{(a-x)^2}$$

$$(3q)(a-x)^2 = qx^2 \qquad \text{or:} \qquad 2x^2 - 6ax + 3a^2 = 0$$

$$x = \frac{6a \pm \left(36a^2 - 24a^2\right)^{1/2}}{4} = \frac{3a \pm \left(3a^2\right)^{1/2}}{2} = \frac{(3)(5\,\mathrm{cm}) \pm (5\,\mathrm{cm})\sqrt{3}}{2}$$

These two answers are 3.2 cm and 11.8 cm, but since the fields are in the same direction at $x = 11.8$ cm this is not a true cancellation, so only $x = 3.2$ cm is correct.

**S-6** *(from PS, problem 3)*

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = k_e\frac{Q}{R^2}\left[\frac{a\hat{x}/2}{R} + \frac{a\hat{y}}{R}\right] + k_e\frac{Q}{R^2}\left[-\frac{a\hat{x}/2}{R} + \frac{a\hat{y}}{R}\right]$$

$$= k_e\frac{Q}{(5a^2/4)^{3/2}}(a+a)\hat{y}$$

**S-7** *(from PS, problem 6)*

$$E_x = -2.0 \times 10^{-3}\,(\mathrm{N/C})$$

$$v_i = 2.0 \times 10^3\,\mathrm{m/s}$$

$$a_x = \frac{qE_x}{m} = \frac{(1.6 \times 10^{-19}\,\mathrm{C})(-2.0 \times 10^{-3}\,\mathrm{N/C})}{9.1 \times 10^{-31}\,\mathrm{kg}} = -3.5 \times 10^8\,\mathrm{m/s^2}$$

$$v_f^2 - v_i^2 = 2ax \Longrightarrow x = \frac{v_f^2 - v_i^2}{2a}$$

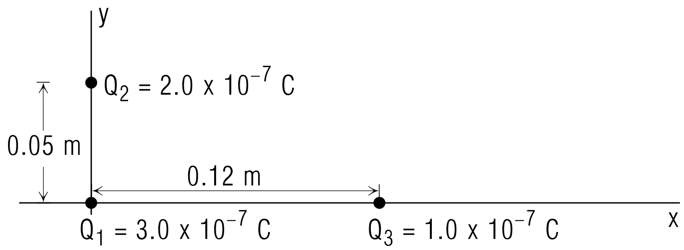
$$v_f = v_i + at \Longrightarrow t = \frac{v_f - v_i}{a}$$

# MODEL EXAM

$k_e = 8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$

1. See Output Skills K1-K3 in this module's *ID Sheet*.

2. Three charges are arranged as shown:




   a. Find the electric field at the point $x = 0.12\,\mathrm{m}$, $y = 0.050\,\mathrm{m}$ due to the three charges located as shown above.

   b. Find the force on charge $q_3$, due to $q_2$ and $q_1$.

3. Find the acceleration of an alpha particle (charge= $+2e$, mass= $M_\alpha$) in a uniform electric field $E\hat{y}$. If its initial velocity $v_0$ is perpendicular to the field, find the equations for its displacement as functions of time.

**Brief Answers**:

1. See this module's *text*.

2. See Problem 15 in this module's *Problem Supplement*.

3. See Problem 16 in this module's *Problem Supplement*.