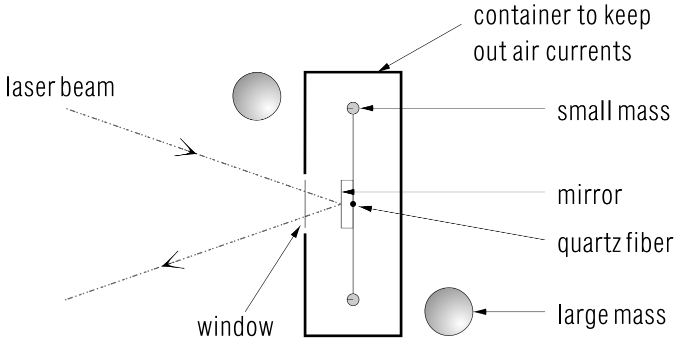
PHYSNET

MISN-0-100

THE CAVENDISH EXPERIMENT



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE CAVENDISH EXPERIMENT
by
P. Signell and V. Ross

Title: **The Cavendish Experiment**

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Input Skills:

1. Find the torque produced about a shaft by a given force and describe the twisting effect produced by that torque (MISN-0-34) or (MISN-0-416).
2. Define the equilibrium point in simple harmonic motion and explain how it is related to the spatial properties of the restoring force (MISN-0-25).

Output Skills (Knowledge):

K1. Describe and sketch the essentials of the Cavendish balance, communicating clearly how the apparatus works.

K2. Describe how the Cavendish experiment can be used to examine the validity of each of the three variables in Newton's law of gravitation.

External Resources (Optional):

1. I. Freeman, *Physics—Principles and Insights*, McGraw-Hill (1968). For availability, see this module's *Local Guide*.

Post-Options:

1. "Newton's Law of Gravitation" (MISN-0-101).
2. "Derivation of Newton's Law of Gravitation" (MISN-0-103).
3. "The Equivalence Principle: An Introduction to Relativistic Gravitation" (MISN-0-110).

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1. Introduction

Newton's law of gravitation is certainly one of the greatest laws of the universe: the one that describes what holds together our earth, holds us to the earth, and holds our earth in its orbit about our sun. Observations indicate that it holds in the incredibly distant reaches of the universe exactly as it holds here on earth. How exciting it is, then, to be able to examine this great law in the laboratory with fairly simple apparatus! We will first describe the background for Cavendish's experiment and then show how it can be used to examine the gravitation law.

2. Historical Overview

2a. Introduction. One of the greatest adventures in the history of mankind has been the determination of the causes of night and day and of the seasons, and the regularities of motion of "the wanderers,"[1] the planets. Careful observations were recorded in many cultures, including that of the American Indian. However, the first statements of the simple mathematical characteristics of planetary motion were three laws proposed by Johannes Kepler[2] who built on Tycho Brahe's careful astronomical measurements and Copernicus's proposal that the sun is at the center of the solar system.

2b. Newton's Law of Gravitation. Then, in 1666, Isaac Newton published his law of universal gravitation.[3] Kepler's three laws for the motions of the planets in our solar system were now replaced by a single law which covered, as well, the force of gravity here on the earth and the motions of planetary moons, galaxies, binary stars, and our sun's constituents. Newton's law said that the gravitational force between two spherically symmetric objects is directly proportional to each of their masses and inversely proportional to the square of the distance between their centers. That is,

$$F = G\frac{mm'}{r^2}\,, \tag{1}$$

where G is some universal gravitational constant, m and m' are the two masses[4] and r is the distance between their centers. Here F is the force with which each of the masses is attracted to the other.[5]

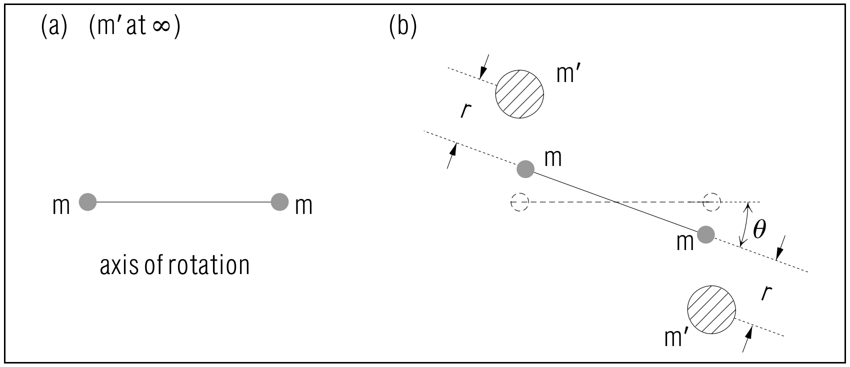

Figure 1. A schematic diagram of the Cavendish apparatus: (a) m' far away; (b) m' up close.

2c. Newton Could Not Determine G. Newton checked his law mainly through ratios of forces[6] since he could not directly measure the gravitational constant G. For example, he set the force on an object at the earth's surface equal to its weight, mg, and found:

$$GM_E = gR_E^2\,, \tag{2}$$

where R_E and M_E are the radius and mass of the earth. The value of R_E was known from the curvature of the horizon but M_E was not accessible to measurement. Thus Newton could only determine a numerical value for the product GM_E.

2d. Cavendish Makes the First Measurement of G. Over a century after Newton's formulation of the law of gravitation, Henry

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[1] So-called in ancient times because their positions moved against the background of stars.

[2] See "Derivation of Newton's Law of Gravitation" (MISN-0-103) for a discussion of Kepler's laws and a simplified version of Newton's derivation.

[3] See "Newton's Law of Gravitation" (MISN-0-101) for applications of the law to people, mountains, satellites, and planets.

[4] Mass can be defined in several ways. One method is given in (MISN-0-14), while the strange equivalence of gravitational and inertial mass is examined in (MISN-0-110).

[5] See (MISN-0-16) for a discussion of Newton's third law and the equality of the two forces.

[6] See (MISN-0-103) for a derivation of one of the ratios used by Newton to check the law of gravitation.

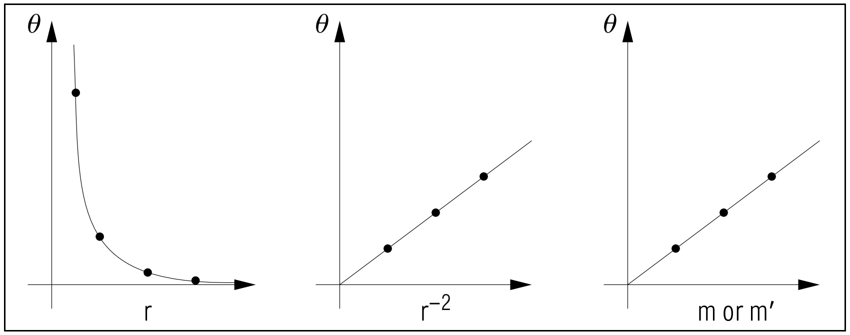

Figure 2. Hypothetical Cavendish data that would confirm Newton's law of gravitation.

Cavendish succeeded in directly measuring the gravitational force between two masses, thus enabling him to evaluate G and hence the masses of the earth,[7] moon[8] and sun.

3. The Cavendish Experiment

3a. Description of the Cavendish Apparatus. A good description of Cavendish's apparatus is given by Freeman[9] : "...A light rod with a small metal ball at each end [was] hung from a fixed point by means of a thin wire. When two massive lead spheres [were] brought close to the small balls, the gravitational forces of attraction [made] the suspended system turn slightly to a new position of equilibrium. The torque[10] with which the suspending wire [opposed] twisting [was] measured in a separate experiment..." Figure 1 shows a top view of the Cavendish apparatus at two stages during a measurement of the gravitational force between known masses. Figure 1a shows the position of the suspended light rod and small masses m when the large lead masses m' are far away ($r = \infty$). The top end of the suspending wire, the end toward the viewer, is rigidly clamped.

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[7] The major motivation of finding M_E was that of determining the earth's average density so as to obtain information about the composition of its interior.

[8] See "Derivation of Newton's Law of Gravitation" (MISN-0-103) for the equation used.

[9] Ira M. Freeman, *Physics — Principles and Insights*, McGraw-Hill (1968). For availability, see this module's *Local Guide*.

[10] For a discussion of torque see "Torque and Angular Momentum in Circular Motion," MISN-0-34.

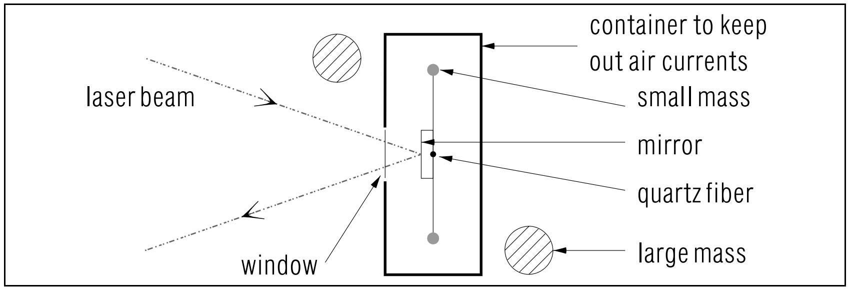

Figure 3. A top-view diagram of the modern student-lab Cavendish apparatus.[]

When the large masses m' are brought so close as to produce a significant force of gravitational attraction on the small masses, the small masses move toward the large masses, causing the rod to rotate. This rotation twists the lower end of the wire (see Fig. 1b).

3b. Gravitational Attraction Balanced by Restoring Torque. In the position shown in Fig. 1b with m' close to m, the gravitational attraction between the m-m' pairs is balanced by the restoring torque (or force) produced by the twisting wire:

$$F_{\text{gravitational}} = -F_{\text{restoring}}\,. \qquad (3)$$

The values of m, m' and r are varied, producing various values for the gravitational force and hence producing various equilibrium angles of twist θ (see Fig. 1b). At this point we could plot the measurements as m vs. θ, m' vs. θ, and r^{-2} vs. θ. However, it is force that occurs in Newton's Law, not θ.

3c. Determining the Restoring Force. Here we describe how one obtains the factor for converting the measured equilibrium angles θ to force values F. It involves measuring, in a separate experiment, the frequency of free oscillations of the apparatus.

In practice experimenters make use of the fact that the suspension receives such a small amount of twisting that the arc-like displacement of each mass from its equilibrium position is linearly proportional to the restoring force in the suspension[11] :

$$F_{\text{restoring}} = -ks = -k\ell\theta\,, \qquad (4)$$

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[11] Here the "restoring force" is the force within the metal of the suspension that resists the twisting. It grows linearly with twist angle, opposing the force causing the twisting. When the twist angle is so large that the two opposing forces are equal,

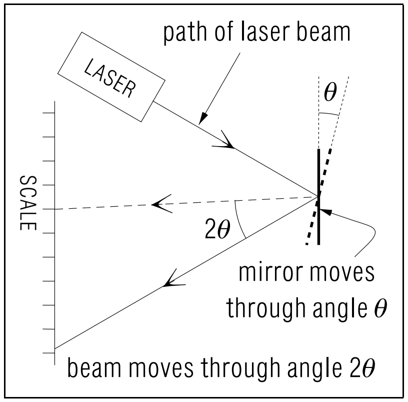

Figure 4. The angular deflection of the laser beam produced by the twisting mirror.

where ℓ is the length of either arm of the suspension and s is the displacement of the mass along an arc. Because of this linearity, when the suspended system is displaced from equilibrium and then released it will exhibit a simple harmonic twisting motion about an equilibrium angle[12] with an angular frequency of oscillation given by[13]

$$\omega^2 = k/m\,. \tag{5}$$

A simple measurement of the angular frequency of oscillation, along with measurements of m and ℓ, then gives the needed proportionality constant for converting θ values to F values:

$$F_{\text{restoring}} = -k\ell\theta = -(\omega^2 m\ell)\theta\,. \tag{6}$$

3d. Examination of Cavendish Data. Data of the type shown in Fig. 2 would support the form of Newton's law of gravitation. That is, since the force is seen to be linearly proportional to m, m', and r^{-2}, the form must be:

$$F = \text{ constant } \times mm'/r^2\,. \tag{7}$$

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the force on the suspension is zero. For more details see "Simple Harmonic Motion" (MISN-0-25) and for further discussion of restoring-force linearity with displacement see "Small Oscillation Technique" (MISN-0-28).

[12] "Simple harmonic motion" about an "equilibrium point" is an oscillating ("back and forth") motion where the position of the object is a sinusoidal function of time. The "frequency" of the oscillation is the number of complete motion cycles per unit time. See "Simple Harmonic Motion" (MISN-0-25).

[13] For a derivation of this relation see "Simple Harmonic Motion" (MISN-0-25) .

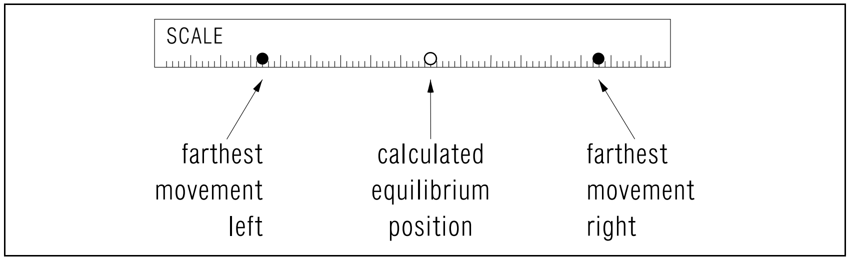

Figure 5. Finding the equilibrium position.

Then the measured force for any one combination of m, m', and r gives the proportionality constant G:

$$F = G\frac{mm'}{r^2}\,. \tag{8}$$

The current best measured value of G is:[14]

$$G = 6.6732(31) \times 10^{-11}\,\mathrm{N\,m^2/kg^2}\,. \tag{9}$$

3e. The Modern Student Cavendish Balance. In a modern student lab apparatus (see Fig. 3), the Cavendish balance is basically the same as the original one, except that the thin wire is replaced by a thin quartz fiber which produces a more consistent restoring torque. A mirror is attached to the quartz fiber and a laser beam is reflected off the mirror and onto a scale.[15] As the small masses oscillate back and forth around the axis, the fiber twists and the reflected laser beam moves back and forth along the scale.

Note that the law of reflection[16] requires the angular displacement of the beam to be twice that of the mirror (see Fig. 4). The scale is sometimes curved to make easier the conversion from the scale reading to the angle θ. By measuring the points traveled farthest to the left and right on the scale, the equilibrium point can be found (see Fig. 5). The large masses are moved near or away from the small masses by an arm which is pivoted in the center so that the distance, r, between the large and small mass will be the same on both ends of the balance beam.

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[14] *Handbook of Chemistry and Physics*, 54th Edition, Chemical Rubber Co., CRC Press (1973).

[15] A laser beam is used because it is pencil-thin.

[16] See "The Rules of Geometrical Optics" (MISN-0-220).

Acknowledgments

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LOCAL GUIDE

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